On the Polarization Analysis of Optical Beams for Use in Quantum Communications between Earth and Space

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Abstract. In this work we will address the transformation of the polarization state of single photons during the transmission along a Space channel and the measures to correct them in order to accomplish Quantum Communication (QC) between Space and Earth.

An open issue in space scale QC is the preservation of polarization states by the telescope and all the involved moving optical components, as well as ensuring the alignment of the polarization basis between the orbiting sender and receiver on Earth. In the following, we will treat in detail this crucial aspect, by modelling the measurement of the polarization properties of the quantum channel, expressed by its Mueller matrix, in the experimental conditions of Ref. [12] with the addition of the control of the outbound state of the photons and the measure of the polarization state of the inbound beam.

Keywords: Satellite quantum communication, polarization analysis, quantum key distribution.

1 Introduction

About a decade ago, several groups endeavored the porting of QC in general and Quantum Keys Distribution (QKD) as first example, outside the cradle of the lab, where QC was initially tested [2], [3].

The final step of this extension of the tract covered with QC is naturally Space, due to the restrictions imposed on the Earth surface by the Earth curvature as well as atmospheric turbulence. Several studies of satellite QC addressed the various aspects involved by the long leg length, the relative motion, the diffraction losses, the background radiation and the error rate ([4,5,6,7,8,9,10]).

The experimental demonstration of the feasibility of single photon exchange between Space and a ground receiver has been also demonstrated in our group recently [12].

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The demonstration of the control of the polarization state along the Space channel is important to pave the way to further steps in Space QC. We believe that the most suitable infrastructure to investigate this topic is the Satellite Laser Ranging System for Geodynamics.

2 Problems in the Transmission

Polarization analysis for an Earth-Space link as first, start with the problem of modeling a real quantum link on space. Physically, we have to deal with a free-space dynamic optical link that crosses the atmosphere. Dynamic means the fact that transmitter and receiver are in relative motion between them.

We can identify the following main issues, that have to be addressed:

- Effects due to atmospheric turbulence that causes attenuation and fluctuations in the signal received.
- Background noise: due to sunlight, moonlight and every source of photons that can be collected by the receiver.
- The relative motions beetween transmitter and receiver, that is the main source of misalignment in the polarization references of transmitter and receiver.
- Non ideal optics that can cause depolarization, attenuation and distortion.

The "Radar Link Equation" given by Degnan [11], is the common representation used to define the noise and attenuation due to atmosphere and non ideal experimental setup, and so the link budget.

Is not trivial the problem of states choosing to optimize the experimental results. Infact, the effects of noise and attenuation on measurements are related to the transmitted states.

An accurate model of the link is the winning key to desing a successfull experiment of polarization analysis.

3 Channel Polarization Analysis

The polarization properties of an optical system can be generally described through the use of the Mueller matrix and the Stokes formalism. The interaction between a polarized beam and a polarizing device can be described by:

$$S = \mathbf{M}S' \tag{1}$$

where S' and S are respectively the Stokes vectors of incident beam and of the emerging beam. $\mathbf{M}$ is the 4x4 Mueller matrix of the optical system. A tipical way for retrieving the Mueller matrix of an optical device is to test the device with n different polarized beam (with $n \geq 4$) and measure the outcoming beam. The input state can be generated through the rotation of a quarter-wave plate at determinated angle θ:

$$J = \begin{bmatrix} 1 \\ cos^2(2\theta) \\ cos(2\theta)sin(2\theta) \\ sin(2\theta) \end{bmatrix} \tag{2}$$

For n different input states the equation (1) can be rewrited in matricial form:

$$\mathbf{S} = \mathbf{MA} \tag{3}$$

where $\mathbf{S}$ is the matrix of output Stokes vectors:

$$\mathbf{S} = \left[S^{(0)} S^{(1)} \cdots S^{(n)} \right] \tag{4}$$

and $\mathbf{A}$ is the matrix of input state vectors:

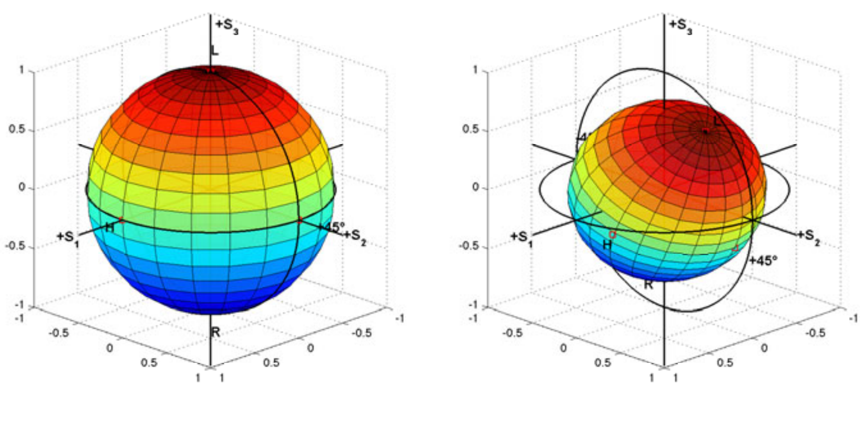
$$\mathbf{A} = \left[J^{(0)} J^{(1)} \cdots J^{(n)} \right] \tag{5}$$

Inverting equation (3) is possible to retrieve the Mueller matrix $\mathbf{M}$:

$$\mathbf{M} = \mathbf{SA}^+ \tag{6}$$

where $\mathbf{A}^+$ is the pseudo-inverse matrix of $\mathbf{A}$.

An easy way to see the effects of an optical system on a polarized beam is to plot its related Poincaré sphere. Knowing the Mueller matrix of the system we can use the relation (1) to obtain the associated Poincaré sphere. In figure (1) is



(a) Input states (b) Output states

Fig. 1. Poincaré spheres

plotted the Poincaré sphere before and after the trasformation of a collimator-radiometer system described by the Mueller matrix in the eq. (7) measured by Howell in [17]:

$$M = \begin{bmatrix} 1.0000 & 0.0338 & 0.1587 & 0.0758 \\ 0.0490 & 0.6955 & 0.0001 & -0.0653 \\ 0.1590 & -0.0001 & 0.8138 & 0.2527 \\ -0.0729 & -0.0753 & -0.2361 & 0.6346 \end{bmatrix} \tag{7}$$

As we can see the output states sphere is rotated and also smaller than the input states one. By modelling the trasformation along the space channel and measuring the Stokes parameters with a suitable polarimeter under development in our group, we will correct for the transformation induced.

4 Feedback Control in Quantum Communication System

An important feature of a quantum communication system is the polarization preservation. This is in general not realized in present systems, due to the non-idealities of a real channel. It's however possible to study the polarization properties of the channel by the use of Mueller matrices formalism and use it to compensate the distortion of the channel. In fact knowing the Mueller matrix of the channel it's possible to prepare an input state so that is received in the desired polarization state. Using the cascade of an half-wave plate and a quarter-wave plate is possible to choose anykind of polarization states in the Poincaré sphere. In figure (2) is represented an example of a quantum trasmission system. In Mueller formalism the matrix of the rotated quarter-wave retarder is:

$$Q(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta & \sin 2\theta cos2\theta & -\sin 2\theta \\ 0 & \sin 2\theta cos2\theta & \sin^2 2\theta & \cos 2\theta \\ 0 & \sin 2\theta & -\cos 2\theta & 0 \end{bmatrix} \tag{8}$$

and the rotated half-wave retarder matrix is:

$$H(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 4\theta & \sin 4\theta & 0 \\ 0 & \sin 4\theta & -\cos 4\theta & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{9}$$

In practice, starting from a vertically polarized state we can choose the appropriated rotation angles to send the desired state. Considering all the trasmitting system together with the channel the equation (1) become:

$$S = \mathbf{MQ}(\theta_{\mathbf{Q}})\mathbf{H}(\theta_{\mathbf{H}})V \tag{10}$$

where V is the Stokes vector of the input vertical polarized state. A simple way to find the two rotation angles is to minimize the norm of the difference between the sent state and the expected received state:

$$(\theta_Q, \theta_H) = \min_{\theta_Q, \theta_H} \parallel S - \mathbf{MQ}(\theta_{\mathbf{Q}})\mathbf{H}(\theta_{\mathbf{H}})V \parallel_F \tag{11}$$

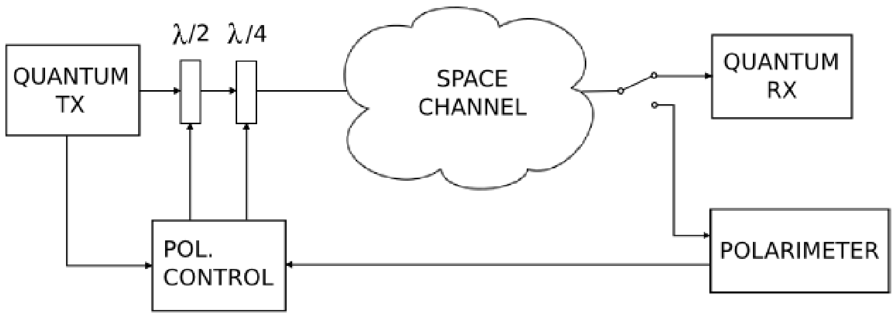


Fig. 2. Scheme of a quantum transmission system

In this way, knowing the Mueller matrix of the channel we can pre-compensate any retardation effect introduced by the channel. In a typical trasmitting system channel probing and information exchange share the same medium, this implies that Mueller matrix measure should not affect the single-photon exchange in the quantum channel. Two possible solution are time-multiplexing and wavelenght-multiplexing proposed by [9].

With the model here proposed and the undergoing experimental measurements, the necessary knowledge of the channel transformations will be available for the first time. With these we can predict the evolution of a time-varing channel and compensate it for changes in the polarization by means of a feedback control.

5 Conclusions

In conclusion, these analysis are aimed to ascertain the causes of incorrect alignment of the polarization reference of an orbiting quantum transmitter and a quantum receiver on Earth. The findings will be used to envisage the feedback system that will correct for these transformations and that will allow the demonstration of the Space QC.

The ability to analize and compensate the polarization state of photons will give us the tools and the basis for quantum communication on free-space on space scale. What we are doing, can be seen like a creation of a communication medium, in wich photons can be used to transfer informations. The problem can be compared, as were invented fiber optics. At that time the problem was the development of a media allowing the propagation of light over long distance. Now our problem is the "development" of a media in wich the photons can propagate preserving their polarization state, and the media is formed by: free-space link, polarization analyzer and compensator.

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