

Spectral Efficiency Using Combinations of Transmit Antenna Selection with Linear Dispersion Code Selection

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Abstract. In this paper the objective is to enhance the spectral efficiency using combinations of transmit antenna selection with Linear Dispersion Code Selection. Both bit error rate minimization and throughput maximization criteria are here examined¹. The performance of the proposed spatial link adaptation scheme is evaluated under low mobility environments concluding that in case of linear receivers the transmit antenna code selection scheme with the combination of adaptive modulation and coding achieves a noticeable SNR gain (up to 3dB) in a large SNR margin (SNR from 6 to 18dB), which could be considered as a potent technique to achieve a smooth transition between diversity and multiplexing in order to maximize the overall system throughput.

Keywords: WiMAX, Spatial time coding, Transmit Antenna Selection, Linear Dispersion Codes, spectral efficiency.

1 Introduction

The use of multiple antennas at the transmitter and at the receiver has demonstrated the benefits of increasing the channel capacity and diversity [1]. Multiple Input Multiple Output (MIMO) systems may exploit the channel diversity and capacity by using different space-time-(frequency) coding techniques. When the channel state information is available at the transmitter (CSIT), the best space time coding technique is the beamforming where the information is transmitted in the strongest eigenmode(s) of the channel and the power is allocated to each eigenmode following the water-filling principle. However, CSIT techniques require that the transmitter estimates the full channel matrix which in some scenarios might become unfeasible.

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On the other hand, if CSI is not available at the transmitter, the well known Space-Time Block Coding (STBC) schemes are preferred. Hassibi and Hochwald proposed in [2] a framework where any kind of linear STBC could be analyzed, classifying such class of space-time codes as Linear Dispersion Codes (LDC). The main advantage from the LDCs is that different tradeoffs between diversity and spatial multiplexing (SM) can be achieved thanks to proper design of the code [2] [3]. As a result, different authors have proposed to use LDCs, since specific LDC codes structures are able to optimize particular channel metrics or enhance certain parameters (i.e. channel capacity, outage probability, bit error rate, etc.) [4][5].

Nevertheless, many research efforts have been done in order to exploit the best from each approach under the concept of Partial CSIT (PCSIT). In this case, the transmitter is provided only with a small amount of information about the channel state (e.g. Frobenius channel norm, channel rank, channel [6] [7] condition number, etc.), thus the transmitter sets up the transmitted signal to the current channel (this is referred as precoding). The simplest scheme of precoding is the Transmit Antenna Selection (TAS) where the best (set of) antenna(s) are selected for transmission. Actually, it has been demonstrated that the TAS scheme gives the same diversity order than without antenna selection at the expense of reducing the coding gain [8]. This effect has been analyzed in [8]-[10] for Spatial Multiplexing, Orthogonal STBCs and LDCs respectively. Yet, another well-known precoding technique which fixes the set of precoding matrices from a limited codebook (known a priori from both transmitter and receiver) has also shown to provide large capacity and link reliability improvement by using a low rate channel feedback providing uniquely the codeword index [11] [12].

As a result, the latest researches have combined both types of precoding (TAS and codebook-based) schemes for further enhance of the system performance. This paper extends the works in [13] [14] by applying both the TAS and codebook-based precoding schemes from a pure LDC perspective. Then, a spatial adaptation scheme for the downlink/uplink is developed where the proper Transmit antenna subset and LDC (from a set of predefined LDCs) are selected for each frame. Two optimization criteria (i.e. minimizing the bit error rate and maximizing the throughput) are evaluated showing that both the diversity order and the system throughput are maximized. In this paper the proposed scheme is referred as the Transmit Antenna and Code Selection (TACS).

The rest of the paper is organized as follows. In Section 2, the system model considered and the LDC code structure is introduced. The proposed TACS space-time adaptations criteria are detailed in Section 3 and the corresponding simulation results are analyzed in Section 4. Finally, conclusions are stated in Section 5, where the main Performance behaviors of the proposed approaches are exposed.

2 System Model

The MIMO system model with M and N transmitter and receiver active antennas respectively is defined by

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}} \mathbf{H} \mathbf{S} + \mathbf{N}, \quad (1)$$

where $\mathbf{S} \in \mathbb{C}^{M \times T}$ and $\mathbf{Y} \in \mathbb{C}^{N \times T}$ are the transmitted and the received signals from each antenna during each channel access, and the channel matrix $\mathbf{H} \in \mathbb{C}^{N \times M}$ is assumed

constant during T periods (i.e. block fading channel model). The transmitted signal has unitary power, and the noise matrix \mathbf{N} follows a circular complex Gaussian distribution with zero mean and unitary standard deviation. The Linear Dispersion Code (LDC) structure subsumes most of the previous Space-Time (ST) codes such as the Bell-Labs Layered Architecture Space Time coding (BLAST), the Alamouti scheme, etc. [2]. Then, considering the LDC framework, the transmitted signal matrix \mathbf{X} has necessarily the following structure

$$\mathbf{S} = \sum_{q=1}^Q (\alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q), \quad (2)$$

where $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{M \times T}$ are the basis matrices, $E\{\text{tr}(\mathbf{S}^H \mathbf{S})\} = MT$, and the values $s_q = \alpha_q + j\beta_q$ are the complex data symbols we want to transmit with $E\{s_q^* s_q\} = 1$. The number of basis matrices is Q , and the spatial multiplexing rate is Q/MT . The rate R achieved by the system is given by $R = Qn/T$ [bits/s/Hz], where n means the number of bits transmitted per each complex symbol.

Then substituting (2) into (1) and applying the vec operator on both sides of the expression, the (real valued) system equation can be rewritten as

$$\underbrace{\begin{bmatrix} \Re(\mathbf{y}_0) \\ \Im(\mathbf{y}_0) \\ \vdots \\ \Re(\mathbf{y}_{Q-1}) \\ \Im(\mathbf{y}_{Q-1}) \end{bmatrix}}_{\underline{\mathbf{y}}} = \sqrt{\frac{\rho}{M}} \mathcal{H} \underbrace{\begin{bmatrix} \alpha_0 \\ \beta_0 \\ \vdots \\ \alpha_{Q-1} \\ \beta_{Q-1} \end{bmatrix}}_{\underline{\mathbf{s}}} + \underbrace{\begin{bmatrix} \mathbf{n}_0 \\ \mathbf{n}_0 \\ \vdots \\ \mathbf{n}_{Q-1} \\ \mathbf{n}_{Q-1} \end{bmatrix}}_{\underline{\mathbf{n}}} \quad (3)$$

where $\underline{\mathbf{s}}$ is the real input symbols vector and $\underline{\mathbf{n}}$ is the real vector noise i.i.d. components $\mathcal{N}(0, 1/2)$ -distributed. The equivalent real valued channel matrix $\mathcal{H} \in \mathbb{R}^{2NT \times 2Q}$ is then given by

$$\mathcal{H} = \underbrace{\begin{bmatrix} \mathbf{I}_N \otimes \mathcal{A}_0 & \mathbf{I}_N \otimes \mathcal{B}_0 & \cdots & \mathbf{I}_N \otimes \mathcal{A}_{Q-1} & \mathbf{I}_N \otimes \mathcal{B}_{Q-1} \end{bmatrix}}_{2NT \times 4MNQ} \times \underbrace{\begin{bmatrix} \mathbf{I}_{2Q} \otimes \underline{\mathbf{h}} \end{bmatrix}}_{4MNQ \times 2Q}. \quad (4)$$

$$\text{With} \quad \mathcal{A}_q = \begin{bmatrix} \Re\{\mathbf{A}_q\} & -\Im\{\mathbf{A}_q\} \\ \Im\{\mathbf{A}_q\} & \Re\{\mathbf{A}_q\} \end{bmatrix}_{2T \times 2M}, \quad (5)$$

$$\mathcal{B}_q = \begin{bmatrix} -\Im\{\mathbf{B}_q\} & -\Re\{\mathbf{B}_q\} \\ \Re\{\mathbf{B}_q\} & -\Im\{\mathbf{B}_q\} \end{bmatrix}_{2T \times 2M},$$

$$\underline{\mathbf{h}} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{N-1} \end{bmatrix}_{2MN}, \quad \hat{h}_n = \begin{bmatrix} \Re\{\mathbf{h}_n\} \\ \Im\{\mathbf{h}_n\} \end{bmatrix}_{2M},$$

where \mathbf{h}_n is the n -th row of the MIMO channel matrix \mathbf{H} .

Typically, the Maximum Likelihood (ML) detection is assumed during the LDCs design. However, it is well-known that the complexity requirements derived from such decoding techniques is extremely high ($\mathcal{O}(2^{Qn})$), making ML unaffordable for high data rates R in real implementations. Furthermore, due to the linear relationship between input and output samples observed in (3), a linear detector is enough to recover the symbols. However, the performance of such linear decoder is far from that offered by the ML. Nevertheless, one important benefit from using a linear decoder is that an equivalent channel can be estimated for each symbol. Hence Adaptive Modulation and Coding (AMC) can be applied on a per symbol basis. In this paper, a linear decoder using a Minimum Mean Square Error (MMSE) equalizer is only considered for evaluation of the TACS scheme.

Then, using a linear MMSE receiver, the Effective Signal to Interference and Noise Ratio (ESINR) per each symbol q is given by

$$ESINR_q^{(MMSE)}(\mathbf{H}) = \frac{\rho}{M \left[\mathbf{H}^H \mathbf{H} + 2\rho^{-1} \mathbf{I}_{2Q} \right]_{q,q}^{-1}} - 1, \quad (6)$$

where $\mathbf{X}_{q,q}^{-1}$ refers to the (q,q) element from \mathbf{X}^{-1} , and ρ is the average Signal to Noise Ratio (SNR). Furthermore, if the mapping applied to the symbols follows a 2^n -QAM constellation, the average pair wise error probability per stream applying the Nearest Neighbor Union Bound can be obtained as follows

$$P_{e,q} \leq 1 - \left(1 - N_e(n) \cdot \mathbb{E} \left\{ Q \left(\sqrt{ESINR_q(\mathbf{H}) \frac{d_{\min}^2(n)}{2}} \right) \right\} \right) \quad (7)$$

where $Q(x) = 0.5 \times \text{erfc}(x/2^{1/2})$, d_{\min}^2 is the squared minimum distance between any two points of the constellation (assuming an unitary average transmission power), and N_e is the average number of nearest neighbors constellation points. For a 2^n -QAM modulation $d_{\min}^2 = 6/(2^n - 1)$ and $N_e = 4 \times (1 - 2^{-n/2})$. In addition, in case all the symbols within the codeword apply the same modulation, the average pairwise error probability for the whole codeword is usually approximated by (assuming $P_e < 10^{-2}$)

$$P_e \leq Q \cdot N_e(n) \cdot \mathbb{E} \left\{ Q \left(\sqrt{ESINR_{\min}(\mathbf{H}) \frac{d_{\min}^2(n)}{2}} \right) \right\}, \quad (8)$$

where $ESINR_{\min} = \min(ESINR_{0}, \dots, ESINR_{Q-1})$ [13].

3 The TACS Space-Time Adaptation Criteria

Then, given the ESNR per stream in (6) and the average pair wise error probabilities in (7) and (8), two different optimization scenarios are studied where both the transmit antenna subset as well as the best LDC from a set of codes are selected (see Fig 1).

In the first scenario, we consider that the same modulation is applied to all the symbols and that the rate R is fixed. In that case, and since transmission power is fixed, we are interested in selecting the transmit antenna subset and LDC code that minimizes the error rate probability (i.e. the bit error rate – BER) while the modulation that is required by each LDC is adapted in order to achieve the cited rate R . In that case, since the Q -function is monotonically decreasing as a function of the input, the optimization problem can be defined as follows

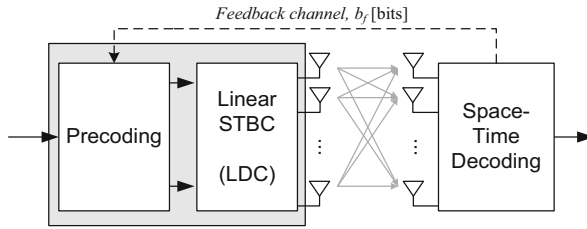


Fig. 1. TACS spatial adaptation scheme and its integration using adaptive LDC code selection

$$\max_{LDC_i, p_i} \min_q \left\{ ESINR_q(H, LDC_i, p_i) d_{\min}^2(n_i) \right\}, \quad (9)$$

where i means the LDC index and p_i the transmitting antenna subset (set of antennas that can be used according to the number of transmitter antennas M_a and the number of antennas required by the LDC). It also noted that the constellation is a function of the LDC.

In the second scenario, the optimization is performed in order to maximize the system throughput considering a certain quality of service requirement (i.e. a maximum Block Error Rate - BLER). In that case, the problem is formulated as follows

$$\max_{LDC_i, p_i, MCS_j} \min_q R(1 - BLER(ESINR_q)) \quad \text{s.t.: } BLER \leq \mu \quad (10)$$

where j means the Modulation and Coding Scheme (MCS) index that maximizes the spectral efficiency for the specific channel state subject to a maximum Block Error Rate (BLER). Actually, the selection of the optimum MCS is carried out assuming that the ESNR is the SNR that would be obtained at the receiver in case having an Additive White Gaussian Noise (AWGN) channel. Under that assumption, there is a direct mapping between each MCS and the obtained BLER for each ESNR.

4 Simulation Results

For the TACS evaluation, the downlink mode of a WiMAX TDD system has been used [15]. The number of available transmitter antennas are $M_a = \{2, 3, 4\}$ whereas the number of receiver antennas is fixed to $N=2$. One user is simulated which is allocated one subchannel per frame. The channel follows a spatial uncorrelated Rayleigh distribution whereas a block fading model is assumed per subchannel (flat in frequency and constant in time). It is assumed that the channel is perfectly known at both transmitter and receiver sides.

The performance of the TACS adaptation scheme in case the throughput is maximized (see Eq. (10)) is analyzed. Then, for such adaptation scheme, the antenna set and the LDC code that maximizes the throughput is selected. In addition, the highest MCS (in the sense of spectral efficiency) that achieves a $BLER < 0.01$ (1%) is also selected. In these simulations the minimum allocable block length according the IEEE 802.16e standard was selected [15] (i.e. the number of subchannels N_{sch} occupied per block varies between 1 and 4). The number of available antennas is $M_a=2$ whereas $N=2$. Linear detection with the MMSE and ML detection are also compared.

The basic set of LDC codes that we have been used for the study are: the *Single Input Multiple Output* code using a Maximum Ratio Combiner (MRC), the *Alamouti* code (referred as G2 in the plots), the *BLAST*-like codes with $M=2$ (referred as Spatial Multiplexing, SM, in the plots) and the *Golden* code. The codeword length for all the codes is $T=2$. Moreover, for the SM case two types of encoding have been tested named vertical encoding (SM-VE) and horizontal encoding (SM-HE). For the vertical encoding, the same MCS is used for all the symbols transmitted within the same codeword, whereas for horizontal encoding each data stream (symbol) may apply a different MCS according to the channel status. Actually, all these codes are part of the standard and can be found in [15]. Consequently, since each i -th LDC from this basic set require at most two transmitting antennas, in case $M_i < M_a$ the best set p of transmitting antennas is selected from the M_a available antennas, and since the order in which the antennas are chosen is relevant we have P_i possible transmitting antennas combinations with

$$P_i = C \binom{M_a}{M_i} = \frac{M_a!}{M_i!(M_a - M_i)!} \tag{11}$$

To solve (9) or (10), an exhaustive search is performed among all the available LDC codes and P antenna sets, despite it would be very interesting testing the performance of the TACS under an incremental or decremental search as those proposed in [8].

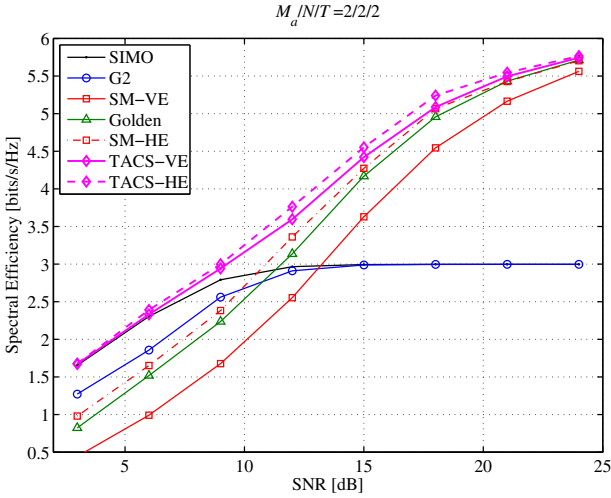


Fig. 2. Spectral efficiency achieved using the TACS scheme ($M_a/N/T=2/2/2$) joint with AMC under the throughput maximization criterion

The performance of TACS under the second optimization case and $M_a=2$ is shown in Fig 2. In this last scenario a PER bound of 1% is fixed and the MCS available are {4,16}-QAM with turbo-coding, with coding rates varying from 1/4 to 3/4. The MCS can be adapted on a per symbol (stream) basis. It is then shown that that at low SNRs (SNR<13dB) the SIMO and Alamouti achieved the highest spectral efficiencies.

However, as the SNR is increased, codes with higher multiplexing capacity are necessary; hence the SM and the Golden code achieve the highest spectral efficiencies. We also observed that the SM with VE implies a loss of around 2 dB compared to the Gold code, but when HE is used, the Gold code is around 0.5 dB worse than SM-HE. Above all, we observe that the TACS scheme with AMC gives the highest spectral efficiency where the highest benefit is obtained in the $8\text{dB} < \text{SNR} < 18\text{dB}$ margin where a smooth transition between both types of codes (codes with $g_s=1$ or $g_s=2$) is carried out.

In case of MMSE receiver (Fig 3 and Fig 5), it is shown that at low SNRs ($\text{SNR} < 13\text{dB}$) the SIMO and Alamouti schemes achieved the highest spectral efficiencies (something that has been already announced in several works [8]). However, as the SNR is increased the codes with higher multiplexing capacity (e.g. the SM and the Golden code) are preferred. We also observed that the SM with VE implies a loss of around 2dB compared to the Golden code, but when HE is used, the Golden code is around 0.5dB worse than the SM-HE.

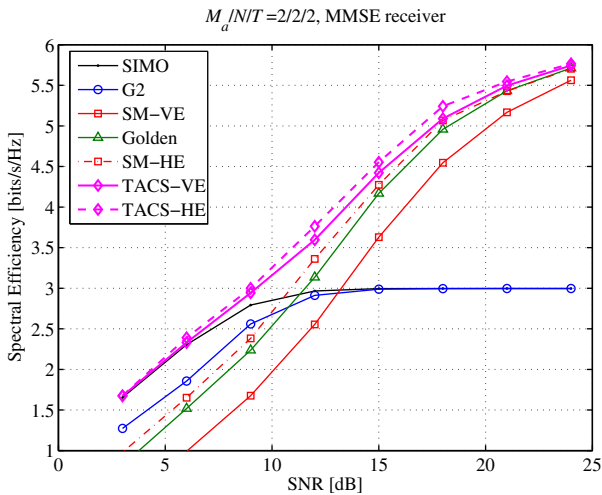


Fig. 3. Spectral efficiency under TACS with throughput maximization criterion with $M_a=2$, $N=2$, adaptive MCS and MMSE receiver for an uncorrelated MIMO Rayleigh channel

To gain further insights of the TACS behavior the statistics of LDC selection as a function of the average SNR are plotted in Fig 4. We can clearly appreciate that at low SNR the preferred scheme is SIMO where all the power is concentrated in the best antenna, while as the SNR is increased full rate codes ($Q=M$) are more selected since they permit to use lower size constellations. Moreover, comparing SM-VE with SM-HE, we can observe that SM-HE is able to exploit the stream's diversity and hence achieves a higher spectral efficiency than with the Golden code. Actually, at average SNR=12 dB, the SM with HE is the scheme selected for most frames, even more than SIMO.

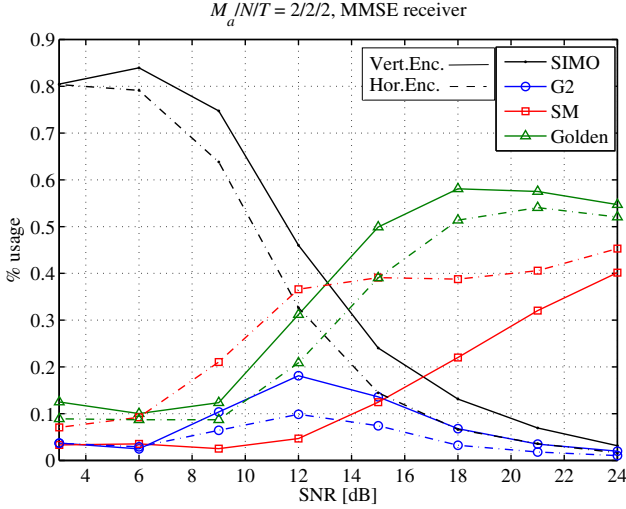


Fig. 4. LDC selection statistics under TACS with throughput maximization criterion with $M_a=2$, $N=2$, adaptive MCS and MMSE receiver for an uncorrelated MIMO Rayleigh channel

The evolution of the measured ESINR for the different streams is depicted in Fig 5 where the x -axis is scaled in channel accesses (i.e. time slots). It can be observed that SIMO is the scheme that achieves the highest ESINR values during the whole simulation time. Then few dBs below we have the Alamouti behavior, since it using both transmitter antennas the Frobenius channel norm dictates the ESINR evolution.

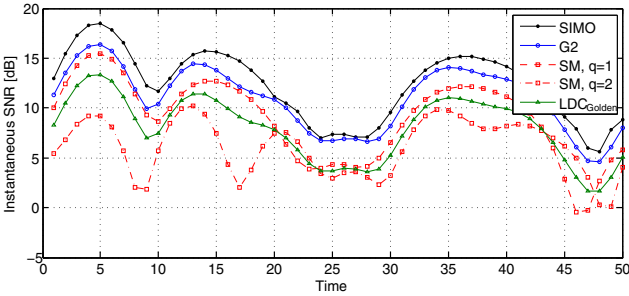


Fig. 5. Evolution of the ESINR per stream for different space-time codes with $M_a=2$, $N=2$, and time-correlated MIMO Rayleigh channel

Then, the ESINR achieved by different streams of the SM mode is plotted separately, where it can be seen that the best stream (SM_q , $q=1$) gets 3dB less than the ESINR using the SIMO scheme, this difference is due to the higher number of transmitter antennas that receive half the transmitting power. The second stream (SM_q , $q=2$) may fall several dB's (up to 10dB) compared to the best stream. This is the main reason why the SM-VE is so limited compared with the SM-HE. Finally, it is observed that the ESINR of the Golden code falls between the ESINR of both SM

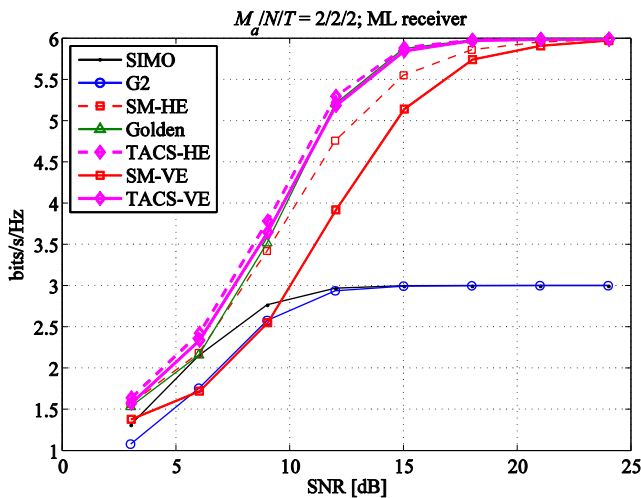


Fig. 6. Spectral efficiency under TACS with throughput maximization criterion with $M_a=2$, $N=2$, adaptive MCS and ML receiver for an uncorrelated MIMO Rayleigh channel

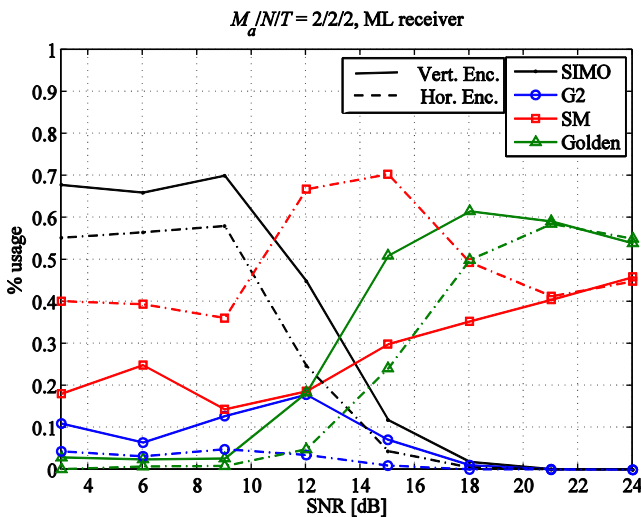


Fig. 7. LDC selection statistics under TACS with throughput maximization criterion with $M_a=2$, $N=2$, adaptive MCS and ML receiver for an uncorrelated MIMO Rayleigh channel

streams ($q=1$, $q=2$), however the Golden code's ESINR is lower than the arithmetic mean of the ESINR of both SM streams.

Next, the performance of the TACS with throughput optimization when using the ML decoder is shown in Fig 6 and Fig 7. Clearly, there is a large gain when using the ML compared to the previous MMSE detector. Something quite relevant is that when using the ML decoder, the Golden code achieves a capacity very close to that achieved by the TACS selection scheme (less than 1dB improvement due to the

TACS). This is a very logical result since the Golden code has been optimized assuming a ML decoder in order to maximize the capacity when $M=T=2$ and the Z-QAM modulation which is the same case as analyzed during the simulations.

If we analyze the LDC selection statistics in Fig 7, similar results as in the MMSE case are obtained. It is observed that at low SNR values the SIMO looks like the preferred scheme, whereas at high SNRs both the Golden and the SM are chosen similarly (60% and 40% respectively). Furthermore, it is observed that at SNR range of 10dB to 18dB using the TACS scheme the SM-HE is the scheme selected most the time (even more than the Golden code). Again, the high spectral efficiencies achieved by the SM-HE in the above SNR range come from the fact that this code is able to exploit the diversity between streams when the AMC mode is applied.

5 Conclusions

As a conclusion, when using TACS with AMC and ML detection the benefits are not so clear even more if we take into account the signalling associated to the TACS scheme. Hence the focus in this case should be to use optimized LDCs codes for each $\{M, T\}$ pair. However, obtained results shown that in case of linear receivers (e.g. MMSE) the TACS scheme with AMC achieves a noticeable SNR gain (up to 3dB) in a large SNR margin (SNR from 6 to 18dB), and also is a good technique to achieve a smooth transition between diversity and multiplexing. Then it seems also logical to consider the TACS scheme with linear receivers for the downlink where computational complexity at the mobile station must be kept as low as possible in order to save the batteries energy.

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