

# Analytical Framework for Contact Time Evaluation in Delay-Tolerant Networks

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**Abstract.** In the last few years, there has been an increasing concern about stochastic properties of contact-based metrics under general mobility models in delay-tolerant networks. Such a concern will provide a first step toward detailed performance analysis of various routing/forwarding algorithms and shed light on better design of network protocols under realistic mobility patterns. However, throughout the variety of research works in this topic, most interests rather focused on the inter-contact time while other contact-based metrics such as the contact time received too little interest. In this paper, we provide an analytical framework to estimate the contact time in delay-tolerant networks based on some recent key results derived from biology and statistical physics while studying spontaneous displacement of insects such as ants. In particular, we analytically derive a closed-form expression for the average value of the contact time under the random waypoint mobility model and then give an approximation for its distribution function.

**Keywords:** Delay-tolerant networks, Performance evaluation, Contact time, Bio-inspired networks.

## 1 Introduction

Delay-tolerant networks (DTN) are complex distributed systems that are composed of wireless mobile/fixed nodes, and they are typically assumed to experience frequent, long-duration partitioning, and intermittent node connection [1,2]. In such networks, communication opportunities appear opportunistic, and an end-to-end path between source and destination may break frequently or may never exist. Due to such special features, many techniques [2] have been proposed for message delivering in DTN with high probability even when there is never a fully connected path between source and destination nodes. Most of such techniques take benefit of opportunities offered by *node mobility*.

The performance of such data delivery techniques depends on the knowledge of traditional networking parameters such as node density, mobility pattern, and transmission range, to name a few. However, since DTNs differ from traditional mobile ad hoc networks in that disconnections are the norm instead of the exception, two additional critical parameters arise here [3]. The first one, which is commonly called the *inter-contact* time or sometimes referred to as the inter-meeting time, can be defined as the time duration between consecutive

points of time where two relay nodes come within transmission range of one another. The second one, called the *contact time* or the contact duration, can be defined as the period of time during which two nodes have the opportunity to communicate. The importance of both parameters stems from the fact that they directly impact the delay and capacity of the network, thereby helping to choose the proper design of various scheduling/forwarding algorithms for DTNs. However, throughout the variety of research works in this topic, most interests rather focused on the inter-contact time while the contact time received too little interest. This is due to empirical observations and assumptions often made in initial works that the contact time is several orders of magnitude less than the inter-contact time [4], thereby making the inter-contact time more crucial. This is our primary motivation that prompted us in this paper to focus on the contact time and try to provide an analytical framework for its estimation.

There have been various research works on the characteristics of the inter-contact time and its impact on the performance of different proposed data forwarding schemes [3,5]. Initial works typically assumed that the CCDF (complementary cumulative distribution function) of the inter-contact time decays exponentially over time under several currently used mobility models such as random waypoint model and simple random walks [6]. Although this assumption is supported by numerical simulations conducted under most existing mobility models in the literature, it is generally conjectured by authors so as to make their analysis tractable [6,7]. However, extensive empirical mobility traces later show that the CCDF of the inter-contact time follows approximately a power law over large time range with exponent less than unit [3,5].

At first glance, this finding suggested a need of new mobility models to produce the power-law property exhibited by real traces, and called for further studies to explain the outright discrepancy in the behavior of the inter-contact time. While attempting to resolve this discrepancy, many research works have recently provided credible evidence that the inter-contact time distribution has, in fact, a mixture of power-law and exponential behavior [8,9]. Specifically, using a diverse set of measured mobility traces, authors in [9] found that the CCDF of the inter-contact time follows closely a power-law decay up to a characteristic time, which confirms earlier studies, and beyond this characteristic time, the decay is rather exponential.

Knowing the inter-contact time allows one to evaluate the end-to-end delay in DTNs under ideal conditions of infinite bandwidth and buffer space. This might be a useful approximation for low traffic scenarios or low-resources data forwarding schemes. However, this is inaccurate when resources are rather limited or when the data forwarding scheme utilizes a lot of resources, which characterize several applications of DTNs. In such a scenario, the contact opportunity can be lost due to several causes such as the lack of buffer space, the limited bandwidth or simply due to MAC contention and interferences. In all these cases, even if a node comes in contact with a relay or even the destination node, it might not be able to transfer data during the contact duration. This calls to include the contact time, in addition to the inter-contact time, for a more accurate analysis of the end-to-end delay.

The remainder of this paper is structured as follows. In Section 2, we introduce basic definitions and assumptions used later to derive interesting results. In Section 3, we focus on some statistical properties of the contact time. Based on the invariance property of random walk-like motions in bounded domains encountered in many fields of science such as biology and statistical physics, we derive a closed-form expression for the average value. Under some assumptions, we give then an approximation for its probability distribution function. Finally, some conclusions are drawn in Section 4.

## 2 Preliminaries

We look in this section at a particular class of mobility models, namely, the random waypoint mobility model. This model is widely used for the design, study and analysis of mobile ad hoc networks. Furthermore, we introduce some useful definitions and notation and state the assumptions we will be making throughout the remaining of this paper in order to study the statistical properties of the contact time.

### 2.1 Random Waypoint Model

In the random waypoint mobility model [10], each node is assigned an initial location in a given area and travels at a constant speed  $\mathbf{v}$  to a destination chosen uniformly in this area. The speed  $\mathbf{v}$  is chosen uniformly in  $[v_0, v_1]$ , independently of the initial location and destination. After reaching the destination, the node may pause for a random amount of time after which a new destination and a new speed are chosen, independently of all previous destinations, speeds, and pause times. The stationary distributions of location and speed in the random waypoint mobility model differs significantly from the uniform distribution. In particular, it has been shown that the probability density function, denoted by  $f_{\mathbf{v}}(s)$ , for the stationary distribution of the speed without pausing is given by

$$f_{\mathbf{v}}(s) = \begin{cases} \frac{1}{s \ln(v_1/v_0)} & \text{if } v_0 \leq s \leq v_1 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

### 2.2 Contact Criteria

There are several criteria to define a contact between two nodes. Each definition depends on the context. We restrict ourselves here to the *Boolean* and *Interference-Based* criteria [11,12] defined below.

Let  $\{A_k, k \in \mathcal{T}\}$  be a set of mobile nodes following some mobility models in a common domain  $\Omega$ , and simultaneously transmitting at some time instant over a certain subchannel. Let  $P_k$  be the power level chosen by node  $A_k$ , for  $k \in \mathcal{T}$ . Let us also denote by  $A_k(t)$  the position of node  $A_k$  at time  $t$ . Consider now two

arbitrary mobile nodes  $A_i$  and  $A_j$ , where  $i, j \in \mathcal{T}$ . Under Boolean model with communication range  $d$ ,  $A_i$  and  $A_j$  are deemed to be in contact at time  $t$  if and only if the distance between them is no more than the communication range. In mathematical parlance, this can be stated as

$$\| A_i(t) - A_j(t) \| \leq d. \quad (2)$$

Note in passing that this criterion establishes a *symmetric* contact relation between nodes  $A_i$  and  $A_j$  if and only if all nodes use a common transmission range to communicate with peer nodes in the network. The major drawback of Boolean model is that it does not allow interferences to be taken into account. Although this is not generally the case for DTNs, it turns out that when the number of nodes increases, the wireless medium becomes more and more solicited. This competition for the channel may prevent successful transmissions even when the distance between  $A_i$  and  $A_j$  is no more than their minimum common transmission range. A natural way to take interferences into account is to add the sum of the interfering signals coming from simultaneous transmissions to the background noise. Then we adopt a different criterion, called the Interference-Based criterion, which can be mathematically expressed as

$$\frac{\frac{P_i}{\| A_i(t) - A_j(t) \|^{\alpha}}}{N_0 + \sum_{\substack{k \in \mathcal{T} \\ k \neq i}} \frac{P_k}{\| A_k(t) - A_j(t) \|^{\alpha}}} \geq \beta, \quad (3)$$

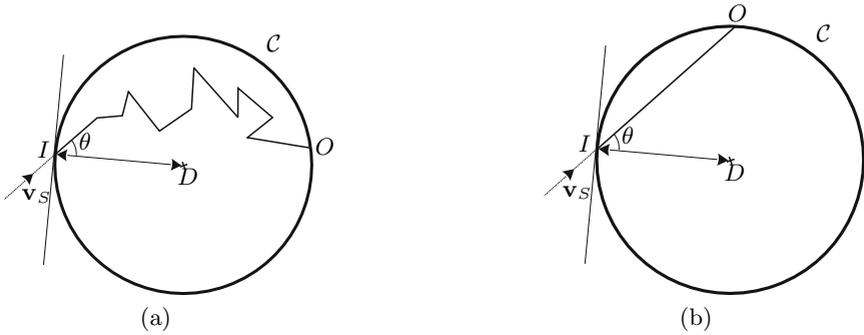
where  $\beta$  is a suitable threshold,  $N_0$  and  $\alpha$  stand for the ambient noise and the path loss exponent respectively. Under the aforementioned contact criteria, the contact time can be formally defined as follows. Let  $A_i$  and  $A_j$  be two nodes moving according to a given mobility model. We assume that they are initially out of contact, and assume they come into contact with each other at time 0. The contact time, denoted by  $\tau_c$  is defined as the time they remain in contact with each other before moving out of contact under both Boolean and Interference-Based criteria. However, for the sake of simplicity, we restrict our analysis in what follows to Boolean criterion.

### 3 Contact Time Analysis

In this section, we study the statistical properties of the contact time under Boolean criterion and using the random waypoint mobility model. We restrict ourselves here to the case where a relay node (or the destination node) is static all the time and thus only the source node is mobile.

#### 3.1 Ant-Based Model Description

We consider two nodes: a mobile source node  $S$  moving at velocity  $\mathbf{v}_S$  and a static relay node  $D$  (that can be also the destination node). Both nodes are assumed



**Fig. 1.** (a) At point  $I$ , the source node enters the transmission region of the relay node at an angle  $\theta$  to the ray  $ID$ , undergoes a random waypoint motion, and then exits at point  $O$ . (b) Source node crosses the transmission region of the relay node without changing direction.

to have the same transmission range denoted by  $d$ . We further suppose that the system is already in the steady-state, which implies that the velocity of the source node is drawn from the stationary velocity distribution characterized by the probability density function given by (1). Furthermore, it is assumed that no specific direction is favored, and therefore, that when node  $S$  enters into contact with node  $D$ , its incident direction is distributed isotropically. The contact time, denoted by  $\tau_c$ , can be defined as the time elapsed from source node's entry into the radio range of relay node  $D$  until its consequent exit. In Figure 1(a), we denote by  $I$  the entrance point of the source node to the connectivity region of the relay node, namely the circle  $\mathcal{C}$  centered at  $D$  with radius  $d$ . Exit point is denoted by  $O$ . Let us also denote by  $\theta$  the angle that velocity  $\mathbf{v}_S$  at  $I$  makes with the ray  $ID$ . We have  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ , otherwise this implies the source node were already in contact with the relay node. Recalling the assumption that incident directions are distributed isotropically,  $\theta$  will be uniformly distributed over  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ . As illustrated in Figure 1(a), relay node  $D$  sees the movement of source node  $S$  as a sequence of epoch segments, where the movement direction and the velocity of the source node may change from epoch segment to another. But, the velocity remains constant in an epoch segment.

Before analyzing the contact time, we draw a parallel between the sought model and a practical animal-biology example that was of interest in many fields of science from biology [13] to statistical physics [14]. Consider ants moving on a horizontal planar surface, and assume a circle is drawn on this surface. It has been shown under isotropic incidence that the mean length of the trajectories inside the circle is independent of the random walks characteristics and is given by a very simple formula.

### 3.2 Mean Contact Time

Considering the random waypoint mobility model as a particular random walk and applying the above key result, a first calculus gives the mean contact time

of the source node as follows  $\bar{\tau}_c = \frac{\bar{L}}{\bar{v}_S} = \frac{\pi d}{2\bar{v}_S}$ . From (1), we can readily calculate the average node speed  $\bar{v}_S$  and thus we obtain  $\bar{\tau}_c = \frac{\pi d}{2(v_1 - v_0)} \ln(v_1/v_0)$ . Many important remarks can be drawn from this key result. First, compared to previous research works [15,16], the above result is obtained without assuming that the source node has a straight line trajectory form while crossing the connectivity region of the relay node. Second, note that this above analysis can be extended somewhat to cover more general connectivity region of any geometric form around the relay node under isotropic uniform incidence. Third, in terms of the obtained mean contact time, there is no fundamental differences between the random waypoint mobility model and any other random walk-like mobility model. Therefore, the mean contact time would be the same.

### 3.3 Distribution of Contact Time

To avoid technical difficulty while deriving such a distribution, we consider in this paper that when the source node enters the connectivity region of the relay node, it keeps the same speed direction. As illustrated in Figure 1(b), this means that when the source node crosses circle  $\mathcal{C}$ , its trajectory is a chord. This assumption can be an acceptable approximation if we consider that the mean epoch distance of the random waypoint motion is higher than the diameter of  $\mathcal{C}$ . Furthermore, we suppose again that incident directions are distributed isotropically. It follows that the randomness of the contact time stems from the interplay of two random variables: on the one hand incidence velocity  $\mathbf{v}_S$  whose probability density function is given by (1), and on the other hand angle  $\theta$  that the chord  $IO$  makes with the ray  $ID$ . Let us first calculate the CDF of the length of the chord  $IO$ , denoted by  $L$ . Clearly, we have  $0 \leq L \leq 2d$ . Note also that  $L$  can be expressed as a function of  $\theta$  and  $d$  according to  $L = 2d \cos(\theta)$ . Then, for all  $0 \leq l \leq 2d$ , we have

$$\Pr\{L \leq l\} = \Pr\{\cos(\theta) \leq \frac{l}{2d}\}.$$

Recalling that  $\theta$  is uniformly distributed over  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ , we find

$$\Pr\{L \leq l\} = 1 - \frac{2}{\pi} \arccos\left(\frac{l}{2d}\right).$$

Therefore, the probability density function of  $L$  can be expressed as follows

$$f_L(l) = \begin{cases} \frac{2}{\pi} \times \frac{1}{\sqrt{4d^2 - l^2}} & \text{if } 0 \leq l \leq 2d \\ 0 & \text{otherwise.} \end{cases}$$

Let us now focus on the CDF of the contact time  $\tau_c$ . Note that  $\tau_c = L/v_S$ . By conditioning on the length  $L$  of the chord  $IO$ , we find

$$\Pr\{\tau_c \leq t\} = \int_0^{2d} \Pr\{\mathbf{v}_S \geq \frac{L}{t} \mid L = l\} \times f_L(l) dl = \int_0^{2d} F_{\mathbf{v}}^c\left(\frac{l}{t}\right) \times f_L(l) dl, \quad (4)$$

where  $F_{\mathbf{v}}^c(v)$  stands for the CCDF of the node source velocity, which can be readily derived from (1), so that

$$F_{\mathbf{v}}^c(s) = \begin{cases} 1 & s \leq v_0 \\ \frac{\ln(v_1) - \ln(s)}{\ln(v_1) - \ln(v_0)} & v_0 \leq s \leq v_1 \\ 0 & s \geq v_1 \end{cases}$$

Remarking that  $v_0 \leq \mathbf{v} \leq v_1$  with probability one, we can calculate the integral involved in (4) by splitting it into three parts over  $[0, tv_0]$ ,  $[tv_0, tv_1]$  and  $[tv_1, 2d]$  so that after elementary calculation, we obtain for all  $0 < t \leq \frac{2d}{v_0}$

$$\Pr\{\tau_c \leq t\} = \frac{2}{\pi} \arcsin\left(\frac{tv_0}{2d}\right) + \frac{2}{\pi \ln(v_1/v_0)} \int_{tv_0}^{tv_1} \frac{\ln(tv_1/l)}{\sqrt{4d^2 - l^2}} dl. \quad (5)$$

It remains now to evaluate the integral involved in (5). Using integration by parts, we obtain

$$\Pr\{\tau_c \leq t\} = \frac{2}{\pi \ln(v_1/v_0)} \int_{tv_0}^{tv_1} \frac{\arcsin(l/2d)}{l} dl.$$

According to [17], we have

$$\int \frac{\arcsin(l/2d)}{l} dl = \frac{l}{2d} + \frac{1}{2 \cdot 3 \cdot 3} \times \left(\frac{l}{2d}\right)^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5 \cdot 5} \times \left(\frac{l}{2d}\right)^5 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} \times \left(\frac{l}{2d}\right)^7 + \dots$$

Recalling that  $0 \leq l \leq 2d$  and remarking that from the third order term, the coefficients involved in all higher order terms vanishes rapidly to zero, as a first approximation we retain only the linear term. Thus, we finally obtain

$$\Pr\{\tau_c \leq t\} \approx \frac{(v_1 - v_0)t}{\pi d \ln(v_1/v_0)} + o(t^3) \quad \text{for} \quad 0 \leq t \leq \frac{2d}{v_0}.$$

## 4 Conclusion

To conclude, we have addressed in this paper some statistical properties of the contact time in DTNs. Our methodology is based on a key result established in statistical physics that when a random walker enters a finite domain under isotropic uniform incidence, the mean length of its trajectories inside the domain depends only on the geometry of the system. This key result allowed us to obtain a closed-form expression for the average value of the contact time under Boolean criterion and using the traditional random waypoint mobility model. In addition, we derived an approximate formula for the probability distribution function of the contact time under the random waypoint mobility model provided that mean length of a jump is higher than the diameter of the contact area. Although this ongoing work reports a new bio-inspired methodology for the analysis of the contact time in DTNs, a number of open questions remain. Indeed, we consider to include some others promising directions in a future revision of this article.

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