

# WiFi Assisted GPS for Extended Location Services

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**Abstract.** This paper shows a lightweight technique for extending the positioning capabilities of a global navigation satellite system to areas characterized by reduced satellite visibility. This technique includes the presence of a WiFi coverage, used to implement the *virtual satellite* concept. In particular we assume that a customer terminal can receive both Global Positioning System (GPS) signals and WiFi beacons broadcasted by an access point (AP). We show that if such beacons include the geo-referenced position of the relevant AP, the suitable usage of this information allows determining the GPS receiver position even if only three satellites are visible. Experimental results show that the achievable performance are similar to that obtainable by a plain GPS receiver using four visible satellites.

**Keywords:** WiFi, GPS, hybrid positioning, performance analysis.

## 1 Introduction

The growing diffusion of location based services using the GPS has driven a lot of research efforts for defining effective solutions for augmenting the GPS coverage [1]. It is known that a GPS receiver can fix its position by processing signals received by at least *four* satellites. In this paper we show a *lightweight* solution for extending the use of the GPS in areas where only *three* GPS satellites are visible. We propose to integrate the information received by a GPS receiver from *three* satellites with an equivalent information obtainable from an AP compliant with the IEEE 802.11 radio technologies (commonly referred to as WiFi). This information consists of the geo-referenced position of a such an AP, through which a fourth *virtual satellite* is constructed so that the receiver position can be fixed. The shown algorithm makes use of the AP position only, and does not include any other information, such as the distance from GPS receiver to AP, the estimation of which is typically critical due to site-dependent signal path loss and multipath propagation.

We stress that we do not propose to integrate the GPS with a WiFi positioning system, as frequently appears in the literature [2]. Our approach is much simpler, and consists of integrating the information received from GPS satellites with a further static information obtainable from an existing WiFi network.

The idea of using less than four satellites for fixing a position on the earth surface is not novel. For example [3] shows a Chinese proposal, based on a satellite system alternative to GPS, working for a limited range of latitudes. [3] includes a derivation of the geometric errors due to the use of a constellation of three satellites. This aspect is indeed the crucial point of using fewer than four satellites. In this regard, the importance of our proposal consists of providing performance similar to that achievable by using four satellites, obtained through a lightweight and easy to implement solution.

The AP coordinates can be easily integrated in WiFi beacons broadcasted periodically, typically every 100 ms. Beacon frames are made of both mandatory and optional components, referred to as Information Elements (IE). Thus, the AP position may be either included in a new, dedicated IE, published within beacons, or embedded in the AP Service Set Identifier (SSID) IE, which usually reports the network name but could, in principle, include any type of information. In addition, if codified into a dedicated IE, it could be also obtainable with the classic probe request/probe response mechanism to save wireless bandwidth ([15] reports a study about the overhead of embedding information into WiFi beacons). In any case, all these approaches does not require to associate to the AP to obtain the information on AP position.

This approach could also be integrated with the use of inertial systems, which are mainly used in vehicular environments in order to improve performance of navigation systems. In fact, inertial system performance degrades very quickly when GPS satellites are not visible and receiver does not move over a straight line [14].

A GPS receiver has to determine four unknowns, which are its own coordinates and clock bias. Thus a system of at least four equations including these unknowns is needed. In our approach three equations consist of the pseudo-range equations of the three visible satellites. The fourth equation is a suitably modified pseudo-range equation using the AP as an equivalent *virtual satellite*. This point is quite critical, since the relevant geometric trilateration could lead to numerical problems. Thus, the selection of a suitable position for the virtual satellite is crucial. The position estimation algorithm works well only if the fourth satellite forms a good geometry with the other three visible satellites, which is typically represented by the so called geometric dilution of precision (GDOP) [9].

This manuscript shows the experimental performance of our proposal. Experiments have been done by using a GPS station ASHTECH Z-12 equipped with a choke ring antenna (J.PSREGANT-DD-E) and a receiver (Topcon ODYSSEY-E) driven by a rubidium atomic clock. The experimental results show that the position fixing capabilities of a GPS receiver using three satellites and implementing our proposal are similar to those of a plain GPS receiver using four satellites if its distance to the AP is within some specific values.

## 2 The GPS Operation

The GPS navigation data are organized in frames [10], including ephemeris and correction clock parameters, by which a receiver can estimate its distance to the sending satellite, referred to as pseudorange. This estimation is done by the difference between the local reception time and the time when the satellite signal has been transmitted. Since this estimation happens at different times, and the receiver clock is not synchronized with the satellite clock, it is necessary to use time difference of arrival (TDOA) techniques to fix the receiver position.

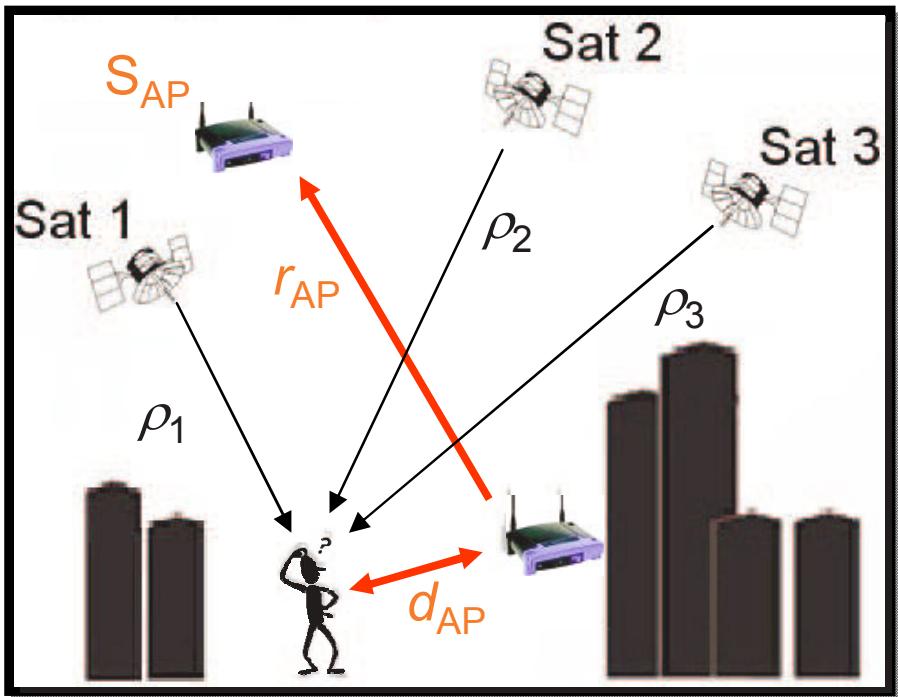
In this way, for each visible satellite a GPS receiver constructs a navigation equation including pseudorange, satellite position, unknown clock bias, unknown receiver coordinates, and other minor error contributions not included in this letter for brevity. A Cartesian reference system with the origin of coordinate axes located at the center of the earth allows it to easily formulate a navigation equation. Let  $\mathbf{S}_i$  be the coordinate vector of the  $i$ th satellite and  $\rho_i$  its estimated pseudorange. If we represent the receiver position coordinate vector as  $\mathbf{X}_{\text{rec}}$  and the receiver clock bias as  $\Delta b$ , we can write:

$$\| \mathbf{S}_i - \mathbf{X}_{\text{rec}} \| - \Delta b = \rho_i, i = 1, \dots, 4 \quad (1)$$

where the operator  $\| \cdot \|$  is the euclidean norm on  $\mathcal{R}^3$ . Bancroft [4] has suggested to model the navigation equation as a hyperbolic equation, and has provided an exact non iterative solution. Through a geometric approach, Abel [5] has investigated the existence and uniqueness of the system solution. Another way to solve the system of pseudorange equations consists of applying the Newton method. Since the number of unknowns is four, three receiver coordinates and clock bias, the minimal number of needed visible satellites is four. The use of more observable data (i.e. more visible satellites) allows improving the estimation accuracy.

## 3 Joint WiFi-GPS Operation

When only three satellites are visible, they are not sufficient to fix the receiver position. Thus, additional information, eventually obtainable from other sources, is needed. We propose to use a geo-referenced WiFi AP which broadcasts its coordinates by including them in broadcasted beacons. If a GPS receiver is equipped by a WiFi network interface, as most of current commercial devices, it can receive and use the information broadcasted by such an AP. In case of different available APs, we assume to use the one closest to the receiver, estimated through the RSSI measures collected by the WiFi card. This way, it is possible to complete the set of equation necessary to solve the receiver position. This additional information could be used in many ways. We propose an algorithm that can reuse the processing algorithms already implemented in GPS receivers. Below we illustrate the processing steps, the relevant approximations and their effect on estimation accuracy.



**Fig. 1.** Construction of the WiFi-based virtual satellite

*Step 1:* The first step consists of a preliminary estimation of the receiver clock bias. To this aim, we approximate the receiver position with the AP position, which is  $d_{AP} = \| \mathbf{X}_{rec} - \mathbf{X}_{AP} \|$  meters away from the GPS receiver (see Fig. 1).

For each visible satellite it is possible to construct a navigation equation where the only unknown is the receiver clock bias, as follows:

$$\| \mathbf{S}_i - \mathbf{X}_{AP} \| - \Delta b = \rho_i, i = 1, \dots, 3 \quad (2)$$

where  $\mathbf{X}_{AP}$  is the coordinate vector of the AP. Thus, three estimates of the receiver clock bias are obtained, and their average is an input to the second step.

*Step 2:* The second step consists of introducing a *virtual satellite*, the coordinates of which are referred to as  $\mathbf{S}_{AP}$ , and constructing a modified system of equations that allows estimating the receiver position.

This new system of equations is made of three navigation equations, relevant to the visible satellites, and a fourth modified navigation equation, relevant to the virtual satellite.

The determination of the position of the virtual satellite is crucial for obtaining a good estimation of the position of the GPS receiver. In fact, the pseudorange measurements are typically affected by both systematic errors (bias) and random

errors. These errors have clearly an impact on the accuracy of the estimated receiver position, expressend by the GDOP, defined as follows.

From (1) we can write the jacobian  $\mathbf{H}$  of  $\rho(\mathbf{X}_{\text{rec}})$ :

$$\mathbf{H} = \begin{pmatrix} \frac{\partial \rho_1}{\partial x}, & \frac{\partial \rho_1}{\partial y}, & \frac{\partial \rho_1}{\partial z}, & \frac{\partial \rho_i}{\partial \delta b} \\ \ddots & \ddots & \ddots & \ddots \\ \frac{\partial \rho_4}{\partial x}, & \frac{\partial \rho_4}{\partial y}, & \frac{\partial \rho_4}{\partial z}, & \frac{\partial \rho_4}{\partial \delta b} \end{pmatrix} \quad (3)$$

Matrix  $\mathbf{H}$  plays a role of measure matrix and if we assume  $\hat{\mathbf{X}}$  to be an estimate of the position and  $\mathbf{X}_{\text{rec}}$  the exact position, the equation of the residue become:

$$\delta\rho \approx \mathbf{H} (\hat{\mathbf{X}} - \mathbf{X}_{\text{rec}}) \quad (4)$$

By inverting this relation we can obtain:

$$(\hat{\mathbf{X}} - \mathbf{X}_{\text{rec}}) = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \delta\rho \quad (5)$$

The matrix  $\mathbf{Cov} = (\mathbf{H}^T \mathbf{H})^{-1}$  is the covariance matrix of position and clock bias. The GDOP is defined as:

$$GDOP = \frac{\sqrt{tr(\mathbf{Cov})}}{\sigma_0} \quad (6)$$

where  $\sigma_0$  is the ranging error variance and the operator  $tr(\mathbf{A})$  is the trace of the matrix  $\mathbf{A}$ .

Similarly, the dilution of precision may also be related to individual geometric and temporal quantities; HDOP, PDOP, VDOP, and TDOP refers to the horizontal, position, vertical, and time dilution of precision, respectively [9].

In order to obtain a reliable position estimation, we need to place the virtual satellite in a position producing a good GDOP. Our approach aims to emulate the GPS operation under unobstructed satellite visibility. Thus we have decided to locate the virtual satellite in a position compliant with the GPS satellite constellation. In other words, the virtual satellite is located by the software of the GPS receiver in a position where a GPS satellite would be visible without the obstructing objects close to the GPS receiver. Thus, the GPS receiver needs to be aware of the satellite ephemerides. This is not a challenging problem, since they could be either collected during the satellite visibility periods, since each satellite broadcasts its own ephemeris and clock correction data, or easily downloaded through already available ephemeris distribution services, accessible through either the WiFi connection or a cellular network, similarly to what happens for the Assisted GPS [13]. Clearly, among the candidate satellite positions we select the one providing the best GDOP value. Hence, the needed fourth navigation equation can be constructed by approximating again the receiver position with the AP position; thus we can replace the pseudo-range with the geometric distance  $r_{AP}$  between the AP and the known (virtual)  $\mathbf{S}_{AP}$  position as follows:

$$\| \mathbf{S}_{AP} - \mathbf{X}_{\text{rec}} \| = r_{AP} \quad (7)$$

where  $r_{AP}$  is the euclidean distance between the AP and the virtual satellite.

The solution of this modified system of equations is an estimation of the GPS receiver position. We have used the iterative Newton method, using as the starting point the preliminary estimation of the receiver clock bias obtained in *step 1* and the known AP position. Below we illustrate the impact of the approximations introduced on the estimation accuracy.

## 4 Results

In order to analyze our positioning algorithm, we have implemented a software tool in C language, used to process the receiver independent exchange format[11] (RINEX).<sup>1</sup> The block diagram of this tool is shown in Fig. 2.

We have processed data obtained by a receiver located at the Topography Lab at the University of Perugia [12]. The accurate world geodetic system 84 (WGS84) coordinates of the receiver have been obtained from the Italy geographic military institute (IGM95) network. Ephemerides, pseudoranges, and ionospheric parameters are extracted and a channel is associated with each satellite. The position of the GPS receiver is available with a resolution of few cm, and we have assumed this position as the "true" reference receiver position for subsequent processing.

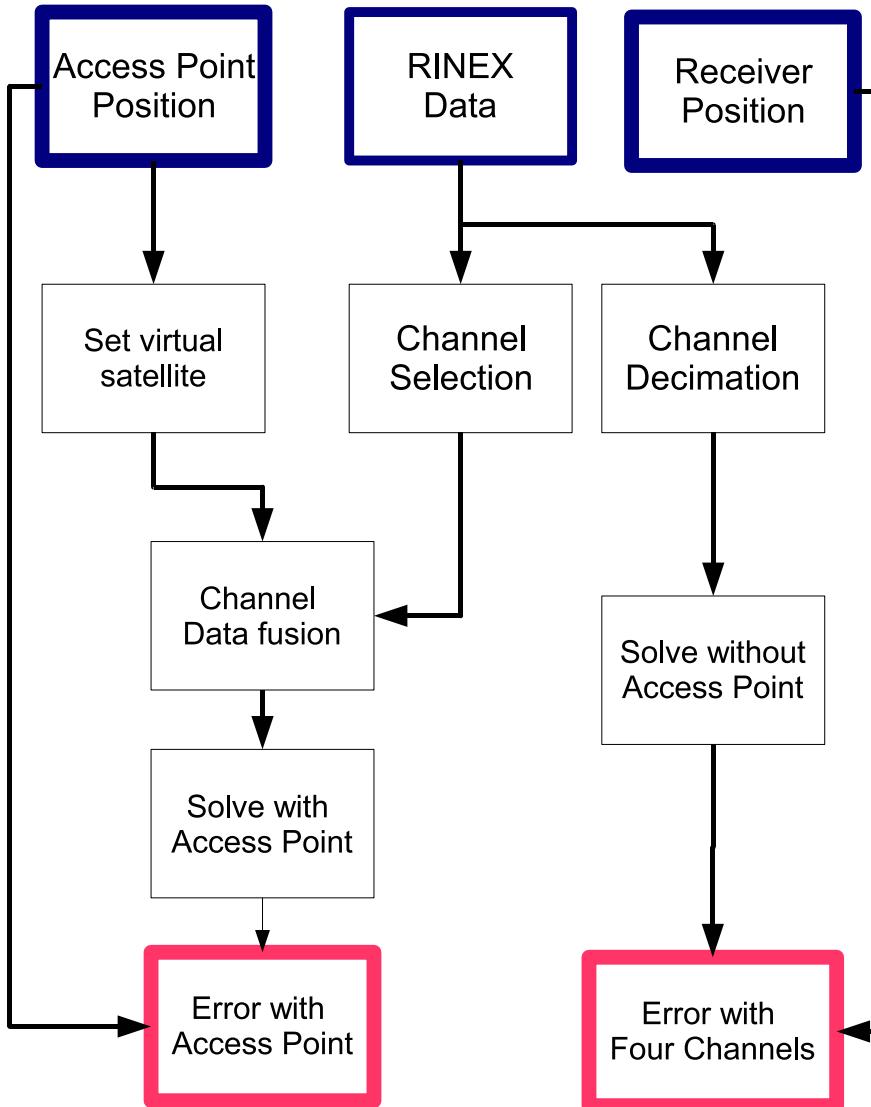
In order to analyze the achievable performance, we have done experiments by randomly selecting four satellites out of the visible ones.

The same set of data has been processed by our hybrid positioning algorithm, which uses three satellites and an AP located at variable distance from the GPS receiver.

For evaluating the location estimation error we have used the great circle distance metric. We have used a spherical model of the earth and assumed that the altitude of the receiver is the same of the known accurate position of the GPS receiver. Since the position estimated is close to the reference position, the great circle arc between these two points can be regarded, with a good approximation, as a straight line. Its length may be taken as a measure of the distance between the estimated position and the accurate position. Experiments have been realized by considering a maximum tolerable GDOP value equal to 3. Above this threshold all relevant satellite positions over the GPS constellation have been discarded. Conversely, all satellite positions obtained from the available ephemerides generating a GDOP value lower than or equal to 3 have been used one by one for locating the virtual satellite and construct the fourth navigation equation. Below we show the results collected during an observation window of one hour, by collecting GPS observations each second. Our algorithm succeeded in finding a solution with the desired GDOP for about 50% of the observations by using three satellites and an AP with a GDOP threshold equal to 3.

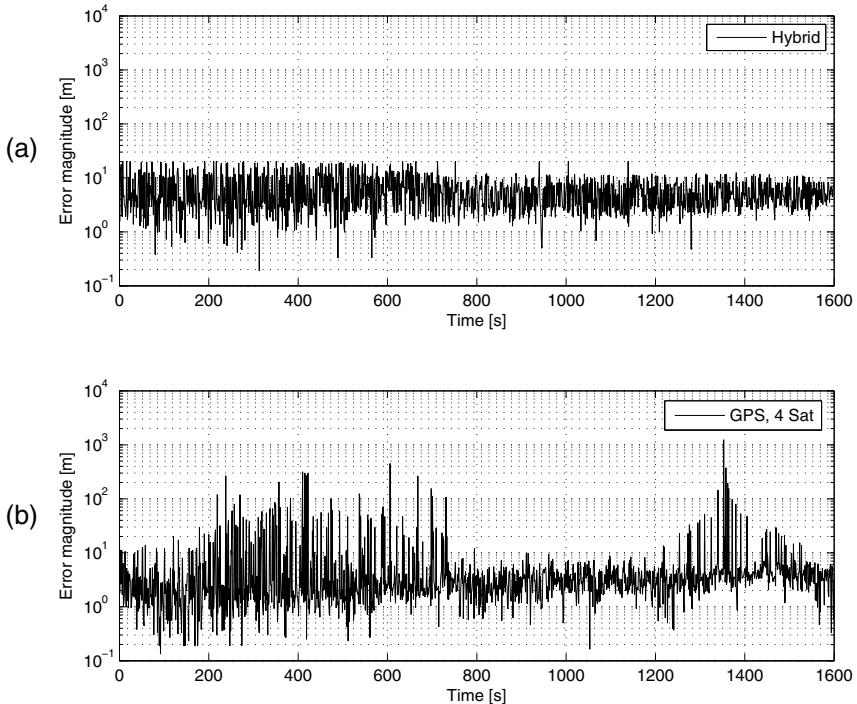
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<sup>1</sup> The RINEX is an internationally accepted data exchange format for GPS receivers. The information is organized in three parts: observation data, navigation data, and meteorological data. Pseudorange values can be obtained by observation data, ephemerides and ionospheric parameters by navigation data.



**Fig. 2.** Flow Graph of data processing

Fig. 3 shows a realization of the estimation of error magnitude obtained by both a hybrid receiver with three satellites and an AP placed at a distance of 20 m (a), and a plain GPS receiver using four satellites (b). It appears that the need of using the only four visible satellites, modeled by selecting four random satellites out of the set of visible ones, can deteriorate performance of a plain GPS receiver, while by suitably positioning the virtual satellite allows obtaining



**Fig. 3.** Error magnitude as a function of time; the AP is 20m away from the GPS receiver

a low GDOP value (up to 3 in our experiments), hence a lower error magnitude. However, when the geometry of the four satellites is good (i.e., low GDOP), the plain GPS receiver outperforms the hybrid one. Nevertheless, we can also observe that the errors induced by our approximations are substantially lower than the distance between the GPS receiver and the AP, which is the error that would be made by simply approximating the receiver position by the AP position. To go further, we have made a statistical analysis on the achieved results. It is worth noting that in the hybrid operation the mean error magnitude, shown in Fig. 4, ranges between 0.2 and 0.3 times the distance between the GPS receiver and the AP.

In addition, when the AP is quite close to the GPS receiver, namely for 10, 20, and 30 m, the average error obtained with the proposed solution is even better than the one relevant to the usage of four satellites, whereas the estimation error is comparable for 40 m. In conclusion, it is evident that performance of the hybrid receiver is equivalent to that achievable by a plain GPS receiver working by four satellites and, in some situations, can be compared also with that relevant to the use of more satellites. Thus, the objective of extending positioning service to areas with difficult satellite visibility is achievable.

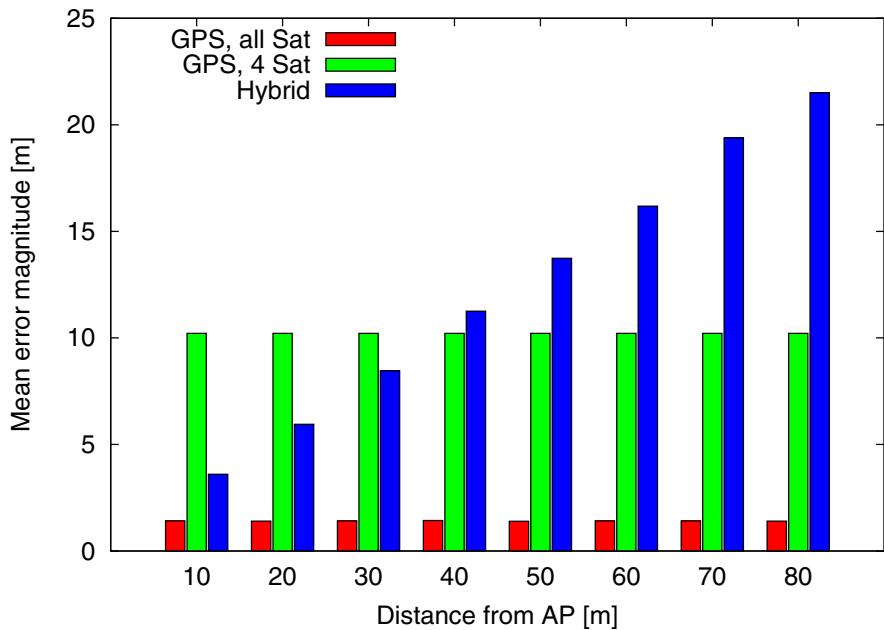


Fig. 4. Mean error at various AP distances from the GPS receiver

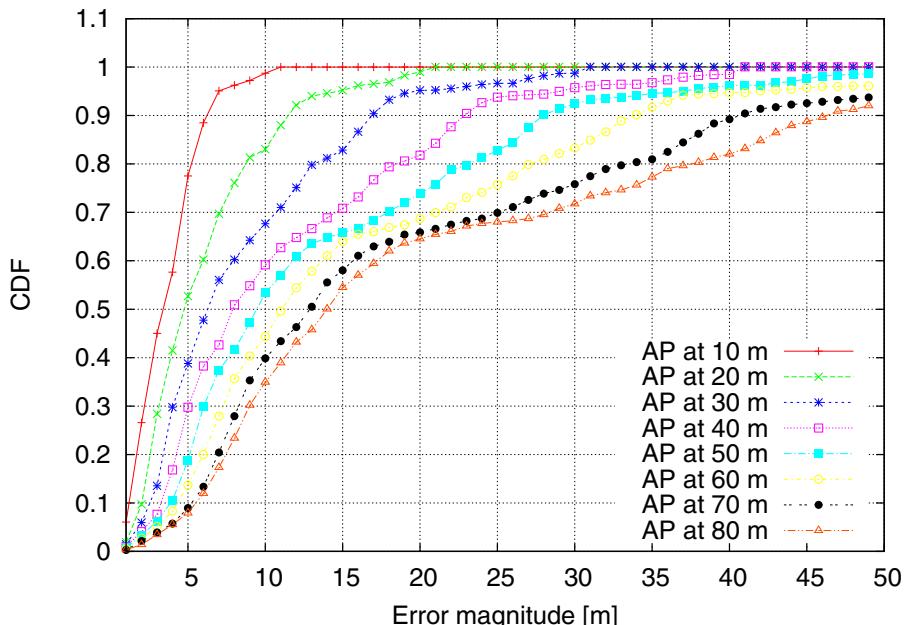
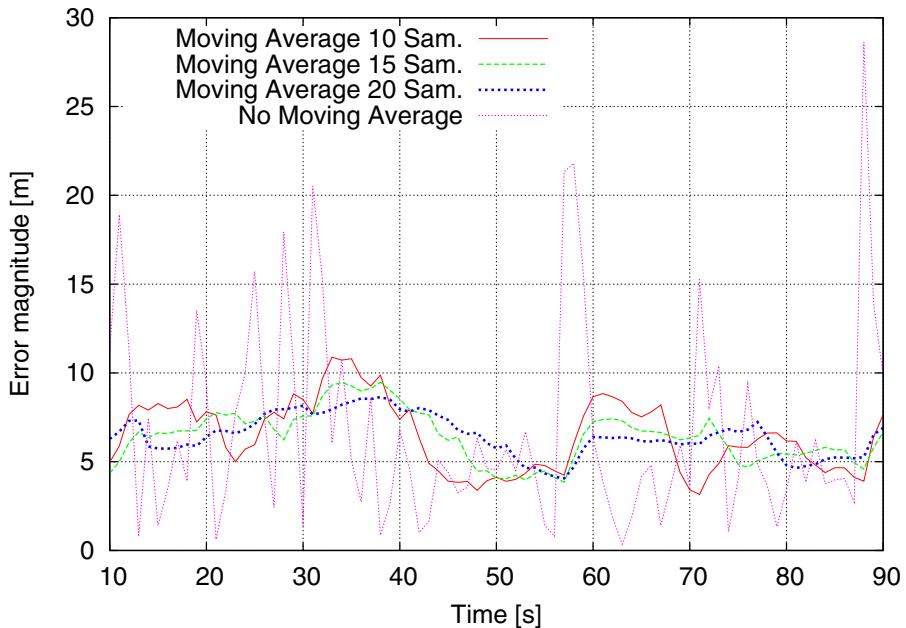


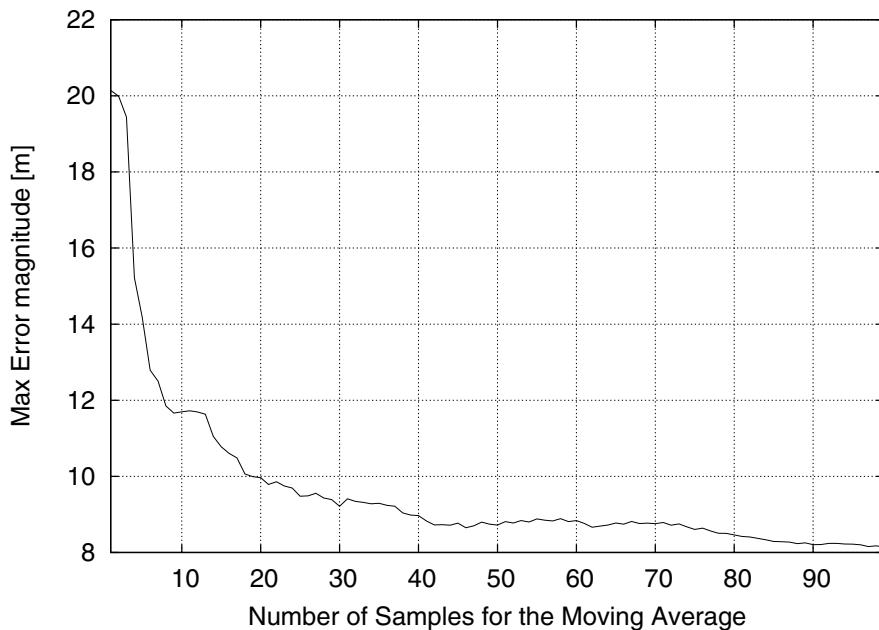
Fig. 5. Cumulative Distribution Functions of error at various AP distances from the GPS receiver



**Fig. 6.** Effect of a moving average filtering on error magnitude; the AP is 20m away from the GPS receiver

Fig. 5 shows the cumulative error distribution function for the AP located at different distances from the GPS receiver. It appears that for all analysed cases, the probability of having an error up to half the distance between the GPS receiver and the AP is about 90%. This is an excellent results when compared with the trivial solution of approximating the position of the GPS receiver with the one of the AP. In addition, this result is not dependent on the distance between the GPS receiver and the AP.

Fig. 6 and Fig. 7 are relevant to the AP located 20 m away of the GPS receiver. Fig. 6 shows the error magnitude versus time, obtained by averaging 10, 15, and 20 subsequent estimates. It is evident that by using a filtering technique (we have used a simple moving average) it is possible to improve the accuracy of the system, since few bad samples are compensated by good ones. Finally, Fig. 7 shows the maximum error versus length of the moving average filter. Clearly, the maximum reachable error is equal to the distance between the GPS receiver and the AP (20 m) and this may happen for a single sample measure. If we average measures over few samples, the maximum error decreases rapidly. For instance, by using three samples (this implies waiting 3s before producing the solution in our testbed) we have obtained a improvement of 20%, and by six samples it increases up to 33%. Asymptotically (in practice by using more than 100 samples), the maximum error converges towards 8 m, which is very close to the average error (5.94 m). Clearly, if samples are provided by the GPS device



**Fig. 7.** Maximum error vs. length of the moving average filter; the AP is 20m away from the GPS receiver

at higher rate than one per second, the performance can be largely improved with reduced latency.

## 5 Conclusion

We have shown a solution for augmenting GPS potentials by the use of WiFi. If a GPS receiver can receive the coordinates of a geo-referenced AP, it can combine three navigation equations, relevant to three visible satellites, and the AP position information. This hybrid approach can provide performance similar to the one achievable by using four satellites, thus providing a significant extension of the area where the GPS can work. The proposed approach highly outperforms the trivial solution of simply approximating the GPS receiver position with that of the AP. In the future we will include the optimal AP selection in case different AP are available and the extension of our approach to two visible satellites.

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