Ground-State Entanglement Gives Birth to Quantum Energy Teleportation

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Abstract. Ground-state entanglement induces emergence of negative-energy-density regions in quantum systems by squeezing zero-point oscillation, keeping total energy of the systems nonnegative. By use of the negativity of quantum energy density, protocols of quantum energy teleportation are proposed that transport energy to distant sites by local operations and classical communication. The energy is teleported without breaking any physical laws including causality and local energy conservation. Because intermediate subsystems of the energy transfer channel are not excited during the protocol execution, the protocol attains energy transportation without heat generation in the channel. We discuss the protocol focusing around qubit chains. In addition, we address a related problem of breaking ground-state entanglement by measurements.

Keywords: Entanglement, quantum teleportation, LOCC, quantum measurement.

1 Introduction

Recently protocols called quantum energy teleportation (QET) have been proposed which transport energy by local operations and classical communication (LOCC), respecting causality and local energy conservation. The protocols can be considered for various many-body quantum systems, including qubit chains [1,2], 1+1 dimensional massless Klein-Gordon fields [3], 1+3 dimensional electromagnetic field [4], and cold trapped ions [5]. The key point of the protocol is that there exists quantum correlation between local fluctuations of different sites in the ground state. The root of this correlation is the ground-state entanglement. By virtue of the correlation, a measurement result of local fluctuation in some site includes information about fluctuation in other sites. By selecting and performing a proper local operation based on the announced information, zero-point oscillation of a site far from the measurement site can be more suppressed than that of the ground state, yielding negative energy density. Here the origin of energy density is fixed such that the expectational value vanishes for the ground state. Such negative energy density appears due to quantum interference effects [6]. Even if we have a region with negative energy density in a system, we have other regions with positive energy density and the total energy of the system remains nonnegative. During the above local operation generating

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negative energy density in the system, surplus energy is transferred from the quantum fluctuation to external systems and can be harnessed.

The organization of this report is as followed: In section 2, the relation between ground-state entanglement and emergence of negative energy density is explained. QET is realized by generating negative energy density at a distant site by LOCC. In section 3, a protocol of this QET is discussed for critical Ising spin chains. In section 4, a related problem of breaking ground-state entanglement by measurements are addressed. In section 5, recent results of QET analysis are summarized for other quantum systems.

2 Ground-State Entanglement and Negative Energy Density

The QET protocol is able to work by virtue of ground-state entanglement and emergence of negative energy density. In what follows, let us concentrate on qubit chain systems and explain the entanglement and the negative energy density. First of all, the Hamiltonian H is given by a site sum of energy density operators T_n, where n denotes site number. The origin of T_n can be shifted so as to satisfy

$$\langle g|T_n|g\rangle = 0 \tag{1}$$

without loss of generality. If each T_n is a local operator at site n satisfying $[T_n,\ T_{n'}] = 0$, all T_n can be simultaneously diagonalized. The ground state $|g\rangle$ becomes separable and an eigenstate for the lowest eigenvalue of each T_n. Clearly, in such a situation, T_n is nonnegative. However, the condition $[T_n,\ T_{n'}] = 0$ is not sustained for cases with interactions between qubits, and entangled ground states are generated. It is noted that a correlation function $\langle g|T_n O_m|g\rangle$ of a separable ground state $|g\rangle$ is given by $\langle g|T_n|g\rangle\langle g|O_m|g\rangle$ for a local operator O_m at site m apart far from n. On the other hand, in the case of the entangled ground state $|g\rangle$, this factorization relation does not hold in general:

$$\langle g|T_n O_m|g\rangle \neq \langle g|T_n|g\rangle\langle g|O_m|g\rangle. \tag{2}$$

This ground-state entanglement induces emergence of quantum states with negative energy density as follows. It turns out first that the entangled ground state $|g\rangle$ cannot be an eigenstate of T_n. The reason is following. If the eigenvalue equation $T_n|g\rangle = \tau|g\rangle$ with a real eigenvalue τ is satisfied, the above correlation function must be written as

$$\langle g|T_n O_m|g\rangle = \tau\langle g|O_m|g\rangle = \langle g|T_n|g\rangle\langle g|O_m|g\rangle,$$

where we have used $\langle g|T_n = \tau\langle g|$ and $\tau = \langle g|T_n|g\rangle$. This obviously contradicts Eq. (2). Therefore the entangled ground state $|g\rangle$ satisfying Eq. (2) is not an eigenstate of T_n. Next let us spectral-decompose the operator T_n as

$$T_n = \sum_{\nu,k_\nu} \epsilon_\nu(n)|\epsilon_\nu(n), k_\nu(n)\rangle\langle\epsilon_\nu(n), k_\nu(n)|,$$

where $\epsilon_\nu(n)$ are eigenvalues of T_n, $|\epsilon_\nu(n), k_\nu(n)\rangle$ are corresponding eigenstates, and the index $k_\nu(n)$ denotes the degeneracy freedom of the eigenvalue $\epsilon_\nu(n)$. Because $\{|\epsilon_\nu(n), k_\nu(n)\rangle\}$ is a complete set of orthonormal basis vectors of the total Hilbert space of the qubit chain, the ground state can be uniquely expanded as

$$|g\rangle = \sum_{\nu, k_\nu(n)} g_{\nu, k_\nu(n)} |\epsilon_\nu(n), k_\nu(n)\rangle,$$

where $g_{\nu,k_\nu(n)}$ are complex coefficients of the expansion. By use of this expansion, Eq. (1) gives an equation as follows:

$$\langle g|T_n|g\rangle = \sum_{\nu, k_\nu(n)} \epsilon_\nu(n) \left| g_{\nu,k_\nu(n)} \right|^2 = 0.$$

Clearly, this equation for $g_{\nu,k_\nu(n)}$ has no solution when the lowest eigenvalue $\epsilon_{\min}(n)$ of T_n is positive. The case with $\epsilon_{\min}(n) = 0$ is also prohibited for the equation because, if so, the entangled ground state $|g\rangle$ would become an eigenstate of T_n with its eigenvalue $\tau = 0$ and contradicts Eq. (2), as proven above. This means that $\epsilon_{\min}(n)$ must be negative. It is thereby verified that there exist quantum states $|\epsilon_{\min}(n), k_{\min}(n)\rangle$ with negative energy density due to the ground-state entanglement. Here it should be stressed that, because of Eq. (1), the eigenvalue of the ground state is zero:

$$H|g\rangle = 0,$$

and H is a nonnegative operator. Therefore, even if we have a region with negative energy density in a system, we have other regions with positive energy density so as to make the total energy of the system nonnegative. In the QET protocol, the negative energy density plays a crucial role as seen in the next section.

3 QET Protocol

By use of the negative energy density, protocols of QET can be constructed. In this section, a QET protocol for a critical Ising spin chain [2] is explained. The Hamiltonian is given by a sum of energy density operator T_n: $H = \sum_n T_n$. The operator T_n is given by

$$T_n = -J\sigma_n^z - \frac{J}{2}\sigma_n^x \left(\sigma_{n+1}^x + \sigma_{n-1}^x\right) - \epsilon, \quad (3)$$

where σ_n^z and σ_n^x are Pauli matrices at site n, J and ϵ are real constants. By fine-tuning ϵ, Eq. (1) is attained. The QET protocol is composed of the following three steps: (i) For the ground state $|g\rangle$, an energy sender A performs a local measurement of σ_A which is a one-direction component of the Pauli spin operator acting on A's qubit. Those eigenvalues of σ_A are $(-1)^\mu$ with $\mu = 0, 1$. Let us write the spectral decomposition of σ_A as

$$\sigma_A = \sum_{\mu=0,1} (-1)^\mu P_A(\mu),$$

where the operator $P_A(\mu)$ are projective operators onto the eigenspaces. In this measurement process, A must input positive amount of energy given by

$$E_A = \sum_{\mu=0,1} \langle g|P_A(\mu)\, H P_A(\mu)\, |g\rangle$$

to the qubit chain. (ii) A announces the measurement result μ to an energy receiver B by a classical channel. (iii) B performs a local unitary operation depending on the value of μ. The unitary operator is defined by

$$V_B(\mu) = I\cos\theta + i(-1)^{\mu}\,\sigma_B \sin\theta,$$

where σ_B is a one-direction component of the Pauli spin operator acting on B's qubit, and the above real parameter θ is fixed so as to extract the maximum energy from the chain. In this analysis, we assume that dynamical evolution of the system induced by H is negligible during short time interval t of the protocol: $\exp[-itH] \sim I$. Hence, the quantum state after step (iii) is written as follows.

$$\rho = \sum_{\mu=0,1} V_B(\mu)\, P_A(\mu)\, |g\rangle\langle g| P_A(\mu)\, V_B^{\dagger}(\mu)\,.$$

Using this state, it can be shown that B extracts positive energy $+E_B$ on average from the qubit chain, accompanied by excitations with negative energy $-E_B$ in the qubit chain around B's site in step (iii). In fact, the expectational value of energy after step(iii) is calculated [1] as

$$\mathrm{Tr}[\rho H] = E_A + \frac{\eta}{2}\sin(2\theta) + \frac{\xi}{2}(1-\cos(2\theta)), \tag{4}$$

where ξ and η are given by

$$\xi = \langle g|\sigma_B H \sigma_B|g\rangle \geq 0,$$
$$\eta = i\langle g|\sigma_A [H,\ \sigma_B]\,|g\rangle.$$

The coefficient η is a two-point correlation function of (semi-)local operators of A and B, and turns out to be real. It is a key point that η does not vanish in general because of the ground-state entanglement. By taking a value of θ defined by

$$\cos(2\theta) = \frac{\xi}{\sqrt{\xi^2+\eta^2}},\quad \sin(2\theta) = -\frac{\eta}{\sqrt{\xi^2+\eta^2}},$$

the minimum value of $\mathrm{Tr}[\rho H]$ with respect to θ is written explicitly as

$$\mathrm{Tr}[\rho H] = E_A - \frac{1}{2}\left[\sqrt{\xi^2+\eta^2} - \xi\right].$$

From the viewpoint of local energy conservation, this result implies that, during the operation $V_B(\mu)$, positive amount of energy given by

$$E_B = E_A - \mathrm{Tr}[\rho H] = \frac{1}{2}\left[\sqrt{\xi^2+\eta^2} - \xi\right] > 0 \tag{5}$$

is transferred from the qubit chain to external systems including the device system executing $V_B(\mu)$. In addition, it is possible to calculate analytically the value of E_B for the critical Ising spin chain as follows [2].

$$E_B = \frac{2J}{\pi}\left[\sqrt{1+\left(\frac{\pi}{2}\Delta(|n_A - n_B|)\right)^2} - 1\right], \tag{6}$$

where $\Delta(n)$ is defined by

$$\Delta(n) = \left(\frac{2}{\pi}\right)^n \frac{2^{2n(n-1)}h(n)^4}{(4n^2-1)\,h(2n)}$$

with $h(n) = \prod_{k=1}^{n-1} k^{n-k}$. The asymptotic behavior of $\Delta(n)$ for large n is given by

$$\Delta(n \sim \infty) \sim \frac{1}{4} e^{1/4} 2^{1/12} c^{-3} n^{-9/4}, \tag{7}$$

where the constant c is evaluated as $c \sim 1.28$. Due to the criticality of this model, E_B decays following not an exponential law but a power law ($\propto |n_A - n_B|^{-9/2}$) for large separation.

4 Breaking Ground-State Entanglement by Measurements

In section 3, we have shown that B obtains energy from the qubit chain by the QET protocol. However, even after the last step (iii) of the protocol, there exists residual energy E_A that A had to first deposit to the qubit chain. Let us imagine that A attempts to completely withdraw E_A by local operations after step (iii). If A succeeded in this withdrawing, the energy gain of B might have no cost. However, if so, the total energy of the qubit chain became equal to $-E_B$ and negative. Meanwhile, we know that the total energy of the qubit chain system must be nonnegative. Hence, A cannot withdraw energy larger than $E_A - E_B$ by local operations at site n_A. This means that, in the QET protocol, B has borrowed energy E_B in advance from the qubit chain on security of the deposited energy E_A. The main reason for A's inability to withdraw is because A's local measurement breaks the ground-state entanglement between A's qubit and all the other qubits. The post-measurement state is an exact separable state with no entanglement. If A wants to recover the original state of her qubit with zero energy density, A must recreate the broken entanglement. However, entanglement generation needs nonlocal operations in general. Therefore, A cannot recover the state perfectly by her local operations alone. This interesting aspect poses a residual-energy problem of the ground-state entanglement broken by measurements. Let us imagine that A stops the QET protocol soon after step (i) of the protocol, and attempts to completely withdraw E_A by local operations. By the same argument as the above, it is shown that this attempt never succeeds because A breaks the ground-state entanglement. Of course, for a long time interval beyond the short time scale that we have considered, local cooling is naturally expected to make residual energy in the qubit chain approaching zero by an assist of dynamical evolution induced by the nonlocal Hamiltonian H. However, in this short time interval, the dynamical evolution is not available. Therefore it is concluded that the residual energy in the qubit chain has its nonvanishing minimum value E_r with respect to A's local cooling processes in short time. In order to make the argument more concrete, let us consider a general local cooling operation of A after step (i) obtaining the measurement result μ. The operation is expressed by use of μ-dependent Kraus operators $M_A(\alpha,\mu)$ satisfying

$$\sum_{\alpha} M_A^{\dagger}(\alpha,\mu)M_A(\alpha,\mu) = I. \tag{8}$$

Then the quantum state after this local cooling by A is given by

$$\rho_c = \sum_{\mu,\alpha} M_A(\alpha,\mu)P_A(\mu)\,|g\rangle\langle g|P_A(\mu)\,M_A^{\dagger}(\alpha,\mu). \tag{9}$$

The minimum value E_r of the residual energy with respect to $M_A(\alpha,\mu)$ satisfying Eq. (8) is written as

$$E_r = \min_{\{M_A(\alpha,\mu)\}} \mathrm{Tr}\left[\rho_c H\right]. \tag{10}$$

Evaluation of E_r is performed analytically in the Ising spin chains [2] and given by

$$E_r = \left(\frac{6}{\pi} - 1\right) J > 0,$$

for the critical chain. Surprisingly, A is not able to extract this energy by any local operation in the short time, though it exists in front of A. Because of the nonnegativity of H, it is easily checked by resuming the QET protocol after the local cooling that E_r is lower bounded by the teleported energy E_B in Eq. (5). In addition, the paper [2] gives a stringent argument that the teleported energy in an extended protocol gives a more tight lower bound of residual energy E_r for general qubit chains.

Finally, a comment is added about recent numerical researches of the ground-state entanglement. As a quantitative entanglement measure, the negativity has been computed between separated blocks of qubit chains [7] (the logarithmic negativity for harmonic oscillator chains [8]) showing that at criticality this negativity is a function of the ratio of the separation to the length of the blocks and can be written as a product of a power law and an exponential decay. In our setting of QET, this suggests that change of the entanglement between A's block and B's block after A's local measurement has a similar rapid-decay dependence on the separation with a fixed block length. Thus it may be concluded that the entanglement between A's block and B's block itself is not essential for QET. Though the entanglement between the two blocks may be rapidly damped, E_B shows a power law decay ($\propto n^{-9/2}$) for large separation n, as seen in Eq. (6)

and Eq. (7). This implies in a sense that almost "classical" correlation between A's block and B' block is sufficient to execute QET for large separation, and is expected to be robust against environment disturbance, contrasting to the entanglement fragility. It should be emphasized, however, that this "classical" correlation is originally induced by the ground-state entanglement characterized by Eq. (2). If the ground state is separable, we have no correlation between the blocks.

5 QET for Other Systems

The QET protocols can be considered for other quantum systems. In [3], a protocol of QET for 1+1 dimensional massless scalar fields is analyzed. Though the nonrelativistic treatment for the qubit chain in section 3 is valid for short-time-scale processes of QET in which dynamical evolution induced by the Hamiltonian is negligible, in this relativistic case, the dynamical effect propagates with light velocity, which is the upper bound on the speed of classical communication. Thus, we generally cannot omit global time evolution. It is also noted that any continuous limit of zero lattice spacing cannot be taken for the protocols in the lattice QET models as long as measurements in the protocols are projective, which becomes an obstacle to obtaining a smooth limit. Therefore, in [3], A makes not a projective but instead a well-defined POVM measurement to the vacuum state of the field. After wavepackets with light velocity excited by A's measurement have already passed by the position of B, B extracts energy from the local vacuum state of the field by a unitary operation dependent on the measurement result announced by A. In [4], two QET protocols with discrete and continuous variables are analyzed for 1+3 dimensional electromagnetic field. In the discrete case, a 1/2 spin is coupled with the vacuum fluctuation of the field and measured in order to get one-bit information about the fluctuation. In the continuous case, a harmonic oscillator is coupled with the fluctuation and measured in order to get continuous-variable information about the fluctuation. In the discrete case, the amount of the extracted energy is suppressed by an exponential damping factor when the energy infused by the measurement becomes large. This suppression factor becomes power damping in the continuous case, and it is concluded that more information about the vacuum fluctuation is obtained by the measurement, more energy can be teleported. In [5], a protocol of QET is proposed for trapped ions. N cold ions, which are strongly bound in the y and z directions but weakly bound in an harmonic potential in the x direction, form a linear ion crystal. The first ion that stays at the left edge of the crystal is the gateway of the QET channel where energy is input. The N-th ion that stays at the right edge of the crystal is the exit of the QET channel where the teleported energy is output. Two internal energy levels of the gateway ion are selected and regarded as energy levels of a probe qubit to measure the local phonon fluctuation. The probe qubit is strongly coupled with the phonon fluctuation in the ground state during short time via laser field and is projectively measured. In the measurement models, the kinetic energy of the gateway ion increases after the measurement, but the kinetic energy of other ions and the potential energy of all the ions remain unchanged. The obtained information is announced through a classical channel from the gateway point to the exit point. The speed of the information transfer can be equal to the speed of light in principle, which is much faster than that of the phonon propagation in the ion crystal. The phonons excited at the QET gateway do not arrive at the exit point yet when the information arrives at the exit point. However, by using the announced information, we are able to soon extract energy from the exit ion. Experimental verification of the QET mechanism has not been achieved yet for any system, and is a quite stimulating open problem.

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