Applying Branching Processes to Delay-Tolerant Networks

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Abstract. Mobility models that have been used in the past to study delay tolerant networks (DTNs) have been either too complex to allow for deriving analytical expressions for performance measures, or have been too simplistic. In this paper we identify several classes of DTNs where the dynamics of the number of nodes that have a copy of some packet can be modeled as branching process with migration. Using recent results on such processes in a random environment, we obtain explicit formulae for the first two moments of the number of copies of a file that is propagated in the DTN, for quite general mobility models. Numerical examples illustrate our approach.

1 Introduction

Delay tolerant networks (DTNs) embrace the concept of occasionally-connected networks [8,11], such as sensor networks, wireless networks with alternating connectivity, etc. In this paper, we address packet forwarding in DTNs where connectivity is low and nodes relay packets of other nodes. We focus on two-hop routing schemes [16] in which a relay node that receives a packet from the source does not relay it further to other intermediate nodes. (Such a restriction may be needed in the context of resource limitations or for security reasons.) We show that various dynamics of packet forwarding in DTNs can be described by multi-type branching processes with immigration operating in a random environment. We then use novel tools from branching processes with immigration in order to derive the two first moments of the number of nodes with a copy of the file.

Related work. Before proceeding to the main results, we present a brief overview of the scientific context of the branching processes methodology and to their applications in networking. The first results on branching processes are often attributed to Galton and Watson and date back to the 19th century. At that time, there was a severe concern among aristocratic families that the surnames were becoming extinct. The disappearance of a name of a family was considered as the death of the family and it was thought that the extinct families were replaced by families from lower social layers [2]. F. Galton posed the question of computing the extinction probability of the names in the Educational Times of 1873 [14]. More precisely, assume that each man in generation n has some

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random number of sons in generation $n + 1$, according to a fixed probability distribution that does not vary from individual to individual. What is then the probability that a family dies out? The Reverend Henry William Watson replied with a solution [27]. Together, they then wrote an 1874 paper entitled "On the probability of extinction of families" [15]. Galton and Watson appear to have derived their process independently of the much earlier work by the French statistician I. J. Bienaymé [6] (1845), which was unknown till it was rediscovered in 1962 by Heyde and Senneta, see e.g. [19].

Branching processes with a random environment have been well studied, both with and without immigration, see [5]. For example, conditions are presented for the extinction when the random environment is stationary ergodic. The stability, strong law of large numbers and central limit theorems for multi-type branching processes with immigration in a random environment have been studied in [20,26]. These processes find applications in very diverse fields, including biological systems and queueing theory. For example, McNamara et. Al [23] consider an asexual species with non-overlapping generations. Individuals born in some year, reach maturity and reproduce one year later and then die. The number of individuals of the different genotypes in the consecutive years constitute a multi-type branching process. Prime examples in queueing theory where branching processes with immigration play a major role, include infinite server queues [10], processor sharing queues [17,24], as well as various polling systems [4,25]. The infinite server queue with random environment has been studied recently in [9,12]. These authors assume a independent exponentially distributed interarrival and service times. The theoretical framework applied here allows for explicit expressions for the first and second moments in the more general setting of general stationary ergodic processes describing the contact processes between pairs of nodes and general independent bounded service time, with a Markovian random environment. It builds on the Theory we developed in [13] and in references therein that allows to compute explicitly the two first moments of the branching process for the case of general stationary ergodic immigration process.

2 Theoretical Framework

First, we briefly present the standard (basic) scalar branching process taking integer values. We then present several extensions, including the vector (multi-type) case. In particular, we introduce the framework of [13] that extends branching processes, and yet provides explicit expressions for the first two moments.

2.1 The Scalar Integer-Valued Case

The standard branching is defined as follows. Let X_n be the number of individuals in generation n. Starting with a fixed X_0, we define recursively

$$X_{n+1} = \sum_{i=1}^{X_n} \xi_n^{(i)} \tag{1}$$

where $\xi_n^{(i)}$ are independent and identically distributed random variables taking non-negative integer values. Define $A_n(m) := \sum_{i=1}^{m} \xi_n^{(i)}$ we can rewrite the above as

$$X_{n+1} = A_n(X_n). \tag{2}$$

Branching processes with immigration are defined through the recursion

$$X_{n+1} = A_n(X_n) + B_n. \tag{3}$$

From equations (1) and (2), A_n obviously possess a divisibility property; for any non-negative integers m, m_1 and m_2 such that $m_1 + m_2 = m$, and for any n,

$$A_n(m) = A_n^{(1)}(m_1) + A_n^{(2)}(m_2)$$

where for each n, $A_n^{(1)}$ and $A_n^{(2)}$ are independent random processes, both with the same distribution as A_n.

This divisibility property naturally leads to the definition of branching processes on a continuous state space. We take this property, together with the non-negativity of A_n as the basis to define the continuous state branching processes. Noting that these properties are satisfied by Lévy processes, we define a continuous state branching process as one satisfying (2) where A_n is a non-negative Lévy process. For references as well as for alternative (equivalent) definitions, see [1,7,21,22] and the references therein.

2.2 General Setting

Consider the sequence of random column vectors $X_n \in \mathbb{R}^M$, adhering to,

$$X_{n+1} = A_n(X_n, Y_n) + B_n(Y_n)\,, \quad n \in \mathbb{Z}\,, \tag{4}$$

The process Y_n and the vector valued processes A_n and B_n correspond to the environment process, the branching process and the immigration process, respectively. The random environment Y_n is a stationary ergodic Markov chain, taking values on a finite state-space $\Theta = \{1, 2, \ldots, N\}$; let $P = [p_{ij}]$ denote its transition matrix. The branching processes $A_n : \mathbb{R}^M \times \Theta \to \mathbb{R}^M$ are independent and identically distributed and further adhere to the following assumptions.

– For each $i \in \Theta$, $A_n(\cdot, i)$ has a divisibility property. Let $x = x^1 + x^2 + \ldots + x^k \in \mathbb{R}^M$, then $A_n(x, i)$ has the following representation,

$$A_n(x, i) = \sum_{l=1}^{k} \hat{A}_n^{(l)}(x^l, i)\,, \tag{5}$$

whereby $\hat{A}_n^{(l)}(\cdot, i)$, $l = 1, \ldots, k$, are identically distributed, but not necessarily independent, with the same distribution as $A_n(\cdot, i)$. Branching processes are those in which $\hat{A}_n^{(l)}(\cdot, i)$, $l = 1, \ldots, k$, are independent.

– For each $i \in \Theta$ and $x = [x_1, \ldots, x_M] \in \mathbb{R}^M$, the first and second order moments of $A_n(\cdot, i)$ can be expressed as follows,

$$\mathrm{E}[A_n(x,i)] = \mathcal{A}_i x \,, \quad \mathrm{E}[A_n(x,i)A_n'(x,i)] = F_i(xx') + \sum_{j=1}^{M} x_j \Gamma_{i,j} \,, \tag{6}$$

whereby $\mathcal{A}_i$ and $\Gamma_{i,j}$ are fixed $M \times M$ matrices and F_i is a linear operator that maps $M \times M$ non-negative definite matrices on $M \times M$ non-negative definite matrices and satisfies $F_i(0) = 0$.

Finally, the immigration process $B_n : \Theta \rightarrow \mathbb{R}^M$ is a stationary ergodic sequence of random functions. The first and second order moments are denoted by $b_i = \mathrm{E}[B_0(i)]$ and $\mathcal{B}_{ij}^{(n)} = \mathrm{E}[B_0(i)B_n(j)]$.

Before proceeding to the main theorems, some notation is introduced. Let $\hat{\mathcal{A}}$ denote the block matrix whose ijth block entry is given by $\mathcal{A}_j p_{ji}$ $(i,j \in \Theta)$. Moreover, the following block vector and block matrix simplify notation,

$$\hat{b} = \sum_{i\in\Theta} \pi_i \begin{bmatrix} p_{i1} b_i \\ p_{i2} b_i \\ \vdots \\ p_{iN} b_i \end{bmatrix} , \; \hat{\mathcal{B}}^{(n)} = \sum_{i\in\Theta} \pi_i \begin{bmatrix} \mathcal{B}_{i1}^{(n)} p_{i1} & \mathcal{B}_{i2}^{(n)} p_{i1} & \ldots & \mathcal{B}_{iN}^{(n)} p_{i1} \\ \mathcal{B}_{i1}^{(n)} p_{i2} & \mathcal{B}_{i2}^{(n)} p_{i2} & \ldots & \mathcal{B}_{iN}^{(n)} p_{i2} \\ \vdots & & \ddots & \\ \mathcal{B}_{i1}^{(n)} p_{iN} & \mathcal{B}_{i2}^{(n)} p_{iN} & \ldots & \mathcal{B}_{iN}^{(n)} p_{iN} \end{bmatrix} . \tag{7}$$

The existence of a stationary solution is now asserted by the following theorem. Theorem 2 then provides expressions for the first and second order moments of this solution. The proofs of these theorems can be found in [13].

Theorem 1. *Assume that (i) $b_i < \infty$ component-wise for all $i \in \Theta$; and (ii) that all the eigenvalues of the matrix $\hat{\mathcal{A}}$ are within the open unit disk. Then, there exist a unique stationary solution X_n^*, for $n \in \mathbb{Z}$ such that $\lim_{n\rightarrow\infty} \|X_n - X_n^*\| = 0$, almost surely, for any initial value X_0.*

Theorem 2. *Assume that the conditions of Theorem 1 are satisfied. The conditional first moment vector is then given by,*

$$\mu = [\mathrm{E}[X_0^* 1\{Y_0 = i\}]]_{i\in\Theta} = (\mathcal{I} - \hat{\mathcal{A}})^{-1}\hat{b} \,. \tag{8}$$

Under the additional assumption that the second order moments of $B_0(i)$ are finite, $i \in \Theta$, the elements Ω_i of the conditional second moment matrix of X_0^* are the unique solution of the system of equations,

$$\begin{aligned} \Omega_l &= \mathrm{E}[X_0^*(X_0^*)' 1\{Y_0 = l\}] \\ &= \sum_{k\in\Theta} \Big(F_k(\Omega_k) + \sum_{j=1}^{M} \mu_k^{(j)} \Gamma_k^{(j)} + \mathcal{B}_{kk}^{(0)} \pi_k + \mathcal{A}_k \Lambda_k + \Lambda_k' \mathcal{A}_k' \Big) p_{kl} \,, \end{aligned} \tag{9}$$

$l \in \Theta$, where Λ_k denotes the kth diagonal (block) element of $\sum_{j=0}^{\infty} \hat{\mathcal{A}}^j \hat{\mathcal{B}}^{(j+1)}$ and with $\mu_k^{(j)}$ the jth element of $\mu_k = \mathrm{E}[X_0^* 1\{Y_0 = k\}]$.

3 DTNs with Variable Number of Nodes

Consider a sparse content distribution network with mobile nodes. At each time slot, new mobile nodes may join or may leave this network. A fixed node spreads some content to other nodes of this network. The goal of the network is to offer access to that content to potential (mobile or fixed) clients that may request it. Whenever the source is within the transmission range of another node, it transmits a packet to that node. A two-hop routing scheme is adopted [16]. A relay node that receives a packet from the source does not relay it further to other intermediate nodes of the network. It only delivers it to a client whenever it encounters one. Time is discrete and at each time n, each node has a probability $p_\theta \geq p > 0$ to meet the source node; this probability also depends on the state $\theta \in \Theta$ of a modulating Markov chain. The chain allows us to model correlation between the channel conditions of different mobiles: it models global fluctuations in the channel conditions that affect the whole system simultaneously. For example, if it rains, then the probability that transmission from the source to a mobile j is successful will dicrese for all mobiles. A measure of the efficiency of the network in distributing the content is then the expected number of mobiles that have a copy of the content (packet) as well as its second moment.

Let W_n denote the number of nodes that have the packet at slot n and let Z_n denote the number of nodes that do not have the packet. We have the following recursion,

$$W_{n+1} = \sum_{j=1}^{W_n} \zeta_{n,1}^{(j)} + \sum_{j=1}^{Z_n} \zeta_{n,2}^{(j)} \nu_n^{(j)}, \; Z_{n+1} = \sum_{j=1}^{Z_n} \zeta_{n,2}^{(j)} (1 - \nu_n^{(j)}) + B_n.$$

Here $\zeta_{n,1}^{(j)}$ is the indicator that the jth node that has the packet leaves the system at slot n, $\zeta_{n,2}^{(j)}$ is the indicator that the jth node that does not have the packet leaves the system at slot n and $\nu_n^{(j)}$ is the indicator that the jth node that does not have the packet, receives the packet at slot n. Finally, B_n denotes the number of new nodes that arrive during slot n. Assuming stationary ergodic arrivals of nodes and independent geometrically distributed residence times with mean T, the theoretical framework applies.

Assuming a Markovian environment with two states, its transition probabilities are characterised by the fraction σ that the environment is in state 1 and by the mean time τ to alternate from state 1 to state 2 and back. Figure 1 depicts the mean number of nodes $\mathrm{E}[W]$ that have the packet and the mean number of nodes $\mathrm{E}[Z]$ that do not have the packet. The left pane plots these means vs. T for different values of τ. The mean number of nodes in the system is fixed to 50 by scaling the mean number of arrivals $\mathrm{E}[B]$ in a slot for increasing T. In state 2, a node receives the packet with probability $p_2 = 0.1$ whereas no transmission is possible in state 1 ($p_1 = 0$). Moreover, for all curves, $\sigma = 90\%$. It is readily observed that the mean residence time of a node has a considerable impact on $\mathrm{E}[W]$. Obviously, if nodes remain longer, they carry the packet for a longer time which explains the increase in the mean number of nodes that carry the packet.

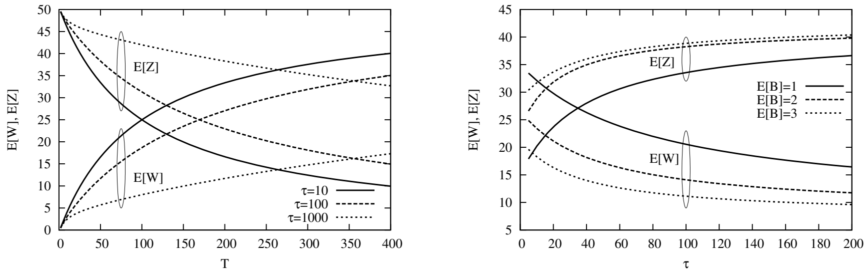

Fig. 1. Mean number of nodes that have the packet and of the number of nodes that do not have the packet vs. T for various values of τ (left) and vs. τ for various values of $\mathrm{E}[B]$ (right).

Further, increasing τ yields lower values of $\mathrm{E}[W]$. This is confirmed by the right pane where $\mathrm{E}[Z]$ and $\mathrm{E}[W]$ are depicted vs. τ for various values of $\mathrm{E}[B]$ and the same parameter settings.

4 Mobility of the Source and the Nodes

We retain the model of the previous section but now replace the channel model by a mobility model. The source node moves according to a random walk through the spatial grid, depicted in Figure 2 (left). In each of the regions of the grid, new nodes arrive according to a stationary ergodic process which then travel through the grid until they leave. If a node is in the same region as the source, the node receives the packet with a fixed (possibly region-dependent) probability.

Let $X_n(k)$ denote the number of nodes in region k at time n with the packet and let $Z_n(k)$ denote the number of nodes without the packet. Further let X_n and Z_n denote the column vectors with elements $X_n(k)$ and $Z_n(k)$, respectively. Let Y_n denote the region where the source node resides at time n — the environment thus tracks the position of the source node — and let $B_n(k)$ denote the number of new nodes that arrive in region k at time n; B_n is a column vector with elements $B_n(k)$. We then have the following recursion,

$$X_{n+1}=\sum_{i=1}^{N}\sum_{j=1}^{X_n(i)}\zeta_{n,1}^{(i,j)}+\sum_{i=1}^{N}\sum_{j=1}^{Z_n(i)}\zeta_{n,2}^{(i,j)}\nu_n^{(i,j)},\ Z_{n+1}=\sum_{i=1}^{N}\sum_{j=1}^{Z_n(i)}\zeta_{n,2}^{(i,j)}(1-\nu_n^{(i,j)})+B_n.$$

Here $\zeta_{n,1}^{(i,j)}$ is a column vector of indicators; its kth element is the indicator that the jth node in region i that has the packet at time n moves to region k. The indicator vector $\zeta_{n,2}^{(i,j)}$ is defined likewise. Its kth element is the indicator that the jth node in region i that does not have the packet at time n moves to region k. Further, $\nu_n^{(i,j)}$ denotes the indicator that the jth node in region i that does not have the packet at time n, receives the packet. Notice that some of the packets may leave the grid as not all packets necessarily move to any of the regions.

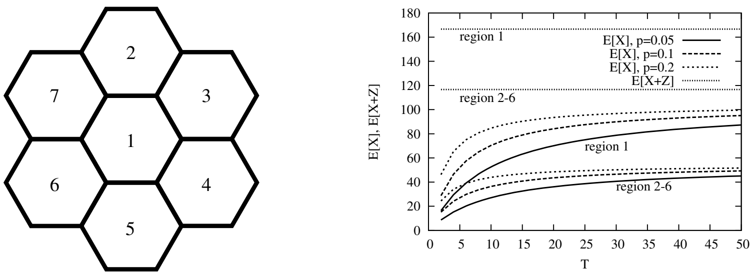

Fig. 2. Spatial grid of the nodes (left) and mean number of nodes with the packet and mean number of nodes vs. the mean residence time in a region (right)

Assuming geometrically distributed residence times (possibly region dependent) and random routing between the regions, the theoretical framework is applicable.

To limit the number of parameters involved, we assume that the mean residence times in the different regions are equal and nodes move to any of the neighbouring regions with probability 1/6, thereby possibly leaving the grid (however, the source node never leaves the grid). A node that does not have the packet which is in the same region as the source node, receives the packet with probability p. The right pane of Figure 2 depicts the mean number of nodes with the packet in the different regions is vs. T for different values of the transmission probability p. The mean numbers of new arrivals in the different regions scale with the residence times of the nodes: $\mathrm{E}\, B^{(i)} = 50/T$ for $i = 1, 2, \ldots, 7$ such that the total mean number of nodes in the different regions $\mathrm{E}[X + Z]$ remains constant. First, notice that by symmetry, the characteristics of regions 2 to 7 are the same. Further, it is clear that longer residence times imply that more nodes receive the packet. Clearly, nodes do not only remain longer in a region but also longer in the grid. Hence, the probability that they receive the packet increases.

5 Conclusions

In this paper, it was shown that some of the dynamics of packet forwarding in DTNs can be described by Markov-modulated branching processes with immigration. The paper illustrates how explicit expressions for the two first moments of the state in DTNs can be obtained. This extends previously known time homogeneous models (without the ramdom environment feature) for which explicit expressions are known for relevant performance measures in DTNs [3,18].

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