Efficient Long Range Communication by Quantum Injected Optical Parametric Amplification

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Abstract. Free-space optical communications over long distances are associated with severe losses while the natural limit for the energy associated to a single bit of information is just one photon. In order to enhance the transmission Efficiency we propose the use of macro qubits consisting of thousand of photons. We investigate the Fidelity of the transmission of a macro-qubit generated by quantum injected optical parametric amplification (QI-OPA) along a lossy communication channel. The realization of a QI-OPA micro-macro Teleportation protocol is considered.

Keywords: Quantum Communication, Quantum Teleportation, Optical Parametric Amplifier.

1 Transmission of Amplified Quantum States over a Lossy Channel

The transmission of quantum states between two or several space-like separated communication stations is an important task for implementing relevant quantum Information, quantum Cryptographic and Teleportation protocols. These ones generally involve the distribution between distant parties of entangled states [1] whose Fidelity is impaired by noise and by losses along the quantum channels, e.g. contributed by the dark counts of the detectors and by the absorptive losses in optical fibers. In particular, the communication resources provided by the present technology limit the distance for faithful entanglement distribution to the order of $100km$ [2]. In order to overcome these problems it has been suggested the use of quantum repeaters, a new technology which still needs significant developments, or the adoption of free-space links [3,4]. Recently the demonstration of the successful transmission of an entangled photon pair over a $144Km$ free-space link has been reported [5]. It has been shown that, even thought the presence of extreme attenuation due to turbulent atmospheric effects, the free-space

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transmission preserves the fidelity of the entangled photon pairs. However the attenuations along the communication channel due to diffraction, atmospheric absorption, turbulence and to device imperfections led to a link Efficiency for single photon communication equal to about to -30 dB.

In this work we propose a relevant improvement of that communication scenario by exploiting the transmission of a macro-qubit produced by a Quantum-Injected Optical Parametric Amplifier (QI-OPA) [6,7] over a lossy communication channel. We expect to attain a large enhancement of the communication Efficiency over the single photon transmission. Luckily enough the QI-OPA generated multiphoton state, consisting of thousands of photons depending on the non - linear (NL) *exponential amplification gain* (g) , has been proved to be a *unitary* and *information preserving* process that keeps all quantum properties of the injected qubit [8]. Furthermore, it has been proved to be quite resilient to decoherence and losses [9]. More specifically, when a *polarization* qubit $|\pm\rangle = 2^{-1/2}(|H\rangle \pm |V\rangle)$, being $|H\rangle$ and $|V\rangle$ the horizontal and vertical photon polarization states, are injected into the QI-OPA amplifier, the output state is expressed as:

$$\left|\Phi^{\pm}\right\rangle = \sum_{i,j=0}^{\infty} \gamma_{ij} \left|(2i+1)\pm,(2j)\mp\right\rangle \tag{1}$$

where $\gamma_{ij} \equiv \sqrt{(1+2i)!(2j)!}(i!j!)^{-1}C^{-2}(-\frac{\Gamma}{2})^{j}\frac{\Gamma}{2}^{i}$, $C \equiv \cosh g$, $\Gamma \equiv \tanh g$, being $g = \chi t$ the NL gain. There $|p+,q-\rangle$ stands for a state with p photons with polarization $\overrightarrow{\pi}_{+}$ and q photons with $\overrightarrow{\pi}_{-}$. The macro-states $|\Phi^{+}\rangle$, $|\Phi^{-}\rangle$ are orthonormal, i.e. $\left\langle {}^{i}\Phi|\Phi^{j}\right\rangle = \delta_{ij}$. Our goal is to investigate whether and how the distinguishability between the two states is modified after the transmission over a lossy channel.

Let us consider the macro-qubit $|\Phi^{+}\rangle$, obtained by the amplification of a single photon qubit state $|+\rangle$:

$$\begin{aligned} |\Phi^{+}\rangle &= \frac{1}{C^2}\sum_{ij}^{\infty}\left(\frac{-\Gamma}{2}\right)^{j}\left(\frac{\Gamma}{2}\right)^{i}\frac{\sqrt{(2j)!(2i+1)!}}{j!i!}|(2i+1)+,2j-\rangle_{b} \\ &= \frac{1}{C^2}\sum_{ij}^{\infty}\left(\frac{-\Gamma}{2}\right)^{j}\left(\frac{\Gamma}{2}\right)^{i}\frac{(\mathbf{b}_{+}^{\dagger})^{\mathbf{2i+1}}}{j!}\frac{(\mathbf{b}_{-}^{\dagger})^{\mathbf{2j}}}{i!}|0\rangle \end{aligned} \tag{2}$$

We consider the case in which the macro qubit propagates along a noisy channel, on the spatial mode $\mathbf{b}$. The losses are modelled by a beam splitter (BS) with transfer function

$$\mathbf{b}_{\pm} = \sqrt{\tau}\mathbf{c}_{\pm} + i\sqrt{1-\tau}\,\mathbf{d}_{\pm} \tag{3}$$

where $\mathbf{b}$ is the propagation spatial mode of the macro-qubit and the *transmittivity* (τ) represents the *efficiency* of the free-space link, see Fig.1. The macro-qubit state transmitted by BS is found to be:

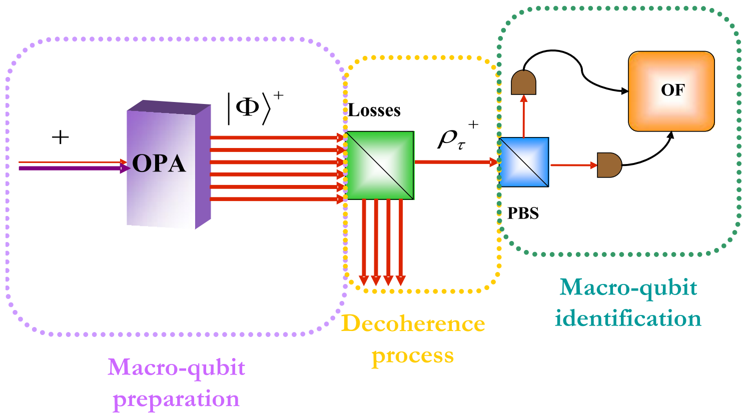

Fig. 1. Conceptual scheme of the proposed experiment: the macro-qubit generated by the QIOPA device undergoes a high losses transmission process, represented by the beam splitter model. The transmitted state is then analyzed and detected through a dichotomic measurement.

$$|\Phi^+\rangle^{out} = \frac{1}{C^2}\sum_{ij}^{\infty}\sum_{k}^{2i+1}\sum_{l}^{2j}\binom{2i+1}{k}\binom{2j}{l}\left(\frac{-\Gamma}{2}\right)^j\left(\frac{\Gamma}{2}\right)^i\frac{1}{j!i!}\left(\sqrt{\tau}\mathbf{c}_+^\dagger\right)^k\left(i\sqrt{1-\tau}\mathbf{d}_+^\dagger\right)^{2i+1-k}$$
$$\left(\sqrt{\tau}\mathbf{c}_-^\dagger\right)^l\left(i\sqrt{1-\tau}\mathbf{d}_-^\dagger\right)^{2j-l}|0\rangle|0\rangle = \frac{1}{C^2}\sum_{ij}^{\infty}\sum_{k}^{2i+1}\sum_{l}^{2j}\left(\frac{-\Gamma}{2}\right)^j\left(\frac{\Gamma}{2}\right)^i\frac{1}{j!}\frac{1}{i!}\frac{\sqrt{\tau}^{k+l}}{\sqrt{k!}\sqrt{l!}}$$
$$\frac{(i\sqrt{1-\tau})^{2i+1+2j-k-l}(2i+1)!(2j)!}{\sqrt{(2i+1-k)!(2j-l)!}}|k+,l-\rangle_d|(2i+1-k)+,(2j-l)-\rangle_c \quad (4)$$

The portion of the state over the spatial mode **d** is lost over the environment while the transmitted state is represented by the density matrix:

$$\rho_\tau^+ = (Tr\rho_{out}^+) = \frac{1}{C^4}\sum_{i,j}^{\infty}\sum_{m,n}^{\infty}\sum_{w=0}^{min\{2i+1,2m+1\}}\sum_{z=0}^{min\{2j,2n\}}(-1)^{j+n}\frac{\Gamma}{2}^{j+n+i+m}\frac{(2i+1)!(2j)!}{j!i!}$$
$$\frac{(2m+1)!(2n)!}{m!n!}\frac{\tau^{i+j+m+n+1-w-z}(1-\tau)^{w+z}}{\sqrt{(2i+1-w)!(2j-z)!}\sqrt{(2m+1-w)!(2n-z)!}}\frac{1}{w!z!}$$
$$|2i+1-w,2j-z\rangle\langle 2m+1-w,2n-z| \quad (5)$$

The measurement of the macro qubit is then realized via a dichotomic strategy involving the action of the O-Filter (OF) device, first introduced by Ref.[8]. We briefly summarize the details of the macro-qubit detection process: the multiphoton state is analyzed in polarization and detected by two photomultipliers. The two intensity signals, proportional to the orthogonally polarized photon numbers n and m, are compared shot-by-shot by the OF electronic device whose filtering action is outlined as follows. When the number of detected photons m_φ bearing the $\boldsymbol{\pi}_\varphi$ polarization, exceeds $n_{\varphi_\perp}$, bearing the $\boldsymbol{\pi}_{\varphi\perp}$ polarization orthogonal to

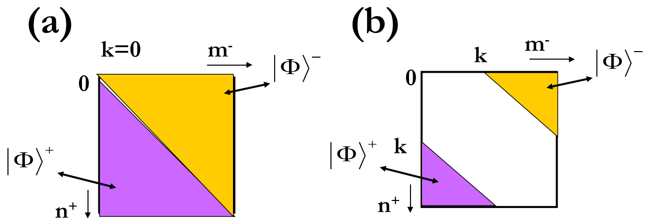

Fig. 2. Macro state identification: (a) If the O-Filter threshold is equal to zero, the measurement on the macro-qubit turns out to be purely dichotomic. (b) For a non-zero k value, the measurement performed on the macro state involves the presence of inconclusive results, that correspond to the non-unbalanced detected pulses.

$\boldsymbol{\pi}_\varphi$, by a ceratin adjustable threshold quantity k, i.e. $m_\varphi - n_{\varphi_\perp} > k$, the (+1) outcome is assigned to the event and the detection of the macro-state $|\Phi^\varphi\rangle$ is assumed. On the contrary, when the condition $n_{\varphi_\perp} - m_\varphi > k$ is met, the (-1) outcome is assigned and detection of $|\Phi^{\varphi\perp}\rangle$ is assumed. Finally, an inconclusive result (0) is obtained when the detected pulses are balanced: $|n_{\varphi_\perp} - m_\varphi| < k$.

The OF based measurement performed on the macro state $|\Phi^+\rangle$, after transmission over the high losses channel, can be represented by the following operator:

$$R^\pm(k) = \sum_{n=k}^{\infty} \sum_{m=0}^{k} |n+, m-\rangle\langle n+, m-| \tag{6}$$

We define the $k-$dependent Visibility $V(k)$ of the macro-state, i.e. the efficiency in discriminating the orthogonal macro-qubits, as:

$$V(k) = \frac{R_{max}(k) - R_{min}(k)}{R_{max}(k) + R_{min}(k)} \tag{7}$$

where $R_{max} = \left\langle R^{+1}(k) \right\rangle_{\rho_\tau^+}$ represent the probability of identifying the macro state after the transmission as $|\Phi^+\rangle$, and $R_{min} = \left\langle R^{-1}(k) \right\rangle_{\rho_\tau^+}$ correspond to the probability of identifying the macro state as $|\Phi^-\rangle$. By increasing the value of k a better discrimination, and hence a higher Visibility can be achieved. The Visibility is related to the Fidelity of the macro state by the relation [10]:

$$F(k) = \frac{V(k) + 1}{2} \tag{8}$$

We are now interested in measuring the macro qubit in a dichotomic way by no O-Filtering, i.e. by choosing the threshold: $k = 0$. In these conditions let's consider the value of the obtained Fidelity F as a function of the non linear gain g of the amplifier. We report in fig.3 the trend of the Fidelity for different values of the transmission Efficiency of the communication channel. By plotting the value of $F(n)$ as a function of the average number of transmitted photons,

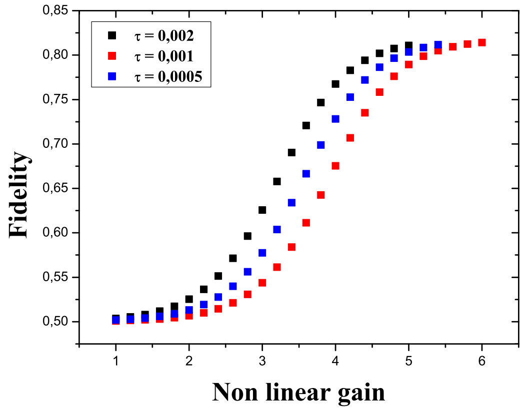

Fig. 3. Trend of the visibility as a function of the non linear gain of the amplifier. We observe that, by fixing the transmission efficiency, the value of the fidelity increases by increasing the gain of the amplifier. These trends are reported for different value of the transmission efficiency.

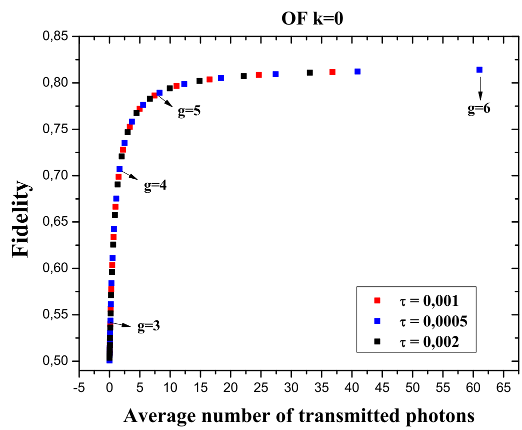

Fig. 4. Trend of the fidelity as a function of the average transmitted photons

we obtain the plot in fig.4, by which we deduce that the asymptotic Fidelity value that can be reached through a dichotomic measurement with: $k = 0$ is $F \sim 0.82$ corresponding to an asymptotic Visibility $V \sim 0.64$. The asymptotic value F is determined by the ratio between the number of detected photons that are correctly attributed to the corresponding macrostate and the number of photons that, still belonging to the same macrostate, are interpreted incorrectly. Of course, by increasing the value of k a large Efficiency and Visibility are

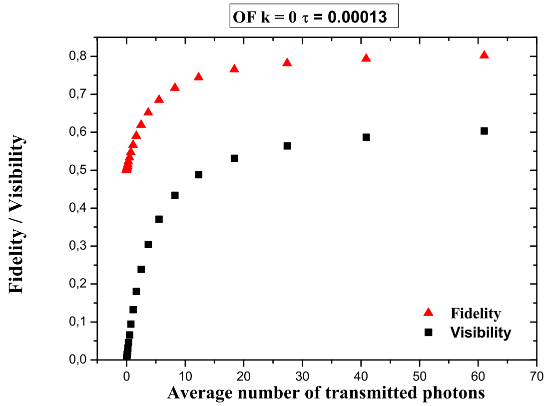

Fig. 5. Trend of the fidelity and visibility as a function of the transmitted photons

attained at the cost of increasing the probability of inconclusive results, i.e. of decreasing the overall measurement Efficiency (QE) of the apparatus.

Similar results are obtained for more realistic simulation parameters as, for instance, by considering a $-30dB$ loss of the transmission channel (as the one presented in Ref.[3]). In that case, by taking into account the reduced quantum efficiency ($QE_p \sim 0.13$) of the IR photomultipliers used to detect the macro-states we may consider the overall efficiency of the transmission process $\tau = 0.00013$. The related trends of the Fidelity and Visibility in these conditions are reported by Fig.5. Our results show that, respect to the single - photon qubit state, the transmission of the macro-qubit over a lossy channel allows to increase the Efficiency of the transmission process to its maximum value for every value of the "channel efficiency", i.e. of the model BS transmittivity τ (see Fig.6). This can be easily obtained by properly increasing the value of the exponential gain g respect to the value of τ. This can be easily done by increasing the power of the pump laser of the QI-OPA system.

2 Micro-Macro Teleportation

As already pointed out, the asymptotic values of Fidelity and Visibility shown by Figures 4 and 5 would hardly allow to perform non locality tests, as required in many QKD applications. As said, increasing values of $F(k)$ and $V(k)$, together with and a decreasing value of QE, can be attained by increasing the value of the threshold $k > 0$. This is shown by fig.7 in which is reported the value of the Fidelity as a function of the OF threshold k (τ and g fixed).

In any case, the reported values of F and V obtained with $k = 0$ are sufficient to implement different quantum information protocols, such as the Teleportation. As it is well known, the latter protocol introduced by Bennett et al. in Ref.[11],

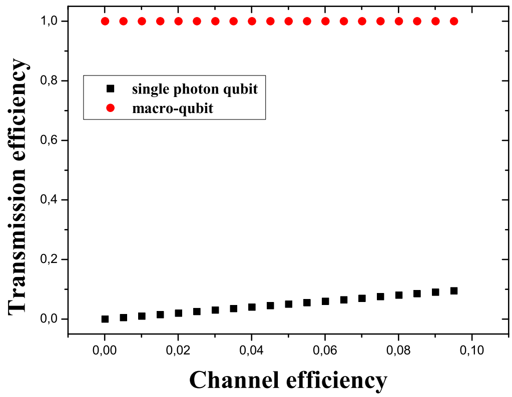

Fig. 6. Comparison between the transmission efficiency of the single photon qubit and of the macro-qubit state

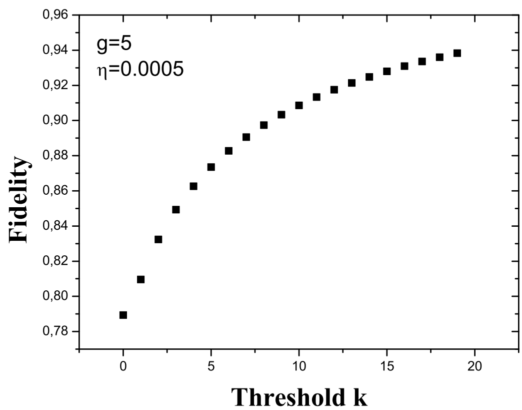

Fig. 7. Trend of the Fidelity as a function of the OF threshold k. A more sophisticated measurement allows to increase the value of the Fidelity, at the cost of decreasing the quantum efficiency of the overall process.

consists of a nonlocal transfert of the state of an unknown qubit onto another far apart particle.

With the present system we could realize the teleportation of a single-photon qubit between the Alice's site and a corresponding photonic macrostate transmitted by a long-range free space link to a Bob's site, Fig.8. More specifically, an EPR source generates a polarization entangled photon pair over the spatial modes $\mathbf{k_A}$ and $\mathbf{k_B}$. The single photon on mode $\mathbf{k_B}$ is amplified by a "phase covariant cloning machine" [7,12].

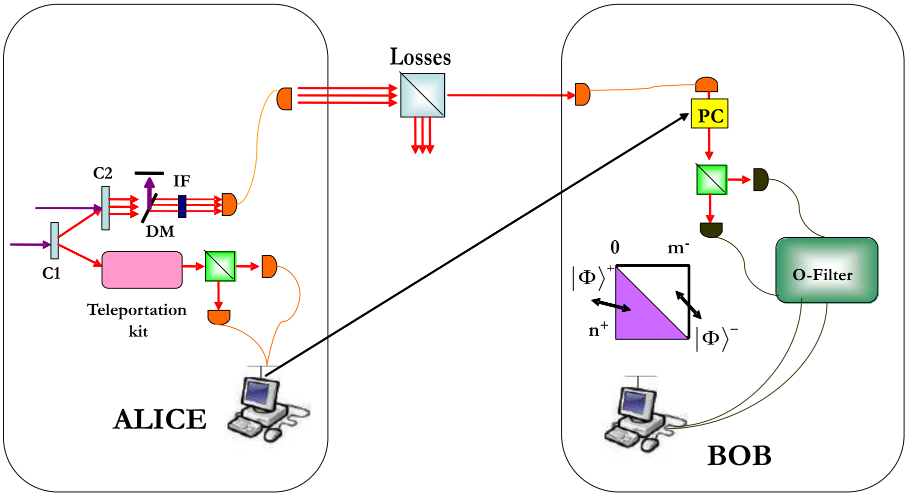

Fig. 8. Experimental scheme for the micro-macroscopic photonic teleportation

At the Alice's site the *Teleportation kit* realizes the conversion of polarization into momentum state, and the qubit to be teleported is encoded into the polarization degree of freedom of the single photon state. The Bell measurement at Alice's site identifies the quantum transformation that will be realized at Bob's site. At last, at Bob's site, a far apart station, the state is analyzed by an O-Filter. By a coincidence procedure between Alice and Bob the success of the Teleportation will be assessed.

Let us analyze in more details the proposed Teleportation experiment. As shown in fig.8 an entangled pair of two photons in the singlet state (9) is produced through spontaneous parametric down-conversion (SPDC) by the NL crystal 1 (C1) pumped by a pulsed UV pump beam:

$$\frac{1}{\sqrt{2}}\left(|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B\right) \tag{9}$$

There the labels A, B refer to particles associated respectively with the spatial modes $\mathbf{k}_A$and $\mathbf{k}_B$. The photon belonging to $\mathbf{k}_B$, together with a strong UV pump beam, is injected into an optical parametric amplifier consisting of a NL crystal 2 (C2) pumped by the beam $\mathbf{k}'_P$. The crystal is oriented for *collinear* operation over the two linear polarization modes, respectively horizontal and vertical. The interaction Hamiltonian of the parametric amplification $\widehat{H} = i\chi\hbar\widehat{a}^{\dagger}_H\widehat{a}^{\dagger}_V + h.c.$ acts on the single spatial mode $\mathbf{k}_B$ where $\widehat{a}^{\dagger}_{\pi}$ is the one photon creation operator associated to the polarization $\overrightarrow{\pi}$ [12]. The overall output state amplified by the OPA apparatus is expressed, in any polarization equatorial basis $\left\{\overrightarrow{\pi}_{\phi} = 2^{-1/2}\left(\overrightarrow{\pi}_{H} + e^{i\phi}\overrightarrow{\pi}_{V}\right), \overrightarrow{\pi}_{\phi\perp} = \overrightarrow{\pi}_{\phi}^{\perp}\right\}$, by the Micro-Macro entangled state:

$$|\Sigma\rangle_{A,B} = 2^{-1/2}\left(\left|\Phi^{\phi}\right\rangle_{B}\left|1\phi^{\perp}\right\rangle_{A} - \left|\Phi^{\phi\perp}\right\rangle_{B}\left|1\phi\right\rangle_{A}\right) \tag{10}$$

where the mutually orthogonal multi-particle "macro-states" are (see eq.1):

$$\left|\Phi^{\phi}\right\rangle_{B} = \sum_{i,j=0}^{\infty} \gamma_{ij} \frac{\sqrt{(1+2i)!(2j)!}}{i!j!} \left|(2i+1)\phi; (2j)\phi^{\perp}\right\rangle_{B}$$
$$\left|\Phi^{\phi\perp}\right\rangle_{B} = \sum_{i,j=0}^{\infty} \gamma_{ij} \frac{\sqrt{(1+2i)!(2j)!}}{i!j!} \left|(2j)\phi; (2i+1)\phi^{\perp}\right\rangle_{B}$$

When an equatorial qubit $|\varphi\rangle = |H\rangle + e^{i\varphi}|V\rangle$ is injected into the amplifier, the ensemble average photon number $N_{\pm}$ on mode $\mathbf{k_B}$ with polarization $\boldsymbol{\pi}_{\pm}$ is found to depend on the phase φ as follows:

$$N_{\pm} = \overline{m} + \frac{1}{2}(2\overline{m} + 1)(1 \pm \cos\varphi) \tag{11}$$

where $\overline{m} = \sinh^2 g$ is the average number of photons emitted by the OPA for each polarization mode in absence of quantum injection.

To show that the Teleportation works for any basis on the "equatorial" Hilbert subspace on the qubit Bloch sphere, we choose the linearly polarized $\{\boldsymbol{\pi}_{+}, \boldsymbol{\pi}_{-}\}$ basis set, and the circularly polarized one $\{\boldsymbol{\pi}_{\boldsymbol{R}}, \boldsymbol{\pi}_{\boldsymbol{L}}\}$set. In order to demonstrate that the experimental results cannot be simulated by a "classical" process, the experimental Teleportation fidelity should be found: [13,14]:

$$\mathcal{F} > \frac{3}{4} \tag{12}$$

The experimental measurement would involve the evaluation of the Visibility by taking into account the coincidences between detectors at Alice's site and the output of the OF device at Bob's site, for different choices of the preparation basis and of the analysis basis. If the Macroscopic state is projected onto the same polarization state as the teleported qubit we would obtain a maximum of coincidences C_{max}. On the contrary when the state is the orthogonal one, we register a minimum of coincidences C_{min}. Hence, in order to demonstrate the teleportation protocol we have to obtain a Visibility as large as:

$$\mathcal{V} = \frac{C_{max} - C_{min}}{C_{max} + C_{min}} > \frac{1}{2} \tag{13}$$

In conclusion, we have analyzed theoretically the transmission of Macro - qubits along lossy channel. We have found that, thanks to the robustness and to the resilience to decoherence of the Macro-state generated by a QI-OPA, a faithful information can be transmitted over a long range channel with an Efficiency nearly equal to one. By adoption of a detection O-Filter with a threshold

$k = 0$ the Fidelity of the state is less than the one achieved by a single photon transmitted state. However, the Fidelity can be largely increased by increasing the value of the OF threshold: $k > 0$.

In summary, we have proposed our Micro - Macro parametric amplification system as an appealing long - range application of the quantum Teleportation protocol. Furthermore, thanks to the high resilience to decoherence of the Macro-qubit, the present scheme can be adopted for general use in free space experiments in an uplink scenario.

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