# Using Redundancy-Free Quantum Channels for Improving the Satellite Communication

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**Abstract.** The quantum based systems could be the next steps in improving the satellite communication. The actual implementation of quantum cryptography systems would be invaluable, allowing for the first time the practical possibility of one-time-pad-encrypted, undecipherable communication, which will offer an essentially new degree of security in future satellite communications. They offer secure key distribution protocols, more efficient coding and communications methods than the classical solutions. However, the classical error coding methods could not be used in a quantum channel, which is required for the quantum communication. There are many quantum error coding algorithms which are based on some redundancy. However, we can construct a channel with zero redundancy error correction. In this paper we introduce three different quantum error correction approaches. The first one is based on eigenvectors and unitary transformations. In the second case we can create a redundancy-free channel using local unitary operation and unitary matrices, while the third one is based on entanglement. All of these can help to set up an efficient quantum channel for the quantum based satellite communication.

**Keywords:** quantum channel, space communication, error correction, redundancy-free.

## 1 Introduction

In last years the quantum theory based on quantum mechanical principles appeared in satellite communication offering answers for some of nowadays' technical questions. The quantum cryptography – cryptography based on quantum theory principles – gives better solutions for communication problems e.g. key distribution than the classical cryptographic methods, which have been found to have vulnerabilities in wired and wireless systems as well. The first *quantum cryptography* protocol, the BB84 [1] was introduced in 1984 and offered a solution for secure key distribution based on quantum theory principles like *no cloning*. The free-space Quantum Key Distribution (QKD) has almost a 20-year-old history. It started with the first published experiment in 1991. The 30 cm long optical path grew to 950 m under nighttime conditions in 1998 [2], and about to 10 km under daylight in 2002 [3], until the distance of 144 km was reached by an international research group in 2006 [4].

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The long distance quantum communication technologies in the future will far exceed the processing capabilities of current silicon-based devices. In current network technology, in order to spread *quantum cryptography*, interfaces able to manage together the quantum and classical channel must be implemented.

In our point of view, the quantum computing algorithms can be used to affirm our free-space communication in the following four ways: [5]

1. *Open-air communication*: we mean usually "horizontal" telecommunication that happens below 100-200 km height. Instead of optical cable air is used for channel.
2. *Earth-satellite communications*: it is usually between 300 and 800 km altitude. Signal encoding and decoding is used to produce quantum error correction that allows operation in noisy environment.
3. *Satellite broadcast*: the broadcast satellite is in orbit at 36,000 km and we want to send data from the satellite to the base stations located on the background. Quantum algorithms can improve the effective bandwidth, thus the brand is better utilized as in traditional cases.
4. *Inter-satellite communication*: the communication between satellites. Any kind of coding and encoding can be used, to increase stability [6].

Free space quantum communication for great distances have been developed and tested successfully. Currently, the quantum cryptographic key generation systems have been realized in metro-area networks over distances on tens kilometres, the free space based QKD solutions can achieve megabit-per-sec data rate communication. Long-distance open-air and satellite quantum communication experiments have been demonstrated the feasibility of extending quantum channel from the ground to a satellite, and in between satellites in free space. The satellite based single photon links already allow QKD on global scale.

## 2 Properties of Quantum Channel

The transmission of classical information over *satellite quantum channel* with no prior entanglement between the sender (Alice) and the recipient (Bob) is illustrated in Fig.1. The sender's classical information denoted by $A_i$ encoded into a quantum state $|\psi_A\rangle$. The encoded quantum states are sent over the satellite quantum channel, In the decoding phase, Bob measures state $|\psi_A\rangle$, the outcome of the measurement is the classical information $B_i$.

A qubit can be described by the two-dimensional Hilbert space $\mathbb{C}^2$, and the operators acting on the quantum system is generated by the Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1}$$

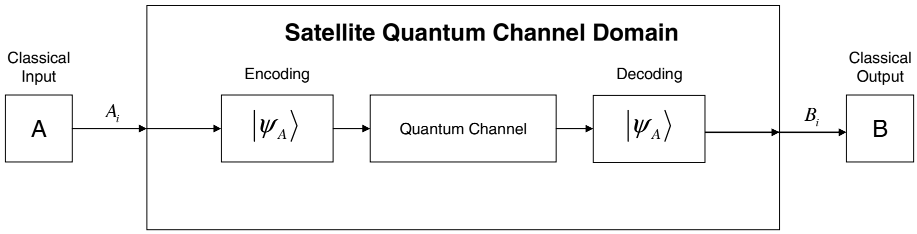


**Fig. 1.** Transmission classical information through the satellite quantum channel

In generally, for a Pauli matrix $\sigma_k$, $Tr(\sigma_k)=0$ and $\sigma_k^2=I$, where $k=x,y,z$. The set of states for a qubit in the computational basis $\{|0\rangle,|1\rangle\}$, is the *eigenbasis* of $\sigma_z$, thus $\sigma_z|0\rangle=|0\rangle$ and $\sigma_z|1\rangle=-|1\rangle$. A generic *pure* state can be given by

$$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle, \tag{2}$$

and the *projector* of the state is $|\psi\rangle\langle\psi|=\frac{1}{2}(\mathbf{1}+\hat{n}\cdot\vec{\sigma})$, where $\hat{n}$ is the *Bloch vector*, and it can be given by $\hat{n}=\left(2\operatorname{Re}\left(\alpha\beta^*\right),2\operatorname{Im}\left(\alpha\beta^*\right),|\alpha|^2-|\beta|^2\right)$. For *pure* state the *norm* of Bloch vector is 1, and these vectors cover the Bloch sphere [1].

The *pure* quantum states of a *two-level* system can be given by unit vectors in spherical coordinates,

$$|\psi\rangle=\cos\frac{\theta}{2}|0\rangle+e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \tag{3}$$

The state $|\psi\rangle$ can be given by state $|+\hat{n}\rangle$, and it is the eigenstate for the eigenvalue *+1* of $\hat{n}\cdot\vec{\sigma}$, with $\hat{n}=\hat{n}(\theta,\varphi)=(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)$ where $\theta\in[0,\pi]$ and $\varphi\in[0,2\pi]$. A $\rho$ *mixed* state can be expressed by a $|\psi\rangle\langle\psi|=\frac{1}{2}(\mathbf{1}+\hat{n}\cdot\vec{\sigma})$ projector on a *pure* quantum state.

The quantum states sent to quantum channel can be represented by their density matrix. We denote by $\mathbf{S}\left(\mathbb{C}^d\right)$ the space of all density matrices of size $d\times d$, and we call it a *d-level* system [1, 7]. A one-qubit system is a *two-level* system [12], and its density matrix can be expressed as

$$\rho=\begin{pmatrix}\frac{1+z}{2} & \frac{x-iy}{2}\\ \frac{x+iy}{2} & \frac{1-z}{2}\end{pmatrix},\ x^2+y^2+z^2\leq 1,\ x,y,z\in\mathbb{R}. \tag{4}$$

Alice's pure quantum state can be expressed by a density matrix $\rho_A$, whose rank is one, while a state with rank two is called *mixed*. According to the noise $\mathcal{N}$ of quantum channel, Alice's sent pure quantum state becomes a mixed state, thus Bob will receive a mixed state denoted by $\sigma_B$. A pure state has special meaning in quantum information theory and it is on the *boundary of the convex object*. A density matrix which is *not pure* is called *mixed state*.

For one-qubit states, the condition for $\rho$ to be pure is simply expressed as $x^2 + y^2 + z^2 = 1$, and it is on the surface of the Bloch ball [8].

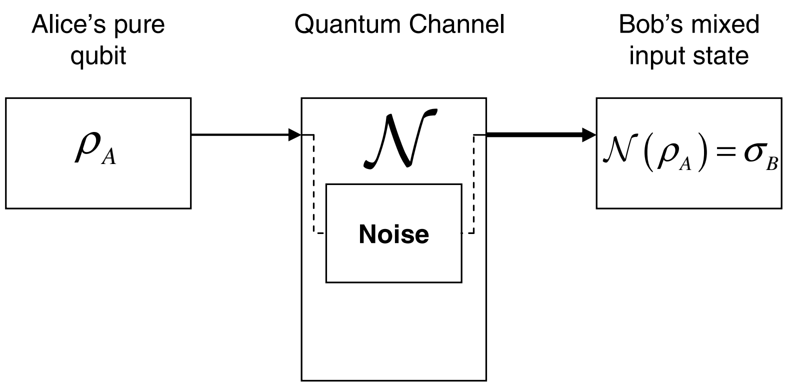


**Fig. 2.** General model of a noisy quantum channel

The map of the quantum channel is a trace-preserving and completely positive map, and it can be given by a linear transform $\mathcal{N}$ which maps quantum states to quantum states. The noise of the channel can be modeled by a linear transform

$$\mathcal{N} : M(\mathbb{C};d) \to M(\mathbb{C};d), \tag{5}$$

where $\mathcal{N}(\rho(\mathbb{C}^d)) \subset \rho(\mathbb{C}^d)$. Thus, if Alice sends quantum state $\rho(x,y,z)$ on the quantum channel, the channel maps it as follows:

$$\left\{(x',y',z') \middle| \rho'(x',y',z') = \mathcal{N}(\rho(x,y,z)),(x,y,z)\right\}. \tag{6}$$

The image of quantum channel's linear transform $\mathcal{N}$ is an ellipsoid. To preserve the condition for density matrix $\rho$, the eavesdropper's cloning transformation $\mathcal{N}$ must be trace-preserving, i.e. $Tr\mathcal{N}(\rho) = Tr(\rho)$, and it must be completely positive, i.e. for any identity map $I$, the map $\mathcal{N} \otimes I$ maps a semi-positive Hermitian matrix into a semi-positive Hermitian matrix. Thus, in our satellite communication based, the channel is modeled by a *TPCP* map [9].

The capacity of the satellite channel $C(\mathcal{N})$ for given noise $\mathcal{N}$, can be defined as follows [9]:

$$C(\mathcal{N}) = \max_{p_1,\ldots,p_n,\rho_1,\ldots,\rho_n} S\left(\mathcal{N}\left(\sum_{i=1}^{n} p_i(\rho_i)\right)\right) + \sum_{i=1}^{n} p_i S(\mathcal{N}(\rho_i)), \tag{7}$$

where $S(\rho) = -\rho \log \rho$ is the von-Neumann entropy.

For a well functioning communication we need a channel coding to handle the errors appearing in a communication channel. In quantum computing the classical error coding methods could not be used. This lead to develop quantum based error correction methods and algorithms. However, they are mostly based on quantum and not classical theorems [10]. In this paper we deal with new three redundancy-free solutions.

## 3 Achieving Redundancy-Free Channel with Eigenvectors

We would like to provide error correction with sending certain amount of qubits over a noisy quantum channel. The qubits are independent, each contains information that needs to be processed. We introduce three different redundancy-free solutions for the quantum communication. In the first one the noise of the quantum channel is modeled by a rotation angle.

The correction of the damaged quantum states is not possible in a classical representation, since the error correction of qubits is realized by unitary rotations.

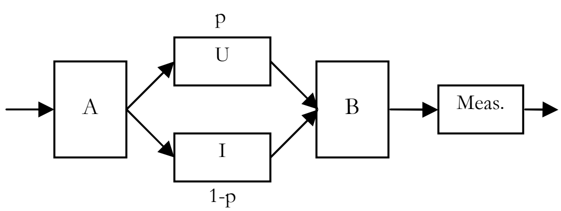


**Fig. 3.** Our channel model. *A* transforms the initial qubits into a special form. B has to produce the inverse of matrix *A*.

Our initial assumption is that the channel rotates the qubit with an $\omega$ degree, that is considered to be constant so far. We wish to create a system where error correction is possible. By this, not a complete restoration is meant. The transmission is considered successful when at the end of the channel the qubit remains in its original state's $\varepsilon$ environment.

To achieve this we mix the qubits and send them over the channel, as shown in Fig 3. What we expect is that at the measurement, the error for one qubit is distributed among the others in its environment (its neighbors). By being so, the error remains in an $\varepsilon$ environment for each qubit.

For the communication we use *n* long qubits so that $2^n = N$, where *n* is the length of the qubits and $N$ is the size of the space. We can construct a classical channel with zero redundancy error correction for any unitary channel, where the information itself is classical, coded into qubits. Let assume that we have a unitary channel with a $U$ unitary transformation, where the $U$ matrix is known. Because $U$ is unitary, it acts on each qubit sent over the channel and changes the qubit. After the successful transmission we have two cases for the eigenvalues

$$\text{I. } U = \begin{bmatrix} e^{j+\alpha} & 0 \\ 0 & e^{j+\alpha} \end{bmatrix} \otimes \begin{bmatrix} e^{j+\alpha} & 0 \\ 0 & e^{j+\alpha} \end{bmatrix} = \begin{bmatrix} e^{j2\alpha} & 0 \\ 0 & e^{j2\alpha} \end{bmatrix} \tag{8}$$

$$\text{II. } U = \begin{bmatrix} e^{j+\alpha} & 0 \\ 0 & e^{j+\alpha} \end{bmatrix} \otimes \begin{bmatrix} e^{j-\alpha} & 0 \\ 0 & e^{j-\alpha} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \tag{9}$$

The description leads to a redundancy-free solution because the classical states are coded into the eigenvectors of the $U$ matrix. With the appropriate selection of the matrix $A$ we can restore one quantum bit sent over the channel without any other (redundant) information. The whole algorithm is described in [11] and [12].

## 4 Redundancy-Free Channel Coding

We consider the redundancy-free implementation of an unitary error correcting operator $\mathcal{R}_\theta$. The protocol achieves the redundancy-free quantum communication using *local unitary operations* and *unitary matrices.*

The whole algorithm is described in [12]. In this paper we introduce how the resources of the error correction could be used and we introduce a novel approach for multi-qubit error correcting.

The *error* of the satellite quantum channel can be modeled by a unitary rotation $\mathcal{R}_\theta^\dagger$, thus the error of the satellite quantum channel can be expressed as an angle $\theta_i \in [0, 2\pi)$. At the beginning of the communication, Alice sends her quantum state $\psi_A$ on the quantum channel, which transforms it to $|d\rangle = \mathcal{R}_\theta^\dagger(\psi_A)$ with given probability *p*. The error of the quantum channel is denoted by $\mathcal{R}_\theta^\dagger$.

In our error-correcting process if Bob tries to read the sent quantum state, he doesn't know the properties of the noise on the quantum channel. In our redundancy-free coding mechanism, Alice's initial state is $\psi_A$, the correction transformation denoted by $\mathcal{R}_\theta$. Bob uses a CNOT to correct the error of the quantum channel. In order to read the sent quantum bits correctly, Bob must rotate the *i*-th data quantum bit by the angle $\theta_i$ in the opposite direction of what the *error of the quantum channel rotated.*

Bob has a chance *not greater* than $\varepsilon = \sin^2(\theta_i)$ to correct the sent states, because he doesn't know the original rotation angle $\theta_i$ of the quantum channel's error on the *i*-th sent qubit. The rotation operation $\mathcal{R}_\theta$ of the error correcting mechanism can be given by the angle $|\theta\rangle$, where

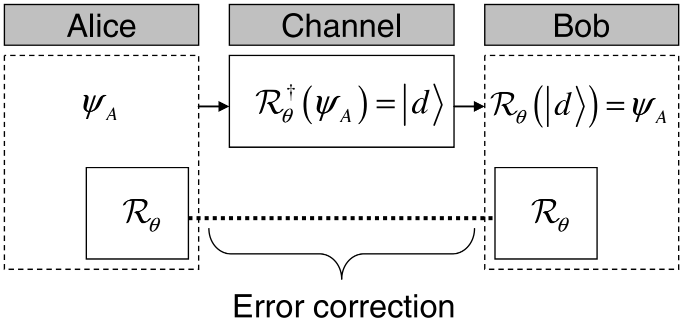


**Fig. 4.** Redundancy free error correction

$$|\theta\rangle = \frac{1}{\sqrt{2}}\left(e^{i\frac{\theta}{2}}|0\rangle + e^{-i\frac{\theta}{2}}|1\rangle\right). \tag{10}$$

The error-correcting method consists of a *control qubit, which* corresponds to the modified qubit $|d\rangle$, and a *target qubit*, which is equal to the *error-correction* angle state $|\theta\rangle$. To correct state $|d\rangle$ to $\psi_A$, Bob uses a simple CNOT transformation, thus our state is transformed to

$$|d\rangle \otimes |\theta\rangle \to \frac{1}{\sqrt{2}}\left(\mathcal{R}_\theta|d\rangle \otimes |0\rangle + \mathcal{R}_\theta^\dagger|d\rangle \otimes |1\rangle\right), \tag{11}$$

and therefore a projective measurement in the $\{|0\rangle, |1\rangle\}$ basis of the correction-state $|\theta\rangle$ will make the modified qubit $|d\rangle$ collapse either into the desired state $\mathcal{R}_\theta|d\rangle$ or into the wrong state $\mathcal{R}_\theta^\dagger|d\rangle$. Bob cannot determine the received state exactly, since he does not know angle of the error $\theta_i$. In this phase, Bob can not be sure whether the *i*-th quantum state $|d_i\rangle$ is identical to the original sent state $|\psi_i\rangle$ or not.

### 4.1 Quantum Probabilistic Channel Decoding

The noise $\mathcal{N}$ on the satellite quantum channel prepares a damaged state $|d_i\rangle = \cos\theta_i'|0\rangle + e^{i\alpha}\sin\theta_i'|1\rangle$. The damaged qubit can be identical to the original qubit $\left(|d_i\rangle = |\psi_i\rangle\right)$, with probability $\cos^2(\theta_i)$, and it differs $\left(|d_i\rangle \neq |\psi_i\rangle\right)$ with probability $\sin^2(\theta_i)$. The damaged state $|d_i\rangle$ can be projected to the original state $|\psi_i\rangle = \cos\theta_i|0\rangle + e^{i\alpha}\sin\theta_i|1\rangle$ successfully with probability

$$p_i = \cos^2\theta_i\cos^2(\theta_i - \theta_i') + \sin^2\theta_i\sin^2(\theta_i + \theta_i'). \tag{12}$$

In our system, the rotation angles are evenly distributed and the *maximum of the average probability* $\overline{p}_i$ can be reached when the angle of state $\theta_i^*$ is equal to zero. Bob gets correct angle $\theta_i$ with probability $p_i = 1 - \frac{1}{2}\sin^2 2\theta_i$, therefore Bob's total probability to receive $n$ quantum states with valid rotation angles $\theta_i$ is $P = \prod_{i=1}^{n}\left(1 - \frac{1}{2}\sin^2 2\theta_i\right)$. As we can conclude, by increasing linearly $n$, Bob's probability of error can be made arbitrarily small.

If we have sent an *N-length* state $\left|\theta^{(n)}\right\rangle$ on the satellite quantum channel, the $n$-qubit length unknown state $\left|\mathcal{R}_\theta^n\right\rangle$ has maximal entropy, thus

$$\int \frac{d\theta}{2\pi}\left|\mathcal{R}_\theta^n\right\rangle\left\langle\mathcal{R}_\theta^n\right| = \left(\frac{\mathbb{I}}{2}\right)^{\otimes n}, \tag{13}$$

where $\mathbb{I}$ is the identify operator. Let assume $p_\theta$ the probability of the successful transformation $\mathcal{R}_\theta\left(\mathcal{R}_\theta^\dagger\left|\psi_A\right\rangle\right) = \left|\psi_A\right\rangle$, which probability is independent of the damaged state $\left|d\right\rangle$.

## 4.2 Resources of Error Correcting

A projective measurement on the basis $\{|0\rangle, |1\rangle\}$ of the error-correcting state will make the damaged qubit collapse either into the desired state $\mathcal{R}_\theta\left|d\right\rangle$ or into the wrong state $\mathcal{R}_\theta^\dagger\left|d\right\rangle$, with each outcome having probability of 1/2. Therefore, Bob applies the gate of Fig.5. to prepare the *bad state* $\mathcal{R}_\theta^\dagger\left|d\right\rangle$ or the right state $\mathcal{R}_\theta\left|d\right\rangle$ with equal probability 1/2.

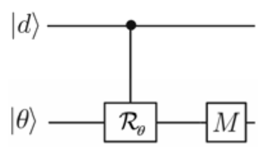


**Fig. 5.** The error correction of a damaged qubit $|d\rangle$ with single-qubit key state $|\theta\rangle$. The angle state is one qubit.

If Bob has an $l$-length qubit string $\otimes_{i=1}^{l}\left|2^{i-1}\theta\right\rangle$ to decode damaged state $\left|d\right\rangle$, Bob's failure probability will be only $\varepsilon = (1/2)^l$. The probability of wrong decoding is decreases exponentially with the size of the $\left|\theta\right\rangle$, the length of the error-correcting string denoted by $l$. Bob takes damaged qubit $\left|d\right\rangle$ as the control-bit, and takes error correcting qubit states $\otimes_{i=1}^{l}\left|2^{i-1}\theta\right\rangle$ as the target, therefore Bob evolves the transformation of $\left|d\right\rangle\otimes_{i=1}^{l}\left|2^{i-1}\theta\right\rangle$ into

$$\frac{1}{\sqrt{2^l}}\left(\sqrt{2^l-1}\,\mathcal{R}_\theta\left|d\right\rangle\otimes\left|right\right\rangle+\mathcal{R}_\theta^{\left(2^l-1\right)\dagger}\left|d\right\rangle\otimes\left|wrong\right\rangle\right), \tag{14}$$

where $\left\langle right\middle|wrong\right\rangle=0.$ The gate for improved decoding is shown in Fig.6.

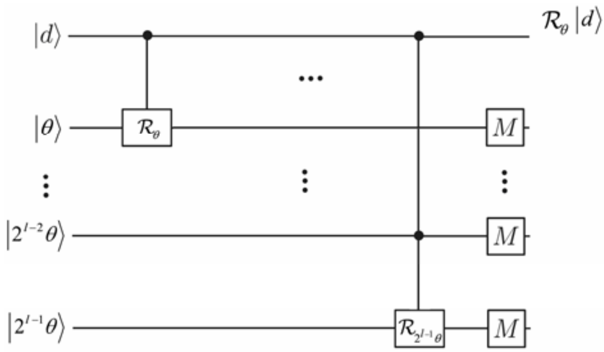


**Fig. 6.** The correction of a damaged state $|d\rangle$ with multi-qubit error-correcting state $\otimes_{i=1}^{l}\left|2^{i-1}\theta\right\rangle$. The error correcting angle is stored in an $l$-length quantum string.

Every rotation transformation $\mathcal{R}_\theta$ succeeds with probability $p=1-(1/2)^l$, with error probability $\varepsilon=(1/2)^l$. In Fig.7. we illustrate the simplified gate for multi-qubit error correction, the short diagonal line on the bottom line represents, that state $\left|\theta\right\rangle$ consists of several quantum bits.

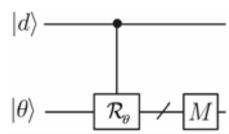


**Fig. 7.** The error correction of a damaged state $|d\rangle$ with state $|\theta\rangle$. The short diagonal line on the bottom line represents, that state $|\theta\rangle$ consists of several quantum bits.

Since Bob has a one-qubit length state for correct the damaged state, Bob fails to perform $\mathcal{R}_\theta\left(\mathcal{R}_\theta^\dagger\left|\psi_A\right\rangle\right)=\left|\psi_A\right\rangle$ with probability $p_1=1/2.$ The average length of the required string is

$$\bar{l}=\sum_{l=1}^{\infty}p_l l=\sum_{l=1}^{\infty}\frac{l}{2^l}=2. \tag{15}$$

Thus, a two qubit error-correcting state for a single-qubit error correction is sufficient, on average.

### 4.3 Multi-qubit Error-Correcting

If Bob receives as $n$-bit length damaged string $|d_1\rangle \otimes \ldots \otimes |d_n\rangle$, he can correct it with the $n$-bit length correction-key $|\theta_1\rangle \otimes |\theta_2\rangle \otimes \ldots \otimes |\theta_n\rangle$ *in one-step*, as it is shown in Fig.8.

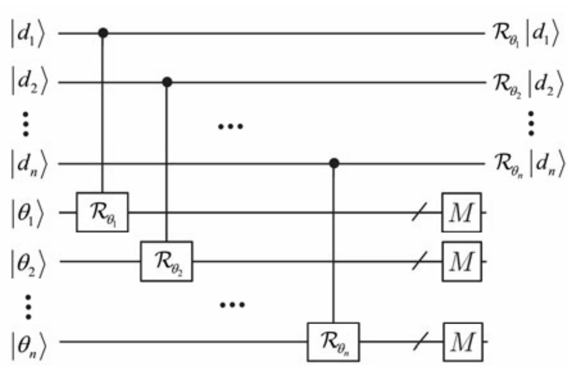


**Fig. 8.** The correction of an $n$-length multi-qubit damaged string $|d_1\rangle \otimes \ldots \otimes |d_n\rangle$ with an $n$-length multi-qubit *string* $|\theta_1\rangle \otimes |\theta_2\rangle \otimes \ldots \otimes |\theta_n\rangle$. The correction state is realized by an $l$-length multi-qubit string $\otimes_{i=1}^{l} |2^{i-1}\theta\rangle$.

Using our method, all the rotation transformations are realized by an $l$-length multi-qubit string, therefore every error correction transformation $\mathcal{R}_{\theta_i}$ on the corresponding damaged qubit $|d_i\rangle$ can be implemented with success probability $1-(1/2)^l$.

## 5 Achieving Redundancy-Free Channel with Entanglement

The entanglement is the ability of qubits to interact over any distance instantaneously. The EPR-states do not exactly communicate, however the results of measurements on each quantum states are correlated. The entangled pure states are those multipartite systems, that cannot be represented in the form of a simple tensor product of subsystem states $|\Psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_n\rangle$, where $|\psi_i\rangle$ are states of local subsystems. An entangled pair is a single quantum system in a superposition of equally possible states, and the entangled state contains no information about the individual particles, only that they are in opposite states. The entangled states cannot be prepared from unentangled states by any sequence of local actions of two distant partners, and the classical communication can not help to generate EPR-states. The Bell states can be defined as $\Psi^{\pm} \equiv \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$ and $\Phi^{\pm} \equiv \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$.

The correction of the damaged state $|d\rangle$ can be realized by a shared EPR-state between Alice and Bob. Our code uses entangled qubits to transfer classical information through the channel. During the communication process Alice encodes the classical information into entangled pairs, then keeps one qubit of the pair and sends the other one to Bob in the quantum channel. Because the information was encoded into an entangled pair, they now share classical information. After receiving the qubit, Bob performs a measurement on it. Because of the properties of entanglement, this makes the measurement on the other qubit of the pair deterministic. When Alice measures its qubit with $t_{waiting}$ time after it was sent, she gains knowledge on what Bob had measured. If the transmission was successful, which means Bob got the correct classical state after measuring, and then Alice sends the next qubit. If the measurement provided a false information, Alice waits a $t_{error}$ time before sending the next qubit, thus letting Bob know that a bit error was occurred during the communication. In that case Bob flips the measured classical bit.

This code does not need any synchronization other than a well chosen $t_{waiting}$. On any noisy channel, because Alice is monitoring the measurement results, a BER of 0% can be achieved without having to send additional qubits through the channel. This makes the efficiency of the code better than any classical and many quantum codes. The transmission on the channel is redundancy-free as well. Since only the measured states carry information, the circuit which produces the entangled pair can be simple, like a CNOT (Controlled NOT) gate. The circuit is shown in Fig.9.

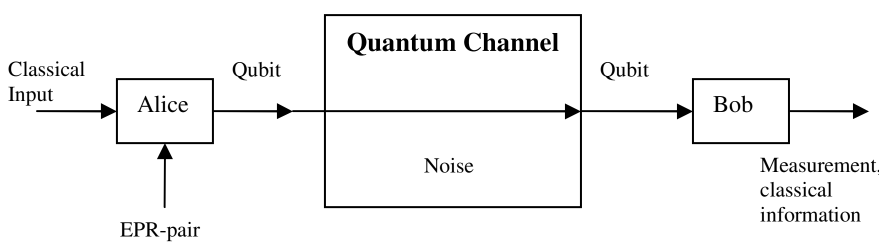


**Fig. 9.** Transmission over a quantum channel with the help of entanglement

The $t_{waiting}$ time can be conciliated during an initialization process, measuring the communication time during sending and receiving back a test message with $l$ length (and with a $t_{timestamg}$). Assuming that the channel is a bidirectional channel (the delays are same in both ways) we can calculate the $t_{waiting}$ as

$$t_{waiting} = \left(t_{sendingAlice2Bob} + t_{sendingBob2Alice}\right)/2 + \varepsilon \tag{16}$$

where $t_{sendingAlice2Bob}$ is the period of sending the message from Alice to Bob, and $t_{sendigBob2Alice}$ is the period of sending the message from Bob to Alice, and $\varepsilon$ is an arbitrarily chosen little number.

For the successful communication the following condition should be set for $t_{error}$

$$\left(t_{waiting} / \lambda\right) < t_{error} << \left(t_{waiting} / 2\right) \tag{17}$$

where $\lambda$ depends on the physical properties of the channel.

However, the loss of a qubit, which happens often in the free-space communication, can cause error during the process. Thus this simple solution can be used in a free space channel only in an advanced form, where Bob knows the sending frequency of Alice, and Bob measures the time differences between two arrived qubits. If the interval between two received qubits is bigger than $t_{waiting} + t_{error} + \varepsilon$, this means that one qubit has been lost during the transmission. With elaborated $t_{waiting}$ and $t_{error}$ time we can provide a redundancy-free solution for quantum transmission, where the interval of the sending period carries the information about the possible error. With this we illustrated an EPR and time based redundancy free error correction method for free-space communication.

## 6 Conclusions

One of the primary requirements of long-distance and free-space quantum communication is the capability of the effective transmission of quantum states in non-ideal, noisy environments. The free-space and satellite quantum channel could be the way to increase significantly the distance limit of current quantum communication systems. The current earthbound free-space quantum channels have the advantage in that they can be combined with satellite quantum communication. In the future, we will be able to overcome the current distance limits in quantum communication by transmitting EPR-states from space to Earth. To exploit the advantages of free-space quantum channels, it will be necessary to use space and satellite technology. The free space optical technology has been combined successfully with entangled pairs and satellite communication.

In classical systems, error correction can be performed only by introduced redundancy within the communication. The error correction capabilities are required for any form of a large scale computation and communication. In classical systems the simplest form to give redundancy to the communication is to encode each bit more than once, however in quantum communication the error correction can be made by much more complex strategies. Currently, many quantum error correction techniques have been introduced to overcome the limitations of quantum theory principles. In these proposals, redundancy is required for successful error correction, because the quantum states are cannot be cloned perfectly, or cannot be measured nondestructively.

In our paper, we have presented a fundamentally new method to realize quantum communication with zero redundancy error correction. Our zero redundancy quantum error correction approaches are based on eigenvectors with unitary transformations, local unitary operations, and entanglement.

The presented redundancy free coding mechanisms can help to set up an efficient quantum channel for the quantum based satellite communication.

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