# Performance of MC-DS-CDMA System in Rayleigh Fading Channel with Non-coherent Combining Schemes and MAI Interference

Ismahene Ikhlef[1], Abdellatif Said[2], Faouzi Soltani[1], and Faouzi Bader[3]

[1] University of Constantine,(UoC), Faculty of Engineering, Department of Electronics,
25000-Constantine, Algeria
[2] United Arab Emirates (UAE) University, UGRU- Math/IT,
PO Box 17172;Al Ain;Abu Dhabi;17172;UAE
[3] Centre Technologic de Telecomunicacions de Catalunya-CTTC,
PMT, av. Canal Olimpic s/n, 08860-Barcelona
`r_messas@yahoo.fr,abdellatifs@uaeu.ac.ae`

**Abstract.** In this paper, we analyzed the multi – carrier direct sequence code division multiple access system when the received signal is divided into predetermined spreading sequences, and the sub - bands are modulated over *M* carriers. Because of the unknown phase of the received signal at each sub-band the receiver cannot coherently combine the outputs of the correlators. We consider in this paper non-coherent combining scheme (equal gain combining) with MAI interferences. The calculation of the probabilities of detection and false alarm for both multicarrier (MC) and single carrier (SC) system in a Rayleigh fading channel demonstrated that the performances of the multicarrier system are better than those with single carrier system. Obtained results demonstrated that the suppression (or the mitigation) of appeared interferences is obtained by the increase of the number of system carriers[1].

**Keywords:** CDMA, multi-carrier DS-CDMA, equal gain combining, non-coherent combing scheme.

## 1 Introduction

The problem of allowing multiple users to simultaneously access a channel without causing an undue amount of degradation in the performance of any individual user is a classical problem in communication systems.

The two most common access techniques are; the frequency division multiple access (FDMA), and the time division multiple access (TDMA), which attempt to solve the problem by spreading the signal in frequency and time respectively.

For instance, in the FDMA access scheme the inter-modulation problem will be the most generated difficulty. However, In the TDMA access scheme the inter-modulation

---

[1] This work has been partially carried out in the framework of the European Celtic project MOBILIA (CP5-016), and in the Spanish National project TSI-020400-2008-82, and partially supported by the project PHYDYAS/FP7-ICT-2007-1-211887

P. Chatzimisios et al. (Eds.): MobiLight 2010, LNICST 45, pp. 1–10, 2010.

problem does not exist, but an accurate synchronization of all the users becomes of paramount importance during the system design.

Furthermore, whether an interference or multipath effect is present (which will be the case in this proposal), degradation in system performance can result in either FDMA or TDMA [1] multiple access schemes. The code division multiple access (CDMA) system seems more suitable in such adverse environments as it offers more robustness against the effects of the multi-user interference degradation. For that reason the CDMA is still considered as a potential candidate for future generation communications system. [1].

In this proposal the authors propose a multi-carrier direct sequence (MC-DS) CDMA system transmitting over a Rayleigh fading channel. The MC-DS-CDMA system has free interference capability which could be achieved among other techniques by correlating the received signal with predetermined spreading sequences. Thus allowing that the inherent processing gain of the system attenuating the interference effects [2], [3]. For that, the use of a chip waveform filter is necessary to reject the narrow band and/or the self interferences.

The serial acquisition performance of a MC-DS-CDMA system has been widely investigated in [4], where each subcarrier signal is subject to frequency-selective fading. The performance of the parallel code acquisition of a MC-DS-CDMA system has been analyzed in [5] for the case where each subcarrier signal is subject to a frequency-non-selective, and modeled as fading or non-fading (with or without partial band interference by computing the probability of error).

In this work, we consider the MC-DS-CDMA system and we study the probabilities of detection and false alarm, we also consider the single carrier SC-CDMA systems in presence of additive white Gaussian noise (AWGN) and multiple access interference (MAI) transmitting over a Rayleigh fading channel with equal gain combining (EGC) equalization. This paper is organized as follows: Section 2 introduces the system model for the proposed multicarrier DS-CDMA system. The statistics of the outputs of the non-coherent correlators and combiners for Rayleigh fading channel are presented in section 3 and 4, as well as the calculation of the probability of detection and false alarm. In section 5, we compared the numerical results for both multicarrier and single carrier CDMA systems using the proposed equal gain combining. Finally, we provide our conclusions in section 6.

## 2 System Description

### 2.1 Transmitter

The block chain of the *k-th* active user's transmitter using CDMA scheme is depicted in Figure 1, where the data $d\left(n/N_D\right)^{(k)}$ is generated as a random binary sequence, where $c_n^{(k)}$ is a pseudo – random (also named as Pseudo Noise-PN) spreading signature sequence with $n=\{1,...N_D\}$, $N_D$ means the spreading gain. We assume that each signature has $N_D$ chips of duration $T_c$ per symbol which represent also the spreading gain code; the $d\left(n/N_D\right)^{(k)}c_n^{(k)}$ sequence is modulated by an impulse train with energy per chips equal to $E_c$. The transmitted signal at the output of the chip wave – shaping filter modulates $M$ subcarriers. The energy per-chip of the modulating signal over each carrier is $\frac{E_c}{M}$. Finally, the spectral spreading is imposed on the complex signal by multiplying it with a spreading code. Therefore, the transmitted signal of the $k$-th user [5]- [6]-[7] has the following expression

$$S_k(t)=\sqrt{\frac{2E_c}{M}}\sum_{n=-\infty}^{+\infty} d_{\left(n/N_D\right)}{}^{(k)} c_n{}^{(k)} h(t-nMT_c)\sum_{m=1}^{M}\cos(\omega_m t+\theta_{k,m}) \tag{1}$$

Where *h(t)* means the impulse response of the chip wave- shaping filter, and $\theta_{k,m}$ is a random phase uniformly distributed over [0, 2π] range. In MC-DS-CDMA systems the modulated subcarriers are orthogonal over the chip duration. Hence, the frequency corresponding to the $m$-th subcarrier is $f_m = f_p + m/T_c$, where $f_p$ is the fundamental carrier frequency.

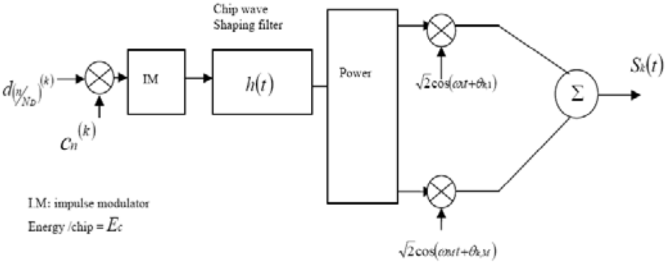

**Fig. 1.** The schematic blocs of the MC-DS-CDMA transmitter

## 2.2 The Channel

The assumed channel is a slowly varying frequency selective Rayleigh channel with a transfer function of the frequency band of the *k-th* user equal to $\alpha_{k,m}\exp(j\beta_{k,m})$, where $\alpha_{k,m}$ and $\beta_{k,m}$ are independent and identically distributed (i.i.d) Rayleigh random variables with a unit second moment, and uniform random variables over [0, 2π] respectively. Consequently, the received signal can be written as [5]:

$$r(t)=\sum_{k=1}^{K}\sqrt{\frac{2E_c}{M}}\left\{\sum_{n=-\infty}^{+\infty} d_{(n+D_k)/N_D}{}^{(k)} c_{(n+D_k)}{}^{(k)} h(t-nMT_c-\zeta_k)\right. \\ \left.\sum_{m=1}^{M}\alpha_{k,m}\cos(\omega_m t+\theta'_{k,m})\right\}+n_w(t) \tag{2}$$

We suppose in our proposal free partial band interference (PBI), $n_w(t)$ means the additive white Gaussian noise (AWGN) with a double side power spectral density (PSD) of $\eta_0/2$ and $\theta'_{k,m}=\theta_{k,m}+\beta_{k,m}$ is uniformly distributed over [0,2π], $D_k$ is the code chip phase sequence of the *k-th* user, $\zeta_k$ means the unknown offset time defined as $\zeta_k = D_k M T_c + \tau_k$. Here $\tau_k$ is the unknown chip delay assumed to be uniformly distributed over $[0, MT_c]$.

### 2.3 The Receiver

The block diagram of the receiver of the $k$-th user is shown in both; Figure 2 and, where there is an in-phase (I) and quadratic (Q) correlators for each one of the $M$ subcarriers. Each one of the in-phase and quadratic non-coherent correlator is composed by a coherent demodulator and a low pass filter (LPF) such that each double frequency terms can be ignored [5] (see Figure 2). The outputs of the low pass filter are sampled and despreaded by a local PN code replica. After the correlation process, the output is squared by a quadratic envelope detector and their outputs are summed in both branches; I and Q. The signal summation results passes through an adaptive gain amplifier for each of the $M$ non-coherent correlators to produce the decision component within the symbol combiner (see Figure 3). The authors propose to handle the acquisition process by means of a sequential method where the incoming signal is correlated serially with all the possible phases of the local PN code replica. The result of this operation is stored within the symbol combiner. We assume that the chip wave-shaping filter has the following characteristics: $X(f) \equiv |H(f)|^2$ satisfied, the Nyquist criterion, $\int_{-\infty}^{+\infty} X(f) \equiv 1$, and $X(f)$ is band limited to whole $W'$ system bandwidth such that $W' \leq \frac{f_{m+1} - f_m}{2}$. These imply that the direct spread waveform does not overlap the adjacent bands and therefore we have free adjacent channel interferences (CI) [5]. To ensure the above filter characteristics, $X(f)$ (see Figure 3) is a raised-cosine filter with the characteristic shown in (3) [8]-[9].

$$X(f) = \begin{cases} \frac{1}{W'} & |f| \leq \frac{W'}{2}(1-\alpha) \\ \frac{1}{2W'}\left\{1 - \sin\left[\frac{1}{2\alpha}\left(\frac{2\pi|f|}{W'} - \pi\right)\right]\right\} & \frac{W'}{2}(1-\alpha) \leq |f| \leq \frac{W'}{2}(1+\alpha) \\ 0 \quad elsewhere & \end{cases} \tag{3}$$

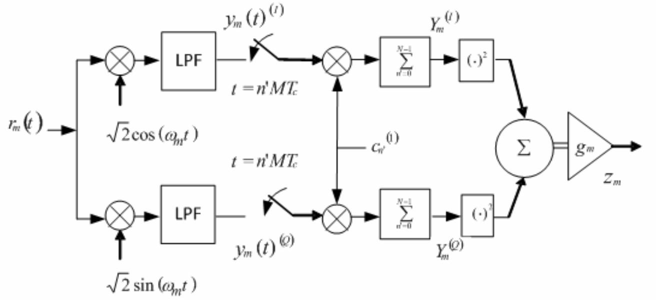

**Fig. 2.** The non-coherent correlator (I,Q) for the each *m-th* subcarrier

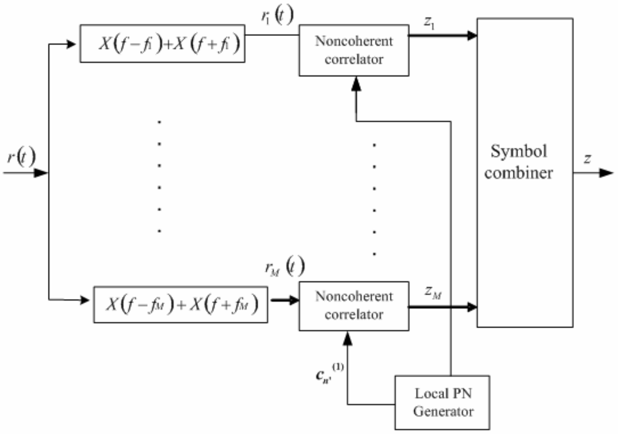

**Fig. 3.** The receiver of the multi-carrier DS – CDMA system

# 3 The Statistics of the Outputs of the Non-coherent Correlator and Combiner

## 3.1 The Statistics of the Non-coherent Correlator

We assume that during the acquisition process no data are transmitted i.e. $d_{(n+D_1)/N_D}^{(1)}=1$, and the outputs of the non-coherent correlator of the *m*-th branch are given by

$$z_m = g_m^{-1}\left\{ \left(Y_m^{(I)}\right)^2 + \left(Y_m^{(Q)}\right)^2 \right\} \tag{4}$$

Where $g_m^{-1}$ represent the adaptive gain control (AGC) over the *m*-th branch assumed perfectly estimated from the average received power and is given by:

$$g_m = \overline{y_m^{(I)}(n'MT_c)^2 + y_m^{(Q)}(n'MT_c)^2} \tag{5}$$

Where $\overline{(.)}$, means the average calculation. The terms $\overline{y_m^{(I)}(n'MT_c)^2}$ and $\overline{y_m^{(Q)}(n'MT_c)^2}$ are; the in phase and the quadratic sampled signals at rate ($n'MT$) respectively (see ) and are equal to:

$$y_m^{(I)} = \alpha_{1,m} S(\zeta_1)\cos\left(\theta'_{1,m}\right) + I_{y_m}^{(I)} + N_{y_m}^{(I)}$$

$$y_m^{(Q)} = \alpha_{1,m} S(\zeta_1)\sin\left(\theta'_{1,m}\right) + I_{y_m}^{(Q)} + N_{y_m}^{(Q)} \tag{6}$$

Where $s(\zeta_1)=\sqrt{E_c/M}\sum_{n'=0}^{N-1}\sum_{n=-\infty}^{+\infty}c_{n'}{}^{(1)}c_{n+D_1}\;x[(n'-n)MT_c-\zeta_1]$ represent the desired detected signal, $I_{y_m}{}^{(I)}$ and $I_{y_m}{}^{(Q)}$ are the interference parts in each of the in phase and quadratic branches respectively. And similar to the interference parts, we have $N_{y_m}{}^{(I)}$ and $N_{y_m}{}^{(Q)}$ which are the experienced noise in the in-phase and the quadratic signals respectively. For the Rayleigh fading channel, the conditional probability density function (p.d.f) on $H_i$ ($i$=0,1) of $z_m$ is given by [5]:

$$p\left(z_m/H_i\right)=\frac{1}{2v'_{m,i}}\exp\left(-\frac{z_m}{2v'_{m,i}}\right)$$
$$where \quad v'_{m,i}=v_{m,i}+\frac{m'^{\,2}_{m,i}}{2} \tag{7}$$

### 3.2 The Outputs of the Non-coherent Combiner

In this paper, we consider one kind of the non-coherent combiner scheme.

#### 3.2.1 Equal Gain Combining (EGC)

In this kind of the symbol combining the output is given by [5]-[10]:

$$z=\sum_{m=1}^{M}z_m \tag{8}$$

For Rayleigh fading channel the characteristic function conditioned on $H_i$ of $z$ in (8) is the product of the all characteristics functions of $z_m$ and is given by:

$$\Psi_{\left(z/H_1\right)}(s)=\prod_{m=1}^{M}\Psi_{\left(z_m/H_i\right)}(s) \tag{9}$$

Where $\Psi_{\left(z_m/H_i\right)}(s)$ is the Fourier transform of the probability density function (p.d.f) of $z_m$ which is obtained from (7). For a Rayleigh fading channel the characteristic function is expressed as

$$\psi_{\left(z_m/H_i\right)}(s)=\frac{1}{1-2v'_{m,i}\,s} \tag{10}$$

The probability density function of z is the inverse Fourier transform of its characteristic function from Cauchy residue theorem; we obtain the p.d.f. function as

$$p\left( z/H_i \right) = \frac{z^{M-1} \exp\left( -\frac{z}{2v'_{m,i}} \right)}{\left(2v'_{m,i}\right)^M Fact(M-1)} \tag{11}$$

The value $2v'_{m,i}$ is hereafter defined.

## 4 Calculation of the Probability of Detection and False Alarm

### 4.1 Probability of Detection

The probability of detection $P_{Dfe}$ is provided by the integration of the probability density function of $z$ in (11) over the normalized threshold $\gamma^*$ ($\gamma^* = \gamma/N$) and the correlation period $N$ [11]. Therefore we obtain the final expression of the probability of detection as

$$p_{Dfe} = \sum_{m=0}^{M-1} \left( \frac{\gamma^*}{(B+a)} \right)^m \frac{\exp\left( \frac{\gamma^*}{(B+a)} \right)}{Fact(m)} \tag{12}$$

With $2v'_{m;i} = N\,(B+a)$, where $B$ and $a$ are calculated as the following

$$B = \frac{\left(1+(K-1)\,v\,\left(1-\alpha/4\right)\right)}{\left(1+K\,v\left(1-\alpha/4\right)\right)}$$

$$a = \frac{N\,v}{\left(1+K\,v\left(1-\alpha/4\right)\right)}$$

$$v = \frac{E_{c1}}{\eta_0} \; with \; E_{c1} = \frac{E_c}{M}$$

Where $\alpha$ means here the roll-off factor.

### 4.2 Probability of False Alarm

In a similar manner of calculation of $p_{Dfe}$, we can obtain the false alarm probability $p_{Fafe}$ as:

$$p_{Fafe} = \sum_{m=0}^{M-1} \left(\gamma^*\right)^m \frac{\exp\left(\gamma^*\right)}{Fact(m)} \tag{13}$$

$$with \;\; v'_{m,0} = v_{m,0} + \frac{m'^{\,2}_{m,0}}{2}$$

## 5 Numerical Results

In this section, we make a comparison between the probability of detection and false alarm in Rayleigh Fading channel with no- partial band interference for both multi-carrier and single-carrier systems. The period of correlation of the single carrier has been chosen equal to $N_1=512$, but for the multi-carrier $N= N_1 /M$, and a roll-off factor $\alpha$ of the raised-cosine filter equal to 0.5 [5]-[8]-[9].

The expression of the probability of detection and false alarm contain too many parameters. Simulation results are obtained by MATLAB software, and by applying linear interpolation over the different points.

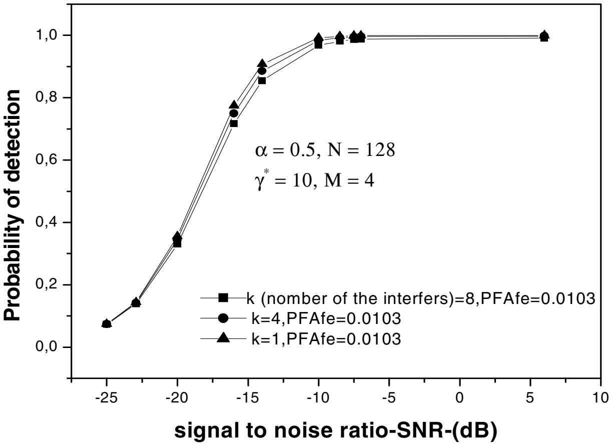

**Fig. 2.** Probability of detection for Rayleigh fading channel for a multi-carrier system ($M\geq2$), with equal gain combining and using different values of *K*.

The significant inconvenient of the CDMA scheme is the multiple access interference (MAI). The effect of the MAI on both probabilities; of detection and false alarm using equal gain combining is shown in

Figure 2. It can be observed that when the number of interferes (undesirable users) increases, the probability of detection decreases. However the probability of false alarm remains constant, this could be explained by the fact that this probability doesn't depend on the value of *K* (active users).

In Figure 5, we can easily see that the minimization of the MAI interference is given by the increasing of the number of the carriers *M* which allows a very good probability of detection and a constant probability of false alarm. The performance of the multi-carrier system (for $M$=2,4) is better than the performance of the single carrier system (for $M$=1 ) because the probability of detection of the first system is greater than the probability of detection of the second system with a low probability of false alarm. Finally, in order to minimize the MAI interference, the use of the equal gain combining in MC-DS-CDMA is more than recommended.

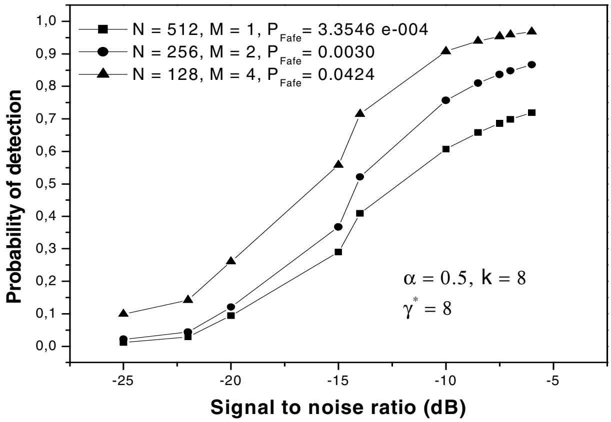

**Fig. 3.** Probability of detection for Rayleigh Fading channel for both multi-carrier (($M\geq2$)) and single carrier ($M$=1) systems with the equal gain combining for different values of $M$, and number of interferes ($K$=8)

## 6 Conclusions

In this paper, we have analyzed the type of combination having an equal gain combining equalization in multicarrier and single carrier systems over a Rayleigh fading channel. We know that interferes (the undesirable users) represent the MAI interference; the results demonstrate that the increase of M minimizes this kind of interference. In the Rayleigh Fading channel, the performance of the multi-carrier system is better than the single carrier system for the equal gain combining.

Based on the obtained results, we recommend the use of the proposed equal gain combiner for multi-carrier CDMA system in the Rayleigh fading channel, especially when the MAI exists in order to minimize it. The increase of the number of subcarrier helps such mitigation.

## References

1. Cook, C.E., Ellersick, F.W., Milstein, L.B., Schilling, D.L.: Spread-Spectrum Communications. IEEE Press, Inc., New York (copyright 1983)
2. Rowitch, D.N., Milstein, L.B.: Convolutional coded multicarrier DS-CDMA in-a multipath channel-part I. IEEE Trans. Commun. 47(10), 1570–1582 (1999)
3. Pickholtz, R.L., Schilling, D.L., Milstein, L.B.: Theory of spread spectrum communications-A tutorial. IEEE Trans. Commun. COM-30(5) (May 1982)
4. Yang, L.L., Hanzo, L.: Serial acquisition performance of single-carrier and multicarrier DS-CDMA over Nakagami-m Fading channels. IEEE. Trans. on Commun. 1(4), 692–702 (2002)
5. Lee, D., Milstein, L.B., Lee, H.: Analysis of multicarrier DS-CDMA code–acquisition system. IEEE Trans. Commun. 47(8), 1233–1243 (1999)

6. Kondo, S., Milstein, L.B.: On the performance of multicarrier DS-CDMA systems. IEEE Trans. Commun. 44, 238–246 (1996)
7. Bingham, J.: Multicarrier modulation for data transmission: An idea whorse time has come. IEEE Commun. Mag. (May 1990)
8. Wong, T.F., Lok, T.M., Lehnert, J.S.: Asynchronous multiple-access interference suppression and chip waveform selection with aperiodic random sequences. IEEE Trans. Commun. 47(1), 103–114 (1999)
9. Nguyen, H.H.: Effect of chip waveform shaping on the performance of multicarrier CDMA systems. IEEE Transactions on vehicular technology 54(3), 1022–1029 (2005)
10. Coulon, F.: Theory and treatment of signals. Edition Georgi. Dunod, Paris (1984)
11. Viterbi, A.J.: CDMA principles of spread spectrum communication. Addison Wesley Longman, Inc., Amsterdam (copyright 1995)