Optimal Channel and Power Allocation for Secondary Users in Cooperative Cognitive Radio Networks

(Invited Paper)

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Abstract. Cognitive radios are a natural evolution of Software Defined Radios (SDRs) that are supposed to be equipped with the ability to learn their RF environment and reconfigurability. A cognitive radio can communicate over a primary user's channel as long as the introduced interference does not degrade the primary Signal-to-Interference-plus-Noise-Ratio (SINR) below its minimum Quality of Service (QoS) requirement. In this paper, we employ cooperation in data transmission in order to increase the secondary transmit power limit. We present an optimal power allocation scheme for secondary users in order to achieve maximum SINR. We show that the optimal channel assignment problem that maximizes the sum-rate can be solved via the so-called Hungarian algorithm at a cubic complexity order. Also, we develop a suboptimal algorithm that permits to solve the channel assignment problem with a quadratic complexity order and with a slight performance degradation compared to that of the optimal solution.

1 Introduction

Most of RF spectrum below 6 GHz is historically owned by licensed users/services. Thus, the spectrum opportunities for the introduction of new wireless services are very limited [1]. With the increase in demand for higher capacities in existing communication systems, as well as for new wireless services, a solution is needed to overcome the problem of saturation of the spectrum. Cognitive radio is suggested as a promising solution after an observation of the spectrum usage, where it turns out that most licensed channels are not used by their owners most of the time, and some channels could handle a higher level of interference based on the Quality of Service (QoS) requirement of their users. According to [2], cognitive radio presents *intelligent* techniques to make efficient use of the spectrum by filling the spatial and temporal spectrum holes, without affecting the performance requirements of primary users. In this context, various research has been made to enhance the performance of cognitive radio systems, and it has been noted that significant

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improvement can be achieved by applying the concept of *cooperation* to cognitive systems. Earlier, [3], [4] and [5] showed that cooperation can overcome the limitations of wireless systems by increasing the spatial diversity.

Previously, node cooperation has been applied for spectrum sensing in cognitive radio networks [6] where cognitive users cooperate to determine the spectral and/or temporal holes in the spectrum, so that cognitive devices will have a better estimate of the channel status, which reduces the excessive interference and collision risk with the primary licensed users [7], [8]. In these existing proposals, once cognitive users estimate the status of a channel, they communicate without cooperation.

In this paper, on the other hand, we present a cooperative design similar to the *cognitive relay* model in [9]: Cognitive users cooperate with primary (licensed) users by relaying the primary signal to its destination. Under certain channel conditions, this cooperation enables the secondary user to achieve a higher SINR without violating the primary user's QoS. In addition, such cooperative communication introduces diversity in the primary link helping the primary user to achieve its required QoS when its channel suffers from severe channel fading.

The motivation behind this model is due to the power constraints that a primary user imposes on a secondary cognitive user. We apply cooperation in order to increase the secondary transmit power. Thus, we develop a power allocation scheme that determines the amount of power spent by every secondary user to send both its private and relayed signals. The proposed power allocation scheme can be used in conjunction with any given channel assignment, and is optimal in the sense of maximizing the secondary transmission rate subject to a given primary QoS requirement. However, we note that the optimal power that maximizes the secondary SINR does not necessarily lead to maximum sum-rate achieved by the joint system made of primary and secondary users. Yet, our objective in this work is to maximize the spectrum utilization over all channels subject to the constraint of minimum primary QoS. Hence, we choose to allocate channels to the secondary users such that the sum-rate is maximized while power is chosen so that secondary SINR is maximized within each channel allocation.

We present two channel assignment methods that have polynomial complexity. The first, the optimal channel assignment method, forms a matching between primary and secondary users subject to maximizing the sum-rate, and is denoted as the Centralized Channel Assignment. The second method is a heuristic algorithm in which primary users are picked randomly and an optimal cooperative secondary user is assigned.

The remainder of this paper is organized as follows. In section 2, we develop the system model. In section 3, we derive the optimal power allocation scheme. Sections 4.1 and 4.2 present the optimal and suboptimal channel assignment methods, respectively. The simulation results are shown in section 5, and we conclude this paper in section 6.

2 System Model

The assumed dynamic spectrum sharing (DSS) cognitive radio system consists of K_p primary users (i.e. K_p licensed channels), K_s secondary transmitters, and 1 primary and 1 secondary receivers (base stations). The users are indexed using the set $\mathcal{K} = \mathcal{K}_p \cup \mathcal{K}_s$, where $\mathcal{K}_p = \{1, ..., K_p\}$ and $\mathcal{K}_s = \{K_p + 1, ..., K_p + K_s\}$ are the indices of the primary and secondary users, respectively. P_k denotes the transmit power of user k to send its *own* signal. In our proposed model, a cognitive secondary user will cooperate with a primary user by sending the primary user's signal in superposition with its own signal. $q_{j,i}$ denotes the transmit power of the cognitive user j ($j \in \mathcal{K}_s$) to send the signal of the primary user i ($i \in \mathcal{K}_p$), i.e. the total transmit power of the cognitive user j is equal to $P_j + q_{j,i}$, where we assume that at any given time each secondary user only cooperates with at most a single primary user. $h_{m,n}$ represents the channel fading coefficient between users m and n, h_{pk} is the channel fading coefficient between user k and the primary receiver, h_{sk} is the channel fading coefficient between user k and the secondary receiver. We denote the instantaneous SINR's of user $k \in \mathcal{K}$ at the primary and the secondary receivers as γ_{pk} and γ_{sk}, respectively. Also, we define $[x]^+ \triangleq \max\{0, x\}$.

In this system, each secondary cognitive user wants to communicate with the secondary receiver on any one of the available K_p primary channels. To achieve this communication, the secondary user will cooperate with the primary user to whom the selected channel belongs.

At any given time, a secondary user is assumed to be only capable of communicating over one chosen channel. The scheduling function $\phi : j \to i$ ($j \in \mathcal{K}_s$ and $i \in \mathcal{K}_p \bigcup \{0\}$) forms a mapping between the cognitive user j and its corresponding cooperative primary channel i. When $\phi(j) = 0$ this will indicate that user j is not cooperating with any primary user. Alternatively, the scheduling function ϕ can be defined using the assignment vector $\Phi = [\phi(K_p + 1), ..., \phi(K_p + K_s)]^T$ which has $[K_s - K_p]^+$ zero elements (representing the secondary users that cannot be assigned to any primary channel when the number of primary channels is limited), and $\phi(u) \neq \phi(v)$ for any $(u, v) \in \mathcal{K}_s \times \mathcal{K}_s$ with $u \neq v$ and $\phi(u)\phi(v) \neq 0$.

Let $b_k^{(l)}$ be the l-th symbol from transmitter $k \in \mathcal{K}$. The transmission of every primary symbol is done in two stages: In the first step, primary i transmits its m-th symbol $b_i^{(m)}$ with a power αP_i (where $\alpha \in [0, 1]$) to secondary user j which generates the estimate $\hat{b}_i^{(m)}$. Secondary users are assumed to be full-duplex devices, so the secondary user is capable of transmitting its m'-th private symbol $b_j^{(m')}$ during the first step at a power that does not degrade the primary QoS. During the second step, primary user i again transmits the same symbol $b_i^{(m)}$ at a power $(1 - \alpha) P_i$ and the secondary cognitive user transmits both $\hat{b}_i^{(m)}$ and its private symbol $b_j^{(m'+1)}$ with respective powers $q_{j,i}$ and P_j. The transmission of the primary symbol in two time stages decreases the transmission rate, but as shown in [4], this decrease in transmission rate can be compensated by the reduction in symbol-error probability under certain channel conditions.

In the following sections, we will make the so-called *genie assumption* [2] which implies that the primary message is known to the cognitive user [10]. Thus, in computing the received SINR and the system throughput, we will assume that the transmission is done in one time slot.

3 Power Allocation Scheme

In cognitive systems, a power constraint is imposed on the secondary user so that the SINR of the incumbent primary user i doesn't drop below its minimal SINR requirement denoted by $\overline{\gamma}_{pi}$ that is determined by the QoS requirement of the primary user on channel i. The SINR at the primary receiver when secondary user j selects channel i (i.e. when $\phi(j) = i$) is:

$$\gamma_{pi} = \frac{P_i h_{pi}^2 + q_{j,i} h_{pj}^2}{P_j h_{pj}^2 + N_0} \ , \tag{1}$$

where N_0 is the average noise power at the receiver. Since this SINR should be greater than the threshold $\overline{\gamma}_{pi}$, by solving for P_j so that $\gamma_{pi} \geq \overline{\gamma}_{pi}$, we obtain the maximum allowable transmit power of the cognitive user j to send its own signal:

$$P_j \leq \min \left\{ \overline{P}_j - q_{j,i}, \left[\frac{P_i h_{pi}^2 - \overline{\gamma}_{pi} N_0 + q_{j,i} h_{pj}^2}{\overline{\gamma}_{pi} h_{pj}^2} \right]^+ \right\} \triangleq \xi_{j,i} \ , \tag{2}$$

where $\overline{P}_j$ is the maximum total transmit power of secondary user j such that $P_j + q_{j,i} \leq \overline{P}_j$. When $\phi(j) = i$, the SINR at the secondary receiver in channel i is:

$$\gamma_{sj} = \frac{P_j h_{sj}^2}{P_i h_{si}^2 + q_{j,i} h_{sj}^2 + N_0} \ . \tag{3}$$

The objective of the power allocation problem is to find the optimal values P_j^* and $q_{j,i}^*$ such that:

$$\left(q_{j,i}^*, P_j^*\right) = \arg \max_{(q_{j,i}, P_j)} \gamma_{sj} \tag{4}$$

subject to:

$$\begin{array}{rl} P_j \leq & \overline{P}_j - q_{j,i} \\ P_j \leq & \frac{q_{j,i}}{\overline{\gamma}_{pi}} + \tau \\ P_j > & 0 \\ q_{j,i} \geq & 0 \end{array} \ , \tag{5}$$

where $\tau = \frac{P_i h_{pi}^2 - \overline{\gamma}_{pi} N_0}{\overline{\gamma}_{pi} h_{pj}^2}$, $i \in \mathcal{K}_p$ and $j \in \mathcal{K}_s$. The shaded area in Fig. 1 represents the feasibility region defined by (5). For any given channel assignment, we

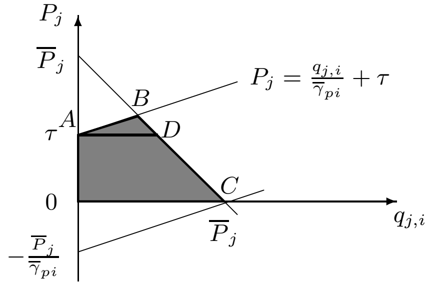

Fig. 1. Feasibility Region for maximizing γ_{sj}

characterize the optimal power allocation solution in (6), and the derivation is shown in Appendix A.

$$\left(q^*_{j,i}, P^*_j\right) = \begin{cases} \left(0, \min\{\overline{P}_j, \tau\}\right) & \text{if } \quad \tau \geq \min\{\overline{P}_j, \tau_1\} \\ \left(\lambda_i, \overline{P}_j - \lambda_i\right) & \text{if } -\frac{\overline{P}_j}{\overline{\gamma}_{pi}} < \tau < \min\{\overline{P}_j, \tau_1\} \\ (0,0) & \text{if } \quad \tau \leq -\frac{\overline{P}_j}{\overline{\gamma}_{pi}} \end{cases} , \tag{6}$$

where $\tau_1 = \frac{P_i h_{si}^2 + N_0}{\overline{\gamma}_{pi} h_{sj}^2}$ and $\lambda_i = \frac{\overline{\gamma}_{pi}}{1+\overline{\gamma}_{pi}} \left(\overline{P}_j - \tau\right)$.

Note that for large $\overline{\gamma}_{pi}$, it is more likely to have $\tau < -\frac{\overline{P}_j}{\overline{\gamma}_{pi}}$ so that the secondary does not get to transmit any signal.

On the other hand, the optimal power allocation for non-cooperative cognitive systems is simply $\left(q^*_{j,i}, P^*_j\right) = \left(0, \min\left\{\overline{P}_j, [\tau]^+\right\}\right)$.

4 Channel Assignment Algorithms

4.1 Centralized Channel Assignment

The cognitive cooperative communications scheme that we introduced in section 2 allows each primary user to cooperate with a secondary user that is sharing its licensed spectrum. The cognitive receiver, which is assumed to know the channel state information (CSI), is assumed to be responsible for assigning a primary channel to each cognitive secondary user.

Since the optimal solution in (6) depends on combination $(i,j) \in \mathcal{K}_p \times \mathcal{K}_s$, some cooperative combinations may lead to a higher secondary SINR than other combinations. Since we are interested in maximizing the transmission rate of the combined spectrum-sharing system, and in driving the primary SINR to its minimum requirement when it drops below its QoS (due to fading for example), we define the objective function $R_s(\Phi) = \sum_{i \in \mathcal{K}_p} R_i$ to be the sum of primary and secondary rates for all users, where R_i is the sum-rate on channel i defined as:

$$R_i \triangleq R_{p,i} + R_{s,i} \triangleq \log_2\left(1 + \gamma_{pi}\right) + \log_2\left(1 + \gamma_{sj}\right) ,$$

where $j = \phi^{-1}(i)$ and $R_{p,i}$ and $R_{s,i}$ are the primary and secondary rates on channel i, respectively.

Thus, the problem of optimal channel assignment is solved by finding the assignment vector Φ^* such that $\Phi^* = \arg\max_{\Phi} \sum_{i \in \mathcal{K}_p} R_i$. Because each primary user can share its spectrum with at most one secondary user at a time, and each secondary user can transmit over one channel at a time, the channel assignment problem becomes similar to the assignment problem in a weighted bipartite graph[1] where primary and secondary users constitute the two disjoint sets of vertices, and the edge weight between primary i and secondary j is equal to R_i. Figure 2 shows an example of a system consisting of $K_p = 4$ primary and $K_s = 4$ secondary users, with the corresponding edge weights R_i. In solving the channel assignment problem, our goal is to find the optimal matching between the elements of the two sets so that we maximize the sum of the weights of the matching edges (so that we maximize the sum-rate $R_s(\Phi)$)[2].

According to [11], this assignment problem is a special case of the Hitchcock problem, and it can be solved by the *Hungarian algorithm* which is proposed by Khun [12]. The *Hungarian algorithm* solves the weighted matching problem for a complete bipartite graph. A complete bipartite graph has the same number of elements in both sets, but according to [11], we can always assume that a bipartite graph is complete by setting the weights of the missing edges to be equal to 0, and [13] shows that we still get the optimal solution for the bipartite graph by applying this modification.

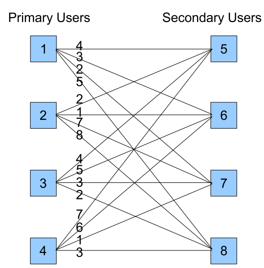

Fig. 2. Bipartite Graph Representation

Algorithm 1 gives the optimal channel assignment using the Hungarian algorithm as described in [13]. In the following, we apply this algorithm to the example in Fig. 2 where $\mathcal{K}_p = \{1, 2, 3, 4\}$ and $\mathcal{K}_s = \{5, 6, 7, 8\}$. We define the weight matrix $\mathbf{W}$ in (7).

[1] A bipartite graph is a graph whose vertices belong to two disjoint sets, such that every vertex is connected to at most one vertex from the other set.

[2] This optimization method can be used to find the optimal channel assignment for cognitive non-cooperative systems by using the non-cooperative optimal power allocation solution given at the end of section 3. In general, it can compute the optimal channel assignment for any cognitive cooperative system after having determined the appropriate power allocation scheme.

In step 1, we initialize $(u_1, u_2, u_3, u_4) = (5, 8, 5, 7)$ and $(v_1, v_2, v_3, v_4) = (0, 0, 0, 0)$. In step 2 we compute $C^{(1)}$ shown in (7). The maximum matching M of G has 3 edges (marked by the stars in $C^{(1)}$) and this matching is not optimal. Thus, in step 4 we form the vertex cover $Q = \{3, 5, 8\}$, to obtain $\epsilon = 1$, and we update $(u_1, u_2, u_3, u_4) = (4, 7, 5, 6)$ and $(v_1, v_2, v_3, v_4) = (1, 0, 0, 1)$. The corresponding $C^{(2)}$ is shown in (7). The maximum matching of G (which maps nodes 1, 2, 3 and 4 to 8, 7, 6 and 5, respectively) has 4 edges and it is the optimal matching for this graph.

$$\mathbf{W} = \begin{bmatrix} 4\,3\,2\,5 \\ 2\,1\,7\,8 \\ 4\,5\,3\,2 \\ 7\,6\,1\,3 \end{bmatrix}, \mathbf{C}^{(1)} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 6 & 7 & 1 & 0^* \\ 1 & 0^* & 2 & 3 \\ 0^* & 1 & 6 & 4 \end{bmatrix} \text{ and } \mathbf{C}^{(2)} = \begin{bmatrix} 1 & 1 & 2 & 0^* \\ 6 & 6 & 0^* & 0 \\ 2 & 0^* & 2 & 4 \\ 0^* & 0 & 5 & 4 \end{bmatrix} \tag{7}$$

We use the code described in [14] to find the maximum weight matching. This code can compute the optimal matching for a 100-by-100 matrix in 120ms when operating on a 2.4GHz processor.

Algorithm 1. Centralized Optimal Channel Assignment

Given $\mathcal{K}_p$ and $\mathcal{K}_s$ (with cardinality k for each set). Let $\mathbf{W} = [w_{ij}] \in \mathbb{R}^{k \times k}$ be the weight matrix where $w_{i,j} = R_i$ with $\phi(j) = i$.
1. Initialize two labels $u_i = \max_{j \in \{1,...,k\}} w_{ij}$ and $v_j = 0$ for $i, j = 1, ..., k$.
2. Obtain the excess matrix $\mathbf{C} = [c_{ij}] \in \mathbb{R}^{k \times k}$ such that $c_{ij} = u_i + v_j - w_{ij}$
3. Find the subgraph G consisting of vertices i and j satisfying $c_{ij} = 0$ and the corresponding edge e_{ij}. Find the maximum matching M in G.
If M is perfect matching with k edges, go to step 5.
4. Let Q be a vertex cover of G, and let $R = \mathcal{K}_p \bigcap Q$ and $T = \mathcal{K}_s \bigcap Q$.
A vertex cover contains at least one endpoint of each edge of a graph.
Find ϵ satisfying $\epsilon = \min\{c_{ij} : x_i \in \mathcal{K}_p - R, y_j \in \mathcal{K}_s - T\}$.
Decrease u_i by ϵ for the rows of R^c and increase v_j by ϵ for the columns of T. Then go to step 2.
5. M is the optimal assignment solution when M is perfectly matched with k edges

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4.2 Heuristic Assignment Method

The Hungarian algorithm presented above solves the optimal matching problem for a complete weighted bipartite graph with $2n$ vertices in $\mathcal{O}(n^3)$ arithmetic operations [11]. Since we can assume that any bipartite graph is complete if we set the weights of the missing edges to be equal to 0, then the complexity order of the optimal channel assignment in our system is $\mathcal{O}\left[(\max\{K_p, K_s\})^3\right]$.

To reduce the computational complexity, in the following we propose a heuristic algorithm (Algorithm 2) similar to [15] with a lower complexity order to solve the channel assignment in large systems. We consider K_p primary and K_s secondary users, and find the channel assignment between these nodes. Algorithm 2 is applied when $K_p \leq K_s$, and an analogous algorithm can be deduced for the case when $K_p > K_s$, as we will show later. As will be shown, this algorithm will have at most quadratic complexity in $\max\{K_p, K_s\}$.

The Algorithm 2 randomly selects a primary user $i \in \mathcal{K}_p$ and its corresponding optimal cooperating cognitive device $j^*(i) \in \mathcal{K}_s$ is found. Then, i and $j^*(i)$ are removed from the sets $\mathcal{K}_p$ and $\mathcal{K}_s$, respectively, and the same procedure is repeated with the remaining elements. In practice, when $K_p \leq K_s$, all available secondary users simultaneously scan a randomly selected primary channel and obtain the CSI and the value of P_i. We assume that the CSI stays fixed for the duration of a block. Once the cognitive secondary system knows the transmit primary power P_i, every secondary user computes the γ_{pi} and γ_{sj} using (1) and (3), respectively. These SINR values can be known after solving for the optimal $q_{j,i}$ and P_j using (6). Next, the set $\{R_i\}_{j \in \mathcal{K}_s}$ is computed and the cognitive user $j^*(i) = \arg\max_{j \in \mathcal{K}_s} R_i$ is selected to cooperate with primary user i.

Similarly, if $K_p > K_s$, a cognitive user is selected randomly from the set $\mathcal{K}_s$, and this user scans all available primary channels and chooses to cooperate with the channel $i^*(j) = \arg\max_{i \in \mathcal{K}_p} R_i$. Then, $i^*(j)$ and j are removed from the sets $\mathcal{K}_p$ and $\mathcal{K}_s$, and the same procedure is repeated until all secondary users are exhausted.

Algorithm 2. Heuristic Assignment Method ($K_p \leq K_s$)

1. Randomly pick a primary user $i \in \mathcal{K}_p$.
2. Calculate $j^*(i) = \arg\max_{j \in \mathcal{K}_s}\{\log_2(1+\gamma_{pi}) + \log_2(1+\gamma_{sj})\}$ when j cooperates with i ($\phi(j) = i$).
3. Remove i and j^* from the set $\mathcal{K} = \mathcal{K}_p \bigcup \mathcal{K}_s$ and repeat the same procedure with the remaining elements until $\mathcal{K}_p = \emptyset$.

Algorithm 2 ensures that all R_i values are considered in the computation. However, it reduces the assignment complexity to the order of $\mathcal{O}(K_p K_s)$, since the number of comparisons is equal to $\sum_{i=0}^{K_p-1}(K_s - i)$ when $K_p \leq K_s$.

5 Numerical Results

We simulate a system consisting of $K_p = 3$ primary users and $K_s = 5$ secondary users. Throughout all simulations, we assume all fading coefficients to be i.i.d. Rayleigh distributed with normalized power $\mathbb{E}[h^2] = 1$. We let $P_i = 1W$, for $i \in \mathcal{K}_p$, and assume $\overline{P}_j$ to be the same for all $j \in \mathcal{K}_s$. The average noise power at the receivers is $N_0 = 0.1W$, and all primary users have the same SINR requirement $\overline{\gamma}_{pi} = \overline{\gamma}_p$. We assume that secondary users have knowledge of the primary message.

In Fig. 3 we plot the average sum-rate $\overline{R}_s$ versus $\overline{\gamma}_p$ subject to fixed $\overline{P}_j$. At any given $\overline{\gamma}_p$, we observe that the value of $\overline{R}_s$ that is achieved by cooperative cognitive systems is higher than $\overline{R}_s$ in non-cooperative cognitive systems. Also,

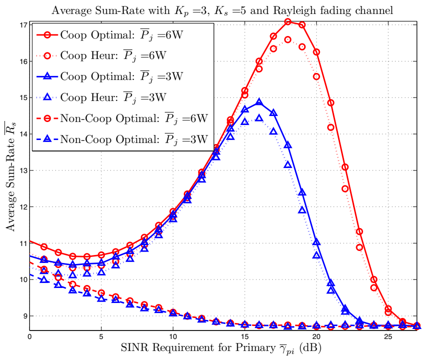

Fig. 3. Average Sum-Rate under Rayleigh fading subject to $\overline{P}_j$

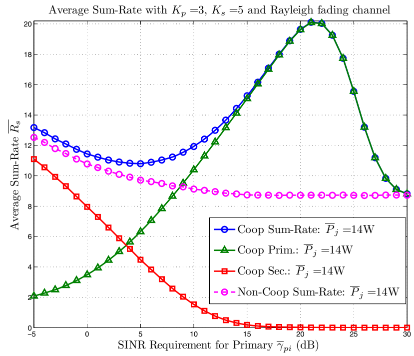

Fig. 4. Average Primary Rate under Rayleigh fading subject to $\overline{P}_j = 14$W

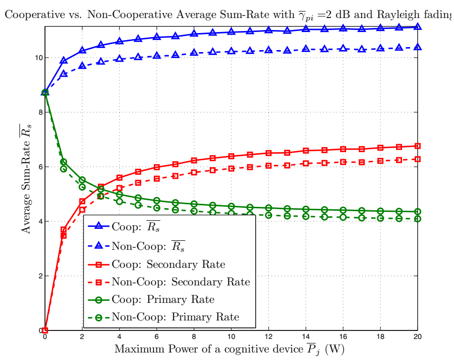

Fig. 5. Elementary Average Sum-Rate under Rayleigh fading

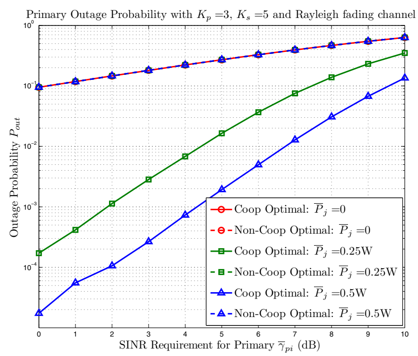

Fig. 6. Outage Probability of Primary Users

the performance of cooperative systems with heuristic assignment method is reasonably close to that of cooperative systems with optimal assignment method. Figure 4 shows that for large values of $\overline{\gamma}_p$, cognitive secondary users do not get to transmit because $\tau \leq -\frac{\overline{P}_j}{\overline{\gamma}_{pi}}$. Also, for $\overline{P}_j = 14W$, the average cooperative sum-rate is decreasing over the interval $[-5 \text{ dB}, 5 \text{ dB}]$ because it is dominated by the decreasing average secondary rate. As the secondary rate approaches 0, the average sum-rate becomes close to $K_p \log_2 \left(1 + \overline{\gamma}_{pi}\right)$ for all $\overline{P}_j$ values that enable the primary to meet its SINR requirement.

Next, in Fig. 5, we plot the averages of R_s, $\sum_{\imath \in \mathcal{K}_p} R_{p,i}$ and $\sum_{\imath \in \mathcal{K}_p} R_{s,i}$ over fading for $\overline{\gamma}_{pi} = 2dB$. For any $\overline{P}_j$, we observe that cooperation increases the average sum-rate for primary and secondary users. In fact, the objective of this optimization is to increase γ_{sj} subject to maintaining $\gamma_{pi} \geq \overline{\gamma}_{pi}$. As we increase

$\overline{P}_j$, the average primary sum-rate decreases to $K_p \log_2 \left(1 + \overline{\gamma}_{pi}\right) = 4.11$, but it does not drop below its QoS requirement. We note also that the average sum-rate of the secondary user is not necessarily equal to 0 when the average sum-rate of the primary is less than its QoS requirement, because whenever the *instantaneous* primary SINR γ_{pi} is greater than $\overline{\gamma}_{pi}$, the secondary user gets to transmit at a non-zero rate, regardless of the *average* primary SINR which could be less than $\overline{\gamma}_{pi}$. Thus the average sum-rate of secondary users in a non-cooperative system is not identically zero when the average sum-rate of primary users is below the QoS requirement.

Next, we plot in Fig. 6 the primary outage probability defined as $P_{out} \triangleq \Pr\left\{\gamma_{pi} < \overline{\gamma}_{pi}\right\}$. This plot shows that cooperation reduces significantly the outage probability of primary users. In the absence of cooperation, the introduction of a cognitive user does not affect the primary outage probability because secondary users are not allowed to degrade the primary QoS requirements at any time. Hence the outage probability curves in Fig. 6 when $\overline{P}_j = 0$ coincide with the outage probability curves in non-cooperative cognitive scenarios with $\overline{P}_j > 0$. However, as can be observed through cooperation, cognitive users help to reduce the primary outage probability, as well as increasing their own transmission rate (as in Fig. 5).

6 Conclusion

In this paper, we have proposed a model that takes advantage of cooperative communications to improve spectrum utilization and primary outage performance of cognitive radio systems. Although cooperation has been widely used in cognitive radio for the purpose of spectrum sensing, our model applies cooperation in data transmission. We showed that our proposed technique could increase the transmission sum-rate of both primary and secondary users by means of increasing the primary channel diversity, and increasing the secondary transmission power limit. We derived an optimal and a heuristic channel assignment algorithm, as well as an optimal power allocation scheme for the proposed system. The channel assignment and power allocation algorithms can be applied independently to cognitive cooperative systems. In this paper, we applied the Centralized Channel Assignment to maximize the average sum-rate of the network, while the power allocation scheme guarantees maximum secondary SINR in every channel.

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A Derivation of the Optimal Power Allocation

The feasibility region in Fig. 1 shows that the optimal solution depends on the range of τ. In order to solve this optimization problem, we first note that γ_{sj} in (3) increases by increasing P_j and by decreasing $q_{j,i}$.

Case 1: If $\tau \geq \overline{P}_j$, the optimal solution is $\left(q^*_{j,i}, P^*_j\right) = \left(0, \overline{P}_j\right)$.

Case 2: If $0 \leq \tau < \overline{P}_j$, and referring to Fig. 1, we see that for any $q_{j,i} \in [0, \overline{P}_j]$, γ_{sj} is maximized when the solution belongs to the segments $[AB]$ or $[BC]$. That's because P_j is maximized (for every $q_{j,i}$) by selecting a feasible point from these segments.

Let $\lambda_i \triangleq \frac{\overline{\gamma}_{pi}}{1+\overline{\gamma}_{pi}} \left(\overline{P}_j - \tau\right)$ be the abscissa of point B. For any $P_j \in \left[\tau, \overline{P}_j - \lambda_i\right]$, we see that all feasible points from the segment $[BD]$ are suboptimal when compared to the points of segment $[AB]$ which has the same values of P_j but with a smaller $q_{j,i}$. Note that we have rejected the region where $P_j < \tau$ because it yields suboptimal γ_{sj} when compared to the point $(0, \tau)$.

Therefore, the optimal solution in this case is on the segment $[AB]$. The secondary SINR expression over this segment is $\gamma_{sj} = \frac{\left(\frac{q_{j,i}}{\overline{\gamma}_{pi}}+\tau\right)h_{sj}^2}{P_i h_{si}^2 + q_{j,i} h_{sj}^2 + N_0}$. Computing its partial derivative with respect to $q_{j,i}$, we get:

$$\frac{\partial \gamma_{sj}}{\partial q_{j,i}} = \frac{P_i h_{sj}^2 h_{si}^2 + N_0 h_{sj}^2 - \tau h_{sj}^4 \overline{\gamma}_{pi}}{\overline{\gamma}_{pi}\left(P_i h_{si}^2 + q_{j,i} h_{sj}^2 + N_0\right)^2} \ . \tag{8}$$

Let $\tau_1 \triangleq \frac{P_i h_{si}^2 + N_0}{\overline{\gamma}_{pi} h_{sj}^2}$. If $\tau < \tau_1$, then $\frac{\partial \gamma_{sj}}{\partial q_{j,i}} > 0$ and the optimal solution is $\left(q_{j,i}^*, P_j^*\right) = \left(\lambda_i, \overline{P}_j - \lambda_i\right)$. Otherwise, the optimal solution will be $\left(q_{j,i}^*, P_j^*\right) = (0, \tau)$.

Case 3: If $-\frac{\overline{P}_j}{\overline{\gamma}_{pi}} < \tau < 0$, and using the same analysis of Case 2, the optimal solution belongs to the line segment formed by the points $\left(-\tau\overline{\gamma}_{pi}, 0\right)$ and $\left(\lambda_i, \overline{P}_j - \lambda_i\right)$. But $\tau < 0 \leq \tau_1$, then γ_{sj} is a monotonically increasing function of $q_{j,i}$ and the optimal solution is $\left(q_{j,i}^*, P_j^*\right) = \left(\lambda_i, \overline{P}_j - \lambda_i\right)$.

Case 4: If $\tau \leq -\frac{\overline{P}_j}{\overline{\gamma}_{pi}}$, we see in Fig. 1 that the problem does not have a feasible solution. This corresponds to the case when the primary QoS could not be met even with the help of the secondary cooperation. In this case, we set $\left(q_{j,i}^*, {P_j}^*\right) = (0, 0)$ so that the secondary user does not relay any amount of power for the primary user unless it is allowed to transmit its own signal for some $P_j > 0$.