On the Maximum Efficiency of Power Amplifiers in OFDM Broadcast Systems with Envelope Following

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Abstract. We suggest a method for determining the efficiency of the power amplifier of an OFDM broadcast system. We present how far the efficiency of conventional power amplifiers can be increased by applying envelope following. The dependency of the efficiency on the probability of clipping is derived. In this context all three possibilities of operating point adjustment are investigated. Finally, we show that by adjusting only the operating point voltage of the power amplifier the efficiency of OFDM broadcast systems can be doubled.

Keywords: OFDM, envelope following, power amplifier, efficiency, broadcast, DVB-T, operating point adjustment.

1 Introduction

The energy required for communication and its price is increasing further and further. In the meanwhile the costs for the energy used by a base station for wireless communication are higher than the investment costs [1,2]. Thus, energy efficiency of communication systems is an important topic on the one hand for the conservation of the environment and on the other hand to reduce costs.

The power amplifier of a wireless communication system contributes significantly to the power consumption. Thus, a highly efficient power amplifier is required for an efficient communication system. Unfortunately, there are several reasons why the efficiency of the power amplifier is lower than intended. Firstly, modern and future wireless communication standards, e.g., DVB-T (Digital Video Broadcasting — Terrestrial) and LTE (Long Term Evolution), require highly linear power amplifiers to obtain their high spectral efficiency. Thus, class A or class AB power amplifiers are typically applied despite their lower maximum efficiency in comparison to the more efficient switched mode power amplifiers. And secondly, those power amplifiers are typically operated in back-off. Back-off is defined as the ratio between maximum output power $P_{\mathrm{out,max}}$

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and instantaneous output power P_{out}. If a power amplifier is operated in back-off the instantaneous output power is lower than the maximum output power. The efficiency of a class A power amplifier is proportional to the output power and is maximal at the maximum output power. Taking this into account it becomes clear that the efficiency in operation is noticeably lower than the already low maximum efficiency of the class A power amplifier. The first reason for the limited efficiency is a systematic one and has to be accepted. But concerning the second reason it is possible to increase the efficiency of a class A power amplifier in back-off by operating point adjustment.

Operating point adjustment has already been used to lessen the drop in efficiency caused by slow changes of the required output power. Such changes can be caused by altering channel conditions. If the channel changes, e.g., if a UMTS (Universal Mobile Telecommunications System) mobile phone gets closer to its base station, the output power has to be reduced to keep the received power constant. This is especially for code division multiple access (CDMA) systems like UMTS necessary but also other communication standards demand a control of the received power. The information about the required output power can be used by the baseband module to control the operating point of the power amplifier. This method is called envelope tracking. Since the changes are slow and the bandwidth is small it is relatively simple to implement an efficient operating point adjustment system. The gain in efficiency can easily be calculated by means of the statistic description of the output power. Figure 1 shows the probability density function of the output power for an UMTS device.

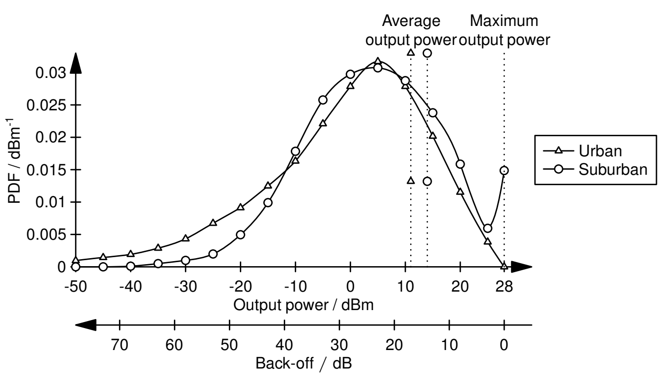

Fig. 1. Probability density function of the output power for an UMTS device [3,4]

Envelope tracking does not take into account the back-off operation caused by the modulation. Furthermore, it cannot be applied for all systems because a feedback about the channel conditions is required and an additional interface between the baseband module and the power amplifier has to be implemented. The envelope following method does not have these drawbacks. In this case the required output power is directly sensed at the power amplifier and the operating point is adjusted in real-time. Thus, the changes of the output power caused by the modulation can be exploited. Thereby, even the efficiency of broadcast systems can be increased. The power amplifier together with the operating point control becomes a stand-alone system, which makes it simple to upgrade existing systems.

In order to investigate the effect of the modulation on the efficiency and how far this influence can be lessen by envelope following, an orthogonal frequency-division multiplex (OFDM) broadcast system like DVB-T is assumed in the further course.

2 System Model

Figure 2 shows a model of the RF front-end applying envelope following. The signals I and Q modulate the carrier and the orthogonal carrier, respectively, which up-converts the spectrum into the radio frequency (RF) domain. The envelope of the RF signal is sensed and forms the reference of the operating point control which adjusts and controls the operating point of the power amplifier accordingly.

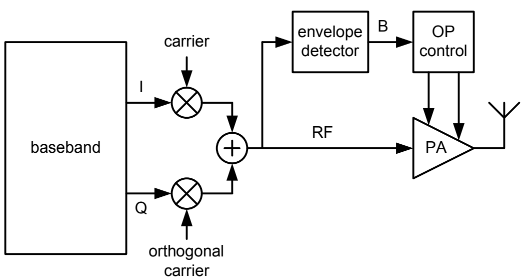

Fig. 2. Model of the transmitter of an OFDM system using envelope following as operating point control method

The envelope detector down-converts the RF signal such that the signal B corresponds to a baseband signal. Thus, an equivalent complex baseband model can be used, which is shown in figure 3. Therein, the envelope detector is replaced by its corresponding mathematical operation, which is

$$B = \sqrt{P} = \sqrt{I^2 + Q^2}. \tag{1}$$

In order to be able to calculate the efficiency, all signals will be regarded as stochastic processes in the further course. The following assumptions and simplifications have to be made to be able to specify properties of the stochastic processes. Figure 4 depicts a block diagram of the baseband module of an OFDM transmitter including important components like the inverse discrete

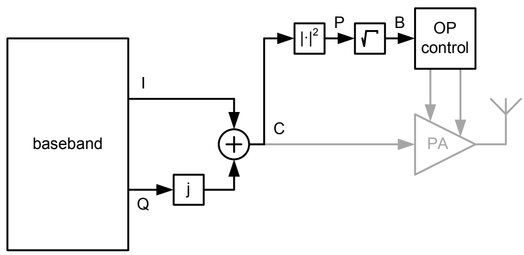

Fig. 3. Equivalent complex baseband model of the transmitter

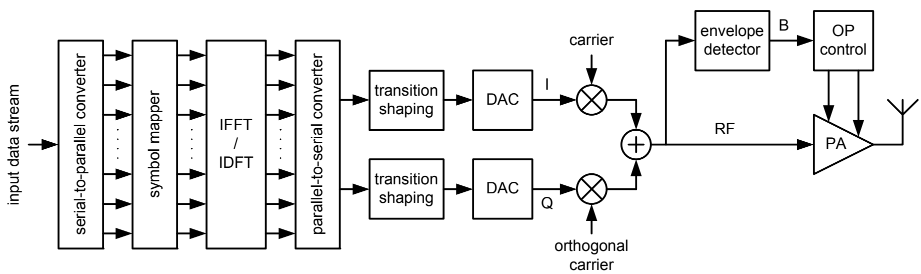

Fig. 4. Block diagram of the baseband module of an OFDM transmitter

fourier transform (IDFT) block and blocks to shape the transitions from one symbol to the next. It is assumed that the impact of those shaping blocks on the signal properties is negligible. This assumption bases on the fact that in a typical OFDM system the signal is modified by the shaping blocks just for a small fraction of time in comparison to the duration of the symbol itself.

3 Signal Properties

In a first step the properties of the processes I and Q will be derived. For OFDM systems it is often assumed that those processes are normally distributed [5,6]. This is a plausible assumption: The signals on the subcarriers can be assumed to be identically distributed processes and with the central limit theorem the superposition of many independent and identically distributed processes results in a normally distributed process.

Each normal distribution is characterized by its expected value μ and its standard deviation σ. Since each subcarrier is zero-mean, the superposition of all subcarriers is also zero-mean. The standard deviation can be determined by using the fact that the sum of the average power of all subcarriers is equal to the average power of the superposition. Thus, we are now looking for the average power $P_{\text{mean,sub}}$ of one modulated subcarrier. Figure 5 exemplarily shows the constellation diagram of a QAM16 modulation. In this case there are four

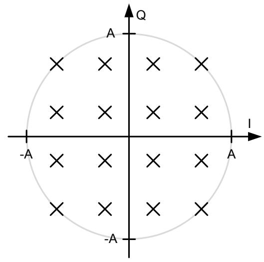

Fig. 5. Constellation diagram of the QAM16 modulation

possible amplitudes a_i for the real and for the imaginary part of the subcarrier. Assuming that each level has the same probability the average power of one modulated subcarrier can be calculated by

$$P_{\text{mean,sub}} = \frac{1}{n} \sum_{i=1}^{n} a_i^2, \tag{2}$$

where n is the number of possible amplitudes. In case of the QAM16 modulation the average power is $\frac{5}{18}A^2$. The average power of one modulated subcarrier can also be calculated using the peak-to-average power ratio of the modulation. The peak-to-average power ratio $PAPR$ is defined by

$$PAPR = \frac{P_{\text{peak}}}{P_{\text{mean}}} = \frac{V_{\text{peak}}^2}{V_{\text{rms}}^2} = CF^2 \tag{3}$$

where P_{peak}, P_{mean}, and CF are the peak power, the mean power, and the crest factor of a signal, respectively. Following this definition the peak-to-average power ratio of the QAM16 modulation is

$$PAPR_{\text{mod,QAM16}} = \frac{\left(1\right)^2}{\frac{4}{16}\cdot\left(1\right)^2 + \frac{8}{16}\cdot\left(\frac{\sqrt{5}}{3}\right)^2 + \frac{4}{16}\cdot\left(\frac{1}{3}\right)^2} = \frac{9}{5}. \tag{4}$$

The peak-to-average power ratios of some other modulations are listed in table 1.

Using the peak-to-average power ratio for the calculation the average power can be determined by

$$P_{\text{mean,sub}} = \frac{P_{\text{peak,sub}}}{PAPR_{\text{mod}}} = \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{PAPR_{\text{mod}}}. \tag{5}$$

Table 1. Peak-to-average power ratios of some modulation schemes

Modulation	QPSK	QAM16	QAM64	QAM256	QAMn^2
$PAPR_{\text{mod}}$	1	$\frac{9}{5}$	$\frac{7}{3}$	$\frac{45}{17}$	$\frac{3(n-1)}{n+1}$
	0 dB	2.6 dB	3.7 dB	4.2 dB	

Now the average power of the processes I and Q can be specified to be

$$P_{\mathrm{mean,I/Q}} = n_{\mathrm{sub}}\, P_{\mathrm{mean,sub}} = \tfrac{1}{2} n_{\mathrm{sub}} \frac{A^2}{PAPR_{\mathrm{mod}}}. \tag{6}$$

Since the processes I and Q are zero-mean, the variance σ^2 is equal to the average power. Thus the standard deviation σ can be calculated by

$$\sigma = \sqrt{P_{\mathrm{mean,I/Q}}} = \sqrt{\tfrac{1}{2} n_{\mathrm{sub}} \frac{A^2}{PAPR_{\mathrm{mod}}}} \tag{7}$$

and the probability density function is finally given by

$$p_{\mathrm{I/Q}}(x) = \tfrac{1}{\sqrt{2\pi}\sigma}\, e^{-\frac{x^2}{2\sigma^2}}. \tag{8}$$

In addition the peak-to-average power ratio of the processes I and Q can be specified to be

$$\begin{aligned} PAPR_{\mathrm{I/Q}} &= \frac{P_{\mathrm{peak,I/Q}}}{P_{\mathrm{mean,I/Q}}} \quad \text{with} \quad P_{\mathrm{peak,I/Q}} = \left(n_{\mathrm{sub}} \frac{A}{\sqrt{2}} \right)^2 \\ &= n_{\mathrm{sub}}\, PAPR_{\mathrm{mod}}. \end{aligned} \tag{9}$$

This shows the fundamental disadvantage of OFDM systems: the peak-to-average power ratio and the crest factor increase with the number of subcarriers and with the square root of the number of subcarriers, respectively.

According to (1) the process P is characterized by $P = I^2 + Q^2$. This implies the the process P is Chi-square distribution [7] with the probability density function

$$p_P(x) = \begin{cases} \frac{1}{2\sigma^2}\, e^{-\frac{x}{2\sigma^2}} & x \geq 0 \\ 0 & x < 0 \end{cases}. \tag{10}$$

This is identical to the probability density function of an exponential distribution. By applying the density transformation the probability density function of the process B can be derived:

$$p_B(x) = \begin{cases} \frac{x}{\sigma^2}\, e^{-\frac{x^2}{2\sigma^2}} & x \geq 0 \\ 0 & x < 0 \end{cases}. \tag{11}$$

This is a special case of the Weibull distribution and also called Rayleigh distribution.

4 Dependency of the Efficiency on the Drive

We already discussed that the operating point of the power amplifier can be adjusted and that thereby the efficiency can be enhanced. In this section the

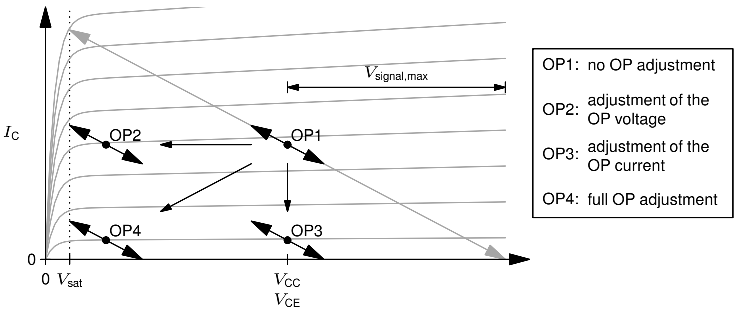

Fig. 6. The different possibilities of the OP adjustment visualized in the output characteristics of a transistor

relation between efficiency and the drive of the power amplifier for the different possibilities of operating point adjustment is derived.

The two main operating point (OP) parameters of a transistor and thereby of the power amplifier are OP voltage and OP current. The OP can be visualized in the output characteristics of the transistor together with the load line. The power amplifier operates linearly as long as the current in the transistor is greater than zero and the voltage across the transistor is greater than the saturation voltage [8]. The maximum drive is illustrated in figure 6 by the gray arrow. In this case the power amplifier delivers its maximum output power. It can be seen that if less output power is needed and thereby the swing is smaller the limitations for linear operation are not reached. This means that the OP is chosen to high for this particular output power and energy is wasted. Thus, one or both OP parameters can be adjusted such that the respective OP parameter(s) are minimized subject to the restrictions mentioned above. This is depicted in figure 6 by means of the three additional operating points (OP2, OP3, and OP4). That way the efficiency can be calculated for each case by means of the power taken from the supply and the power delivered to the load, which is always

$$P_{\text{out}} = \frac{V_{\text{signal}}^2}{2R_{\text{load}}}. \tag{12}$$

For the case that the OP is not adjusted, the power P_{DC} taken from the supply is independent from the signal and is

$$P_{\text{DC}} = V_{\text{CC}}\, I_{\text{C,OP}} = (V_{\text{signal,max}} + V_{\text{sat}})\, \frac{V_{\text{signal,max}}}{R_{\text{load}}}. \tag{13}$$

The efficiency is given by

$$\eta_{\text{no}} = \frac{P_{\text{out}}}{P_{\text{DC}}} = \frac{1}{2}\, \frac{(V_{\text{signal}}/V_{\text{signal,max}})^2}{1 + V_{\text{sat}}/V_{\text{signal,max}}}. \tag{14}$$

This equation shows that the maximum efficiency η_{max} of a class A amplifier is 50% which can theoretically be achieved if the saturation voltage is set to zero.

For the deviation of this equation it was assumed that the resistance of the load line is equal to the resistance of the connected load. This implies for instance an infinite output resistance of the transistor. Since this is practically not the case the parameter η_{max} is introduced to take aditional losses into account. Thus, the efficiency in this case and for all the other cases can be determined to be

$$\text{OP1:} \quad \eta_{\text{no}} = \eta_{\text{max}} \frac{(V_{\text{signal}}/V_{\text{signal,max}})^2}{1 + V_{\text{sat}}/V_{\text{signal,max}}} \tag{15}$$

$$\text{OP2:} \quad \eta_{\text{V}} = \eta_{\text{max}} \frac{V_{\text{signal}}/V_{\text{signal,max}}}{1 + V_{\text{sat}}/V_{\text{signal}}} \tag{16}$$

$$\text{OP3:} \quad \eta_{\text{C}} = \eta_{\text{max}} \frac{V_{\text{signal}}/V_{\text{signal,max}}}{1 + V_{\text{sat}}/V_{\text{signal,max}}} \tag{17}$$

$$\text{OP4:} \quad \eta_{\text{VC}} = \eta_{\text{max}} \frac{1}{1 + V_{\text{sat}}/V_{\text{signal}}}. \tag{18}$$

In order to understand the impact of the saturation voltage Figure 7 illustrates these functions in a simplified way. Thereby, the curve for no OP adjustment shows the typically back-off behaviour of a class A power amplifier whose efficiency is proportional to the output power. If either the OP current or the OP voltage are adjusted it can be seen that the drop in efficiency can be reduced. In case that the OP voltage and the OP current are adapted the efficiency is almost kept constant till the losses caused by the saturation voltage are significant in comparison to the output power.

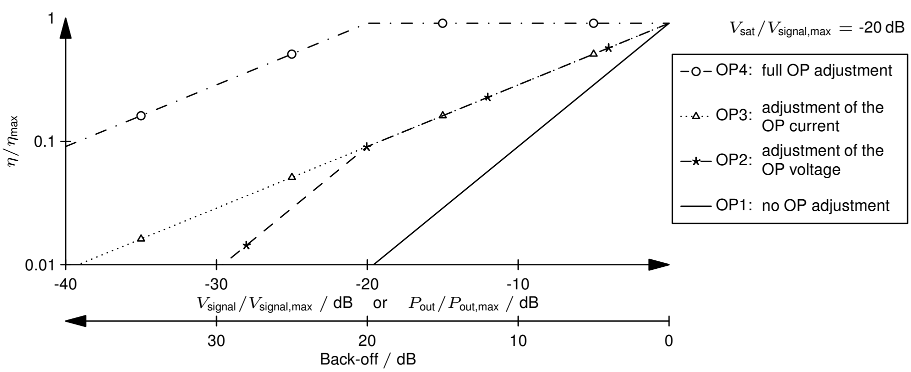

Fig. 7. Simplified dependency of the normalized efficiency versus the normalized output power for the different possibilities of the OP adjustment

5 Efficiency of the System

The average efficiency of the system can be determined by using the derived properties of the stochastic process in combination with the derived formulas for the efficiency. In order to do so it have to be considered that it is practically not possible to design an OFDM system that the full dynamic range of the signal, which is described by the crest factor, is in the range of linear operation. Thus, depending on the systems there is a certain probability of clipping. Clipping means that more output power and more signal swing is demanded from the power amplifier than its maximum output power and maximum signal swing, respectively. The probability of clipping can be influenced by the ratio between maximum output power of the power amplifier and its average output power.

The maximum voltage swing $V_{\text{signal,max}}$ at the output of the power amplifier is set by given constraints like breakdown and saturation voltage. For a desired probability of clipping a proper value for the parameter σ of the Rayleigh distributed voltage envelope of the signal has to be chosen (cf. figure 8). This influences the mean voltage of the signal. In order to achieve the required mean output power the OP current and the load line have to be adapted. Raising the OP current for a given OP voltage is equivalent to increasing the maximum output power of the power amplifier.

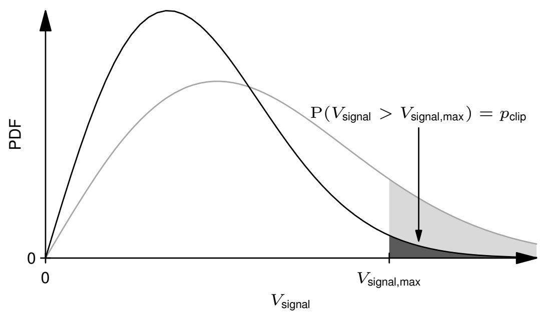

Fig. 8. Influence of the parameter of the Rayleigh distribution on the probability density function of the voltage envelope for a high (gray) and a low (black) probability of clipping, respectively for low and a high ratio between maximum and average output power

The resulting average efficiency versus the probability of clipping can be determined. Thereto, the probability density function of the signal envelope has to be transformed using the relationship between the amplitude and the efficiency derived above to get the probability density function of the efficiency. Afterwards, the average value of the efficiency can be evaluated making use of the transformed density function. The result is depicted in figure 9. The improvement in comparison to a system without operating point adjustment is illustrated in figure 10.

Regarding the clipping probability there are two extreme cases. The first is the operation with 100% clipping, which theoretically is the most efficient because the power amplifier always operates in saturation. Thus, OP adjustment has no benefit, which can be seen in figure 9 where all curves converge to one point. Furthermore, this is not practically relevant because the signal is completly

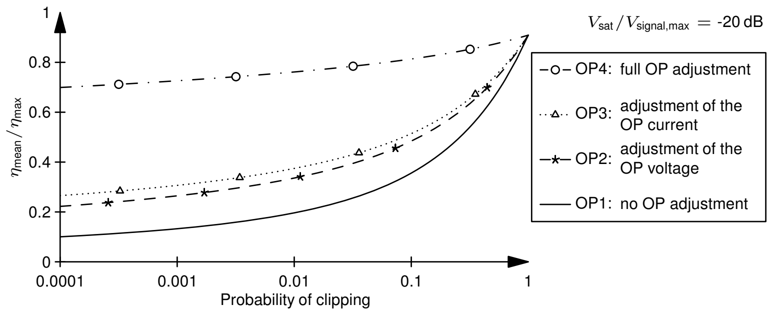

Fig. 9. Average efficiency versus the probability of clipping

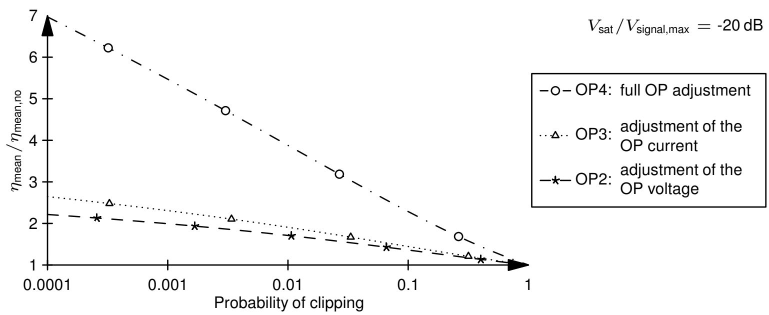

Fig. 10. Benefit of the OP adjustment versus the probability of clipping

distorted which results in a bit error ratio of 0.5. The other extreme case is an operation with almost no clipping, i.e., a very low bit error ratio. In this case the efficiency drops significantly for all kinds of operating point adjustment. For the design of a practical system there is a trade-off between efficiency and bit error ratio.

The designer of a power amplifier needs to know the required maximum output power $P_{\text{out,max}}$. This value can be derived by using the average output power $P_{\text{out,mean}}$ and the probability of clipping p_{clip} which are set by the system designer. Since the power of the signal is exponentially distributed, which was derived in section 3, the probability of clipping is

$$\begin{aligned} p_{\text{clip}} &= \mathrm{P}\left(P_{\text{out,mean}} > P_{\text{out,max}}\right) \\ &= e^{-\frac{P_{\text{out,max}}}{P_{\text{out,mean}}}} = e^{-PAPR_{\text{PA}}} \end{aligned} \tag{19}$$

which can be rearranged to

$$PAPR_{\text{PA}} = -\ln\left(p_{\text{clip}}\right). \tag{20}$$

For typical systems the peak-to-average power ratio $PAPR_{\text{PA}}$, for which the power amplifier is designed, is about 7 dB which is relatively low in comparison to the peak-to-average power ratio of a DVB-T signal of approximately 35 dB an which results in a probability of clipping of around 0.7%.

Due to several drawbacks the adjustment of the OP current is rather difficult. Thus, adjusting only the OP voltage in combination with envelope following is often applied [9]. Even for this case it can be seen that the efficiency of OFDM broadcast systems can be increased by a factor of two.

6 Conclusion

We theoretically analyzed the maximum efficiency of the power amplifier of an OFDM broadcast system with envelope following. Thereto, we investigated the efficiency of OFDM system without operating point adaption, with OP voltage, with operating point current, and with full operating point adjustment. We showed that adapting the operating point is beneficial even for broadcast systems. Thereby, a gain in efficiency by the factor of two for the modulation of a single operating point parameter and a factor of four for the modulation of both operating point parameters can be achieved. Since the calculated relations based on idealized models the gain in efficiency might be lower in practical implementations. Additionally, we showed that there is a trade-off between efficiency and signal quality by means of the probability of clipping and we depicted how the requirements on the power amplifiers are related to the system parameters.

The derivations were done having a broadcast system in mind. But all considerations can also be applied for bidirectional wireless communication systems. Therefore, the results of the statistic investigations on the output power level for the particular standard, which was illustrated for UMTS in figure 1, have to be taken additionally into account.

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