Optimal Expected Discounted Reward of a Wireless Network with Award and Cost*

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Abstract. In this paper, we extend our previous optimization investigation on a single cell (IEEE Transactions on Wireless Communications, vol. 8, no. 2, pp. 1038-1044, 2009) to a whole network with multiple cells and further consider the call admission control (CAC) based on the total expected discounted reward. Here, the system will get the award for admitting a call, but will incur a cost for rejecting a call and incur a cost for holding a call. Call's routing between cells in the network is characterized probabilistically. Under several realistic assumptions on network in general, we consider the reserved channel scheme in the literature and applied the theory of continuous time Markov decision process to derive the optimal policy. We figure out the optimal CAC policy of when to admit or reject an arrival call (either new call or handoff call) in order to achieve the maximum total expected discounted reward for the scheme. Our numerical analysis confirms the correctness of our result for this whole network investigation. The result of this paper could be applied in designing wireless networks for optimal performance, and be extended to the multiple classes of calls in the deign of future wireless mobile multimedia networking.

Keywords: Reserved Channel Scheme, Optimal Call Admission Control, Control Limit Policy.

1 Introduction

In cellular wireless networks, the calls are normally divided into two groups: new calls and handoff calls. When a user moves from one cell to another, the base station in the new cell must take responsibility for this new arriving call and all calls previously already established the connections. Since premature termination of established connections is usually more objectionable than rejection of a new connection request, it is widely believed that a wireless network must give higher priority to the handoff connection requests as compared to new connection requests. Many different admission control strategies have been discussed in

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literature [1,2,3,4] to provide priorities to handoff requests without significantly jeopardizing the new connection requests. The basic idea of these admission control strategies is establish a scheme for admitting or rejecting an arriving call, with priority to handoff calls in each cell with limited resources and is widely used because of its simplicity.

In this paper, we consider the call admission control problem in a wireless network with multiple cells, and there is routing probability consideration among all cells. What makes the decision difficult is that, to achieve some sense of optimality, one needs to consider the future status of the network resources and the pattern of the future arrival requests to accept or reject the current incoming calls. Here, he optimization problem on maximum reward is modeled as a Continuous Time Markov decision process (CTMDP), [4,5,6,7]. To achieve the optimality on the expected rewards, the CTMDP model is described as an infinite-horizon problem with discounting. Due to the finite state space and action space, by using the Rate Uniformization technique [5], the CTMDP model can be transformed to a discrete MDP model, thus the theorems and algorithms for MDP models could be applied. The Value Iteration Method is used to solve this CTMDP problem, not in the way of Linear Programming as in [8,9]. We would like to pint out that in our recent paper [10], we considered the optimization problem in a single cell. However, the current paper is a major step to extend our previous result to the **whole network**.

The rest of this paper is organized as follows. Section 2 discusses the modeling and Section 3 describes the structure of optimal policy for the RCS schemes. Numerical analysis for the schemes are discussed in Section 4 and the final Section 5 is a conclusion for this paper.

2 Model Development

A. Assumptions: The assumptions and notations for this wireless cellular network using Reserved Channel Scheme [11,12] are as follows.

1. The network consists of a number of N cells.
2. New calls are generated in cell i according to a Poisson process with rate λ_i, $i = 1, 2, \ldots, N$. The requested call connection time (RCCT) of a new call at cell i is exponentially distributed with means $1/h_i$.
3. The call residence time in cell i, which is defined as the length of time a call stays in the cell i and which depends on the velocity and the direction of the mobile terminal, is exponentially distributed with means $1/r_i$.
4. The probability that a call moves from cell i to a neighboring cell j, given that it moves to a neighboring cell before the call is completed, is $p_{i,j}$, where $\sum_{j=1}^{N} p_{i,j} = 1$. Clearly, $p_{i,i} = 0$ and cell j is a neighboring cell of i if and only if $p_{i,j} > 0$.
5. There are $C_1, \ldots, C_N$ channels in each cell of the network. The reserved channels for handoff calls are $G_1, \ldots, G_N$ in each cell of the network. The new call or handoff request are rejected if there are not enough channels.

6. Accepting a new call in cell i would contribute R_i units of reward to the system, rejecting a handoff call into cell i would cost ϕ_i units of reward to the system. Let the number vector of calls in cells be $\mathbf{n} = (n_1, n_2, \cdots, n_N)$, n_i is the numbers for calls in cell i. The system incurs a holding cost rate $f(\mathbf{n})$ per unit time.

B. Objective Function: In CTMDP models, a decision rule prescribes a procedure for action selection in each state at a specified decision epoch. Decision rules range in generality from deterministic Markovian to randomized history dependent, depending on how they incorporate past information and how they select actions. Deterministic Markovian decision rules specify the action choice when the system occupies state s at decision epoch t. A policy π specifies the decision rule to be used at every decision epoch. It provides the decision maker with a prescription for action selection under any possible future system state or history. A policy is stationary if, for each decision epoch t, the decision $d_t = d$ is the same, which can be denoted by d^∞. For each policy π, let $v_\alpha^\pi(s)$ denote the total expected infinite-horizon discounted reward with α as the discount factor, given that the process occupies state s at the first decision epoch. Our objective is to find an optimal policy π that can bring the maximum total expected discounted reward $v_\alpha^\pi(s)$ for every initial state s, i.e the objective function is,

$$v_\alpha^\pi(s) = E_s^\pi \left\{ \sum_{k=0}^{\infty} e^{-\alpha t_k} r(s_k, a_k) \right\}, \tag{1}$$

where t_k is the time point of system at epoch k ($s_0 = s$), s_k is the state of system at epoch k, a_k is the action to take at state s_k, and $r(s_k, a_k)$ represents the reward received during epoch k when taking action a_k in state s_k.

C. Construction of Models: Based on these assumptions, we can build the SMDP model for this wireless cellular network using Reserved Channel Scheme as follows:

- **State Space:** Let the state variable consists of number of calls in the system, the status of calls leaving or arriving to the system. So state space $S = \{< \mathbf{n}, b >\}$, where $b \in \{D_i, H_{ij}, A_i\}$, $n_i \leq C_i, i, j = (1, 2, \cdots, N)$, and $i \neq j$. Here b stands for the last call event, D_i means a departure from cell i, H_{ij} stands for a handoff request from cell i to cell j, A_i is an arrival of a new call in cell i.
- **Action Space:** In states $\langle \mathbf{n}, D_i \rangle$, set a_C as the action to continue, thus $A_{\langle \mathbf{n}, D_i \rangle} = \{a_C\}$. In states $\langle \mathbf{n}, A_i \rangle$ and $\langle \mathbf{n}, H_{ij} \rangle$, set a_R as the action to reject the call and a_A as the action to admit, so $A_{\langle \mathbf{n}, A_i \rangle} = \{a_R, a_A\}$ and $A_{\langle \mathbf{n}, H_{ij} \rangle} = \{a_R, a_A\}$, $i, j = (1, 2, \cdots, N)$, and $i \neq j$.
- **Decision epochs:** At each decision epoch, let $\tau(s, a)$ be the sojourn time starting from state s taking action a. Let $F(t|s, a)$ denotes the probability that the next decision epoch occurs within t time units, given that the decision maker chooses action a in state s,

$$P(\tau(s, a) \leq t) = F(t|s, a) = 1 - e^{-\beta(s,a)t}, t \geq 0.$$

For each state $s = \langle \mathbf{n}, b \rangle$ and action a, let $\beta_0(\mathbf{n}) = \sum_{i=1}^{N} \left[\lambda_i * 1_{(n_i<C_i-G_i)} + n_i(r_i+h_i)\right]$, so $\beta(s,a)$ can be written as

$$\beta(s,a) = \begin{cases} \beta_0(\mathbf{n}_i),\ b = D_i, a = a_C, \\ \beta_0(\mathbf{n}),\ b = A_i, a = a_R, \\ \beta_0(\mathbf{n}^i),\ b = A_i, a = a_A, \\ \beta_0(\mathbf{n}_i),\ b = H_{ij}, a = a_R, \\ \beta_0(\mathbf{n}_i^j),\ b = H_{ij}, a = a_A, \end{cases} \tag{2}$$

where $\mathbf{n}_i = (n_1, n_2, \ldots, \max(n_i - 1, 0), \ldots, n_N)$, $\mathbf{n}^i = (n_1, n_2, \ldots, n_i + 1, \ldots, n_N)$,

$$\mathbf{n}_i^j = \begin{cases} \mathbf{n}, i = j, \\ (n_1, n_2, \ldots, n_i - 1, \ldots, n_j + 1, \ldots, n_N), i < j, \\ (n_1, n_2, \ldots, n_j + 1, \ldots, n_i - 1, \ldots, n_N), i > j, \end{cases}$$

$i, j = (1, 2, \cdots, N)$. Here $1_{(\cdot)}$ is the indicator function.

– **Transition Probabilities:** Let $q(z|s,a)$ denote the probability that the system occupies state z in the next epoch, if at the current epoch the system is at state s and the decision maker takes action $a \in A_s$. For states $s = \langle \mathbf{n}, D_i \rangle$, $a = a_C$, the state transition probability, $q(z|\langle \mathbf{n}, D_i\rangle, a_C)$, is

$$\begin{cases} \lambda_k * 1_{((n_k - 1_{(k=i)})<C_k-G_k)}/\beta_0(\mathbf{n}_i), & z = \langle \mathbf{n}_i, A_k \rangle, \\ (n_k - 1_{(k=i)})h_k/\beta_0(\mathbf{n}_i), & z = \langle \mathbf{n}_i, D_k \rangle, \\ (n_k - 1_{(k=i)})r_k p_{kl}/\beta_0(\mathbf{n}_i), & z = \langle \mathbf{n}_i, H_{kl} \rangle, \end{cases} \tag{3}$$

where $k, l = (1, 2, \cdots, N)$ and $k \neq l$. For states $s = \langle \mathbf{n}, A_i \rangle$, admitting a new call arrival in cell i would increase the calls in the system from $\mathbf{n}$ to $\mathbf{n}^i$, rejecting a new call arrival in cell i would keep the calls in the system unchanged, so we have that $q(z|\langle \mathbf{n}, A_i\rangle, a_A)$ is

$$\begin{cases} \lambda_k * 1_{((n_k + 1_{(k=i)})<C_k-G_k)}/\beta_0(\mathbf{n}^i), & z = \langle \mathbf{n}^i, A_k \rangle, \\ (n_k + 1_{(k=i)})h_k/\beta_0(\mathbf{n}^i), & z = \langle \mathbf{n}^i, D_k \rangle, \\ (n_k + 1_{(k=i)})r_k p_{kl}/\beta_0(\mathbf{n}^i), & z = \langle \mathbf{n}^i, H_{kl} \rangle, \end{cases} \tag{4}$$

where $k, l = (1, 2, \cdots, N)$ and $k \neq l$. So, we can see that

$$q(z|\langle \mathbf{n}, A_i\rangle, a_R) = q(z|\langle \mathbf{n}^i, D_i\rangle, a_C), i = (1, 2, \cdots, N),$$
$$q(z|\langle \mathbf{n}, A_i\rangle, a_A) = q(z|\langle \mathbf{n}^i, A_i\rangle, a_R), i = (1, 2, \cdots, N).$$

For states $s = \langle \mathbf{n}, H_{ij} \rangle$, allowing a handoff request H_{ij} would change the number of calls in system from $\mathbf{n}$ to $\mathbf{n}_i^j$, rejecting such request would leave $\mathbf{n}_i$ calls in the system for all $i, j = (1, 2, \cdots, N)$, we have

$$q(z|\langle \mathbf{n}, H_{ij}\rangle, a_A) = q(z|\langle \mathbf{n}^j, D_i\rangle, a_C), i \neq j,$$
$$q(z|\langle \mathbf{n}, H_{ij}\rangle, a_R) = q(z|\langle \mathbf{n}, D_i\rangle, a_C), i \neq j.$$

– **Reward Functions:** Because the system state does not change between decision epochs, the expected discounted reward starting from state s taking action a satisfies

$$\begin{aligned} r(s,a) &= k(s,a) + c(s,a)E_s^a\left\{\int_0^{\tau(s,a)} e^{-\alpha t}dt\right\}, \\ &= k(s,a) + c(s,a)E_s^a\left\{[1-e^{-\alpha\tau(s,a)}]/\alpha\right\}, \\ &= k(s,a) + \frac{c(s,a)}{\alpha+\beta(s,a)}, \end{aligned} \tag{5}$$

where

$$k(s,a) = \begin{cases} 0, & b = D_i, a = a_C, \\ R_i, & b = A_i, a = a_A, \\ 0, & b = A_i, a = a_R, \\ 0, & b = H_{ij}, a = a_A, \\ -\phi_j, & b = H_{ij}, a = a_R, \end{cases} \tag{6}$$

and $c(s,a)$ is the holding cost rate function if the system is at state s and takes action a. It can be defined as

$$c(s,a) = \begin{cases} -f(\mathbf{n}_i), & b = D_i, a = a_C, \\ -f(\mathbf{n}^i), & b = A_i, a = a_A, \\ -f(\mathbf{n}), & b = A_i, a = a_R, \\ -f(\mathbf{n}_i^j), & b = H_{ij}, a = a_A, \\ -f(\mathbf{n}_i), & b = H_{ij}, a = a_R. \end{cases} \tag{7}$$

Thus, from equation (1) and (5) the objective function $v_\alpha^\pi(s)$ can be written as

$$E_s^\pi\left\{\sum_{k=0}^{\infty} e^{-\alpha t_k}\left[k(s_k,a_k) + \frac{c(s_k,a_k)}{\alpha+\beta(s_k,a_k)}\right]\right\}. \tag{8}$$

3 Optimal Policy

Based on the assumptions, for the admission control problem, both the state space S and the action space A_s are finite, the reward function $r(s,a)$ is also finite. From *Theorem 11.3.2* of [5], the optimal policy is a stationary deterministic policy d^∞, so the problem can be reduced to finding a deterministic decision rule d. For each deterministic decision rule d, let $q_d(z|s) = q(z|s,d(s))$, $r_d(s) = r(s,d(s))$ and $\beta_d(s) = \beta(s,d(s))$, from equation (8) we have,

$$\begin{aligned} v_\alpha^{d^\infty}(s) &= r_d(s) + E_s^\pi\{e^{-\alpha\tau(s,d(s))}v_\alpha^{d^\infty}(s_1)\}, \\ &= r_d(s) + \sum_{z\in S}\left[\int_0^\infty \beta_d(s)e^{-[\alpha+\beta_d(s)]t}dt\right] q_d(z|s)v_\alpha^{d^\infty}(z), \\ &= r_d(s) + \frac{\beta_d(s)}{\alpha+\beta_d(s)}\sum_{z\in S} q_d(z|s)v_\alpha^{d^\infty}(z). \end{aligned} \tag{9}$$

We use rate uniformization technique to calculate $v_\alpha^{d^\infty}(s)$. Based on the assumptions, our process fits the condition of *Assumption 11.5.1* of [5], which is $[1 - q(s|s,a)]\beta(s,a) \le c, \forall s \in S, a \in A_s$, here $c = \sum_{i=1}^{N}[\lambda_i + C_i * (r_i + h_i)]$ is a constant. So, we can define a uniformization of our process with components denoted by $\sim$. Let $\tilde{S} = S$ and $\tilde{A}_s = A_s$, we have

$$\tilde{q}(z|s,a) = \begin{cases} 1 - \frac{[1-q(s|s,a)]\beta(s,a)}{c}, & z = s, \\ \frac{q(z|s,a)\beta(s,a)}{c}, & z \neq s. \end{cases} \tag{10}$$

For the reward functions, we have

$$\tilde{r}(s,a) \equiv r(s,a)\frac{\alpha + \beta(s,a)}{\alpha + c}. \tag{11}$$

From *Proposition 11.5.1* [5], for each d^∞ policy and $s \in S$, we have

$$\tilde{v}_\alpha^{d^\infty}(s) = v_\alpha^{d^\infty}(s). \tag{12}$$

From equations (9) and (12), the optimality equation of $v(s)$ for maximum $v_\alpha^\pi(s)$ would have the form of

$$v(s) = \tilde{v}(s) = \max_{a \in A_s}\{\tilde{v}(s,a)\} = \max_{a \in A_s}\left\{\tilde{r}(s,a) + \frac{c}{c+\alpha}\sum_{z \in S}\tilde{q}(z|s,a)v(z)\right\} \tag{13}$$

After uniformization, the transition process from one state to another can be described by a discrete-time Markov chain which allows fictitious transitions from a state to itself.

From equations (5), (6), (7), (13) and (14), we have for states with $b = D_i$, there is only one action a_C, the reward is

$$\tilde{r}(\langle \mathbf{n}, D_i\rangle, a_C) = \frac{-f(\mathbf{n}_i)}{\alpha + c}. \tag{14}$$

From equations (3), (10), the transition probability $\tilde{q}(z|\langle \mathbf{n}, D_i\rangle, a_C)$ is

$$\begin{cases} \lambda_k * 1_{((n_k - 1_{(k=i)}) < C_k - G_k)}/c, & z = \langle \mathbf{n}_i, A_k\rangle, \\ (n_k - 1_{(k=i)})h_k/c, & z = \langle \mathbf{n}_i, D_k\rangle, \\ (n_k - 1_{(k=i)})r_k p_{kj}/c, & z = \langle \mathbf{n}_i, H_{kj}\rangle, \\ (c - \beta_0(\mathbf{n}_i))/c, & z = \langle \mathbf{n}, D_i\rangle, \end{cases} \tag{15}$$

for $k, j = (1, 2, \cdots, N)$ and $i \neq j$. From equations (13), (14) and (15), we have

$$\begin{aligned} v(\langle \mathbf{n}, D_i\rangle) = \frac{1}{\alpha + c}[&-f(\mathbf{n}_i) + \sum_{k=1}^{N}\lambda_k v(\langle \mathbf{n}_i, A_k\rangle) * 1_{((n_k - 1_{(k=i)}) < C_k - G_k)} \\ &+ \sum_{k=1}^{N}(n_k - 1_{(k=i)})h_k v(\langle \mathbf{n}_i, D_k\rangle) \\ &+ \sum_{j=1}^{N}\sum_{k=1}^{N}(n_k - 1_{(k=i)})r_k p_{kj} v(\langle \mathbf{n}_i, H_{kj}\rangle) \\ &+ (c - \beta_0(\mathbf{n}_i))v(\langle \mathbf{n}, D_i\rangle)]. \end{aligned} \tag{16}$$

In the same way, for states $s = \langle \mathbf{n}, A_i \rangle$, from equations (3), (4) and (13), we have

$$\begin{aligned}\tilde{v}(\langle \mathbf{n}, A_i \rangle, a_R) = & \frac{1}{\alpha + c}[-f(\mathbf{n}) + \sum_{k=1}^{N} \lambda_k v(\langle \mathbf{n}, A_k \rangle) * 1_{(n_k < C_k - G_k)} \\ & + \sum_{k=1}^{N} n_k h_k v(\langle \mathbf{n}, D_k \rangle) + \sum_{j=1}^{N} \sum_{k=1}^{N} n_k r_k p_{kj} v(\langle \mathbf{n}, H_{kj} \rangle) \\ & + (c - \beta_0(\mathbf{n})) v(\langle \mathbf{n}, A_i \rangle)], \\ = & \frac{1}{\alpha + c}[(\alpha + \beta_0(\mathbf{n})) v(\langle \mathbf{n}^i, D_i \rangle) + (c - \beta_0(\mathbf{n})) v(\langle \mathbf{n}, A_i \rangle)], \end{aligned} \tag{17}$$

and,

$$\begin{aligned}\tilde{v}(\langle \mathbf{n}, A_i \rangle, a_A) = & \frac{1}{\alpha + c}[(\alpha + \beta_0(\mathbf{n}^i)) R_i - f(\mathbf{n}^i) \\ & + \sum_{k=1}^{N} \lambda_k v(\langle \mathbf{n}^i, A_k \rangle) * 1_{((n_k + 1_{(k=i)}) < C_k - G_k)} \\ & + \sum_{k=1}^{N} (n_k + 1_{(k=i)}) h_k v(\langle \mathbf{n}^i, D_k \rangle) \\ & + \sum_{j=1}^{N} \sum_{k=1}^{N} (n_k + 1_{(k=i)}) r_k p_{kj} v(\langle \mathbf{n}^i, H_{kj} \rangle) \\ & + (c - \beta_0(\mathbf{n}^i)) v(\langle \mathbf{n}, A_i \rangle)], \\ = & \frac{1}{\alpha + c}[(\alpha + \beta_0(\mathbf{n}^i))(R_i + v(\langle \mathbf{n}^{ii}, D_i \rangle)) \\ & + (c - \beta_0(\mathbf{n}^i)) v(\langle \mathbf{n}, A_i \rangle)]. \end{aligned} \tag{18}$$

Also, we have

$$v(\langle \mathbf{n}, A_i \rangle) = \begin{cases} \max\left[\tilde{v}(\langle \mathbf{n}, A_i \rangle, a_R), \tilde{v}(\langle \mathbf{n}, A_i \rangle, a_A)\right], & n_i < C_i - G_i, \\ \tilde{v}(\langle \mathbf{n}, A_i \rangle, a_R), & n_i \geq C_i - G_i. \end{cases}$$

From equation (16), it is seen that $v(\langle \mathbf{n}^i, D_i \rangle) = v(\langle \mathbf{n}^j, D_j \rangle)$, for $i, j \in (1, 2, \cdots, N)$. So the value of $v(\langle \mathbf{n}^i, D_i \rangle)$ is not dependent on i, but is dependent on $\mathbf{n}$. Let $g(\mathbf{n}) = v(\langle \mathbf{n}^i, D_i \rangle)$, from equations (17) and (18), the calculation of $v(\langle \mathbf{n}, A_i \rangle)$ can be simplified to

$$v(\langle \mathbf{n}, A_i \rangle) = \begin{cases} \max\left[g(\mathbf{n}), R_i + g(\mathbf{n}^i)\right], & n_i < C_i - G_i, \\ g(\mathbf{n}), & n_i \geq C_i - G_i. \end{cases} \tag{19}$$

Similarly for events $b = H_{ij}$, we have

$$v(\langle \mathbf{n}, H_{ij} \rangle) = \max\left[v(\langle \mathbf{n}, D_i \rangle) - \phi_j, v(\langle \mathbf{n}^j, D_i \rangle)\right]. \tag{20}$$

Let $\beta_1(\mathbf{n}) = \sum_{i=1}^{N} (\lambda_i + n_i(r_i + h_i))$. If $(n_k - 1_{(k=i)}) \geq C_k - G_k$, which means the arrival of new calls in cell k can only be rejected, we have $v(\langle \mathbf{n}_i, A_k \rangle) = v(\langle \mathbf{n}, D_i \rangle)$. Equation (16) can be transformed to

$$v(\langle \mathbf{n}, D_i\rangle) = \frac{1}{\alpha + c}[-f(\mathbf{n}_i) + \sum_{k=1}^{N} \lambda_k v(\langle \mathbf{n}_i, A_k\rangle)$$
$$+\sum_{k=1}^{N}(n_k - 1_{(k=i)})h_k v(\langle \mathbf{n}_i, D_k\rangle)$$
$$+\sum_{j=1}^{N}\sum_{k=1}^{N}(n_k - 1_{(k=i)})r_k p_{kj} v(\langle \mathbf{n}_i, H_{kj}\rangle)$$
$$+(c - \beta_1(\mathbf{n}_i))v(\langle \mathbf{n}, D_i\rangle)]. \quad (21)$$

Definition: A function $f : R^k \to R$ is supermodular if

$$f(x \vee y) + f(x \wedge y) \geq f(x) + f(y),$$

for all x, y $\in R^k$, where $x \vee y$ denotes the componentwise maximum and $x \wedge y$ the componentwise minimum of x and y. If $-f$ is supermodular, then f is called submodular.

Theorem: If $v(\langle \mathbf{n}, D_i\rangle)$ is a submodular function on $\mathbf{n}$, then the optimal admission control policy with RCS scheme for new call arrivals and handoff calls are control limit policies.

Proof: We already know that the optimal policy is a stationary deterministic policy. Let $\triangle v_i(\langle \mathbf{n}, D_i\rangle) = v(\langle \mathbf{n}^{ii}, D_i\rangle) - v(\langle \mathbf{n}^i, D_i\rangle)$ and $\triangle v_i^j(\langle \mathbf{n}, D_i\rangle) = v(\langle \mathbf{n}^j, D_i\rangle) - v(\langle \mathbf{n}, D_i\rangle)$. So $\triangle v_i(\langle \mathbf{n}, D_i\rangle)$ and $\triangle v_i^j(\langle \mathbf{n}, D_i\rangle)$ are nonincreasing, we have the decision rule for the new call arrivals as

$$d(\langle \mathbf{n}, A_i\rangle) = \begin{cases} a_A, \triangle v_i(\langle \mathbf{n}, D_i\rangle) > -R_i, \\ a_R, \triangle v_i(\langle \mathbf{n}, D_i\rangle) \leq -R_i. \end{cases}$$

For the handoff requests,

$$d(\langle \mathbf{n}, H_{ij}\rangle) = \begin{cases} a_A, \triangle v_i^j(\langle \mathbf{n}, D_i\rangle) > -\phi_j, \\ a_R, \triangle v_i^j(\langle \mathbf{n}, D_i\rangle) \leq -\phi_j. \end{cases}$$

So, if $d(\langle \mathbf{n}, A_i\rangle) = a_R$, since

$$v(\langle \mathbf{n}^i, D_i\rangle) + v(\langle \mathbf{n}^{iix}, D_i\rangle) \leq v(\langle \mathbf{n}^{ix}, D_i\rangle) + v(\langle \mathbf{n}^{ii}, D_i\rangle),$$

which means $\triangle v_i(\langle \mathbf{n}^x, D_i\rangle) \leq \triangle v_i(\langle \mathbf{n}, D_i\rangle))$, so we have $d(\langle \mathbf{n}^x, A_i\rangle) = a_R$. Similarly if $d(\langle \mathbf{n}, H_{ij}\rangle) = a_R$, we have $d(\langle \mathbf{n}^x, H_{ij}\rangle) = a_R$, and so on as x, i, j goes through $1, 2, \ldots, N$. Consequently the optimal policy for both new calls and handoff requests is a control limit policy (or threshold policy).

Remark: We observe that the function $v(\langle \mathbf{n}, D_i\rangle)$ will not be a submodular function on $\mathbf{n}$ if the cost function is not a supermodular function on $\mathbf{n}$. Please see function $f_4(\mathbf{n})$ in the section of numerical analysis. Therefore, in order to make the optimal admission control policy with RCS scheme for either new call arrivals or handoff calls to be a control limit policy, we must carefully select the cost function from the class of supermodular functions.

4 Numerical Analysis

Without loss of generality, we suppose there are $N = 3$ cells in the wireless network. The routing probabilities among cells are set in Table 1. For simplification, let the discount factor $\alpha = 0.1$, and set the other parameters for analysis as in Table 2 to study the performance of the RCS scheme. We get the $v(s)$ values for states in the following Tables.

Table 1. Routing Probablities

Cell	1	2	3
1	0	0.4	0.6
2	0.5	0	0.5
3	0.8	0.2	0

Table 2. Parameters Setting

Cell	C_i	λ_i	h_i	r_i	R_i	ϕ_i	G_i
1	5	4	6	4	2	1	2
2	5	2	4	5	1.6	1.1	1
3	5	1	3	2	1.5	1.2	0

In Table 2, λ is the arrival rate of new calls, h is the call connection rate, r is the cell residence rate, R is the reward for new calls, ϕ is the cost for rejecting a handoff call, and G is the number of reserved channels.

Next, we show the performance of the RCS scheme with different cost functions. First, let us set the cost function be $f_1(\mathbf{n}) = 4n_1^2 + 3n_2^2 + 2n_3^2$, which is a supermodular function. The corresponding $v(\langle \mathbf{n}, D_3 \rangle)$ are shown in Table 3. It can be seen that $\triangle v_1(\langle (3, n_2, 1), D_3 \rangle)$, $n_2 = 0, \ldots, 5$ are less than $-R_1$ (R_1=2), which means that the system would reject cell-1 new call arrivals if there are already 3 calls in cell-1.

Table 3. RCS scheme: Function 1

$v(\langle \mathbf{n}, D_3 \rangle)$	$n_2 = 0$	1	2	3	4	5
$n_1 = 0$	34.7039	33.8302	32.6263	31.1079	29.2705	27.1087
1	33.5482	32.6423	31.4261	29.9008	28.0585	25.8929
2	31.9847	31.0542	29.8283	28.2975	26.4513	24.2828
3	30.0533	29.1045	27.8714	26.3365	24.4875	22.3168
4	27.7431	26.7785	25.5391	24.0007	22.1492	19.9765
5	25.0446	24.0665	22.8216	21.28	19.4263	17.252

Set the cost function to $f_2(\mathbf{n}) = 2(n_1+n_2+n_3)^2$, which is also a supermodular function. The corresponding $v(\langle \mathbf{n}, D_3 \rangle)$ are shown in Table 4. It can be seen that $\triangle v_1(\langle (4, 2, 1), D_3 \rangle)$ and $\triangle v_1(\langle (3, 3, 1), D_3 \rangle)$ are less than $-R_1$, so if there are 2

Table 4. RCS scheme: Function 2

$v(\langle \mathbf{n}, D_3\rangle)$	$n_2 = 0$	1	2	3	4	5
$n_1 = 0$	39.1698	38.3064	37.23	35.9378	34.4342	32.7217
1	38.1851	37.099	35.8102	34.3136	32.6092	30.7056
2	36.9665	35.6818	34.1923	32.4959	30.6012	28.5016
3	35.5502	34.0692	32.382	30.496	28.4058	26.107
4	33.9247	32.262	30.3884	28.3084	26.0198	23.5199
5	32.1286	30.2752	28.2083	25.9307	23.4414	20.7389

calls at cell-2, the system stops accepting cell-1 arrivals when there are 4 calls in cell-1, but if there are 3 calls at cell-2, the system stops accepting cell-1 arrivals when there are 3 calls in cell-1.

Set the cost function to $f_3(\mathbf{n}) = 2(n_1 + 1) * (n_2 + 1) * (n_3 + 1)$, which is a supermodular function. The corresponding $v(\langle \mathbf{n}, D_3\rangle)$ are shown in Table 5. It can be seen that all the $\triangle v_1(\langle \mathbf{n}, D_3\rangle)$ are greater than $-R_1$, so the system would always accept new call arrivals in cell-1 if there are free space.

Table 5. RCS scheme: Function 3

$v(\langle \mathbf{n}, D_3\rangle)$	$n_2 = 0$	1	2	3	4	5
$n_1 = 0$	29.6495	28.9132	28.1433	27.3726	26.564	25.7666
1	28.812	27.9492	27.0791	26.2057	25.2985	24.3877
2	27.9251	26.964	25.9916	25.0148	24.0073	22.9906
3	27.0115	25.9447	24.8719	23.7918	22.6844	21.5697
4	25.9856	24.8219	23.6537	22.4783	21.2794	20.0791
5	24.9774	23.7152	22.4498	21.1788	19.8864	18.5936

Finally, set the cost function to $f_4(\mathbf{n}) = 4n_1^2 + 4n_2^2 + 4n_3^2 - 8n_1n_2$. Let $x = (1, 0, 0)$ and $y = (0, 1, 0)$, so we have $f(x \vee y) = f(x \wedge y) = 0$, and $f(x) = f(y) = 4$. Based on the definition of supermodular function, $f_4(\mathbf{n})$ is not a supermodular function. The values of $v(\langle \mathbf{n}, D_3\rangle)$ are shown in Table 6. If $\triangle v_2(\langle \mathbf{n}, D_3\rangle)$ are less than $-R_2$ ($R_2 = 1.6$), the corresponding action for new call arrivals in cell-2 is to reject.

Table 6. RCS scheme: Function 4

$v(\langle \mathbf{n}, D_3\rangle)$	$n_2 = 0$	1	2	3	4	5
$n_1 = 0$	36.0884	35.2532	34.0446	32.4941	30.5935	28.3406
1	35.02	34.5633	33.715	32.5176	30.977	29.0896
2	33.6422	33.5472	33.0549	32.1976	31.0001	29.4636
3	31.9552	32.2213	32.079	31.5552	30.6671	29.4635
4	29.9821	30.5827	30.7694	30.5569	29.9566	29.0432
5	27.7088	28.6533	29.1793	29.2955	28.9977	28.3771

The actions for cell-2 new call arrivals of cost function 4 are shown in Table 7. In Table 7 '1' stands for the action to accept and '0' is for action reject. It is

Table 7. RCS scheme: for states $n_3 = 0$

$a(\langle \mathbf{n}, A_2 \rangle)$	$n_2 = 0$	1	2	3	4	5
$n_1 = 0$	1	1	1	0	0	0
1	1	1	1	1	0	0
2	1	1	1	1	0	0
3	1	1	1	1	0	0
4	1	1	1	1	0	0
5	1	1	1	1	0	0

seen that when there no calls in cell-1, the systems starts to reject the new call arrivals of cell-2 when there are 3 calls in cell-2, but it starts to reject the new call arrivals of cell-2 when there are 4 calls in cell-2 if there are more than 1 calls in cell-1, so the optimal policy is not a control limit policy.

Based on the four cost functions studied, for those states with no calls in cell-3, if the cost functions are supermodular functions, from Table 3 to Table 5 we can see that the values of $\triangle v_1(\langle \mathbf{n}, D_3 \rangle)$ and $\triangle v_2(\langle \mathbf{n}, D_3 \rangle)$ are nonincreasing in both $n_1 \downarrow$ and $n_2 \rightarrow$ directions, which fits our conclusion of the conjecture. Similar results can also be achieved for new calls and handoff calls to all other states. But if the cost function is not a supermodular function, as shown in Table 6, the resulting $v_3(\langle \mathbf{n}, D_3 \rangle)$ may not be a submodular function, consequently the optimail policy is not a control limit policy.

5 Conclusion

In this paper, we consider a wireless network with multiple cells, with routing probabilities between each cells. Assume that the arrival process of new calls in each cell is a Poisson process, the call connection time and cell residence time follow exponential distributions, accepting each new call would contribute some units of reward to the system, rejecting a handoff call would bring some cost to the system, and the system incurs a holding cost per unit time for the calls in the system. The result of this paper could easily be used in designing wireless networks for optimal performance, and be extended to the multiple classes of calls in the deign of future wireless mobile multimedia networking.

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