Manipulating Frequency Entangled Photons

Laurent Olislager[1], Johann Cussey[2], Anh Tuan Nguyen[3], Philippe Emplit[1],
Serge Massar[4], Jean-Marc Merolla[5], and Kien Phan Huy[5]

[1] Service OPERA-Photonique, CP 194/5, Université Libre de Bruxelles,
Avenue F.D. Roosevelt 50, 1050 Brussels, Belgium
[2] SmartQuantum SA, Espace Phœnix,
Route du Radôme, 22560 Pleumeur-Bodou, France
[3] Service de Physique et Electricité, Institut Meurice,
Haute Ecole Lucia de Brouckère, Avenue Emile Gryzon 1, 1070 Brussels, Belgium
[4] Laboratoire d'Information Quantique, CP 225, Université Libre de Bruxelles,
Boulevard du Triomphe, 1050 Brussels, Belgium
[5] Département d'Optique P.M. Duffieux, Institut FEMTO-ST,
Centre National de la Recherche Scientifique, UMR 6174,
Université de Franche-Comté, 25030 Besançon, France

Abstract. A parametric down conversion source pumped by a
monochromatic laser will produce frequency entangled photon pairs. We
demonstrate this by an experiment in which five-dimensional frequency
entanglement is manipulated at telecommunication wavelengths using
commercially available components such as electro-optic phase modula-
tors and narrowband frequency filters. A theoretical intuition for this
approach is developed by introducing the notion of *frequency bin* entan-
glement. We conclude by showing that using this method one can in
principle violate the CHSH, the CGLMP, and a new – as yet unnamed
– Bell inequalities.

Keywords: Quantum Optics, Entanglement, Frequency Bin.

1 Introduction

Entanglement is one of the most fascinating aspects of quantum mechanics,
useful both for fundamental tests of physical principles and for applications
such as Quantum Key Distribution (QKD). Many different kinds of photonic
entanglement have been produced, including entanglement in polarization [1,2],
momentum [3], angular momentum [4], time-energy [5,6,7] and its discretized
version called *time bins* [8,9]. In the present work we show how energy – or
equivalently frequency – entangled photons can be manipulated directly in the
frequency domain.

Previous work on time-energy entanglement was mainly based on Franson's
original proposal [10] which is based on three key ideas. First, a continuous
pump laser produces time entangled photon pairs: the emission time of each
photon is uncertain, but both photons are emitted simultaneously. Second, one
uses measurements that resolve the time of arrival of the photons. This leads to

the concept of *time bin*: two photons whose arrival time cannot be distinguished by the detectors belong to the same time bin. Third, different time bins are made to interfere by using unbalanced Mach-Zehnder interferometers. Together, these ideas provide a powerful platform to investigate and manipulate quantum entanglement, yielding seminal works such as long distance violation of Bell inequalities [7] and entanglement based QKD [11].

Here we use the same time-energy degree of freedom as in [5,6,7,8,9,10,11], but the way it is manipulated is very different. However at the conceptual level there is an instructive parallel between our approach and that of Franson. First, a narrowband pump laser produces frequency entangled photon pairs: the frequency of each photon is uncertain, but the sum of the frequencies is well defined. Second, we use narrowband filters that resolve the frequency of the detected photons. This leads to the concept of *frequency bin*: two photons whose frequency is so close that they cannot be distinguished are said to lie in the same frequency bin. Third, different frequency bins are made to interfere by using electro-optic phase modulators.

Our approach is inspired by, or related to, several earlier proposals for manipulating qubits in the frequency domain [12,13,14,15,16,17,18]. The experimental techniques we use follow closely those of QKD systems in which the quantum information is encoded in frequency sidebands of an attenuated coherent state [12,13,14,15]. Such systems allow efficient transmission of quantum information at telecommunication wavelengths and constitute a commercial alternative to time bin based QKD. The quantum state can be manipulated with either intensity [12,13] or phase [14] modulators. Here we transpose the setup of [14] to the entangled photon case, thereby showing how phase modulators can manipulate frequency entangled photons. In view of the proven success of this method for QKD, we believe this is a very promising approach for manipulating entangled photons, particularly in the context of long distance quantum communication at telecommunication wavelengths.

In the following we first describe our experiment and give the principle of our method. After presenting our experimental results, we discuss how, using this approach, one should be able to violate Bell inequalities.

2   Experimental Setup

Our experiment is schematized in Fig. 1. Photon pairs are generated by parametric down conversion in a 4 cm long Periodically Poled Lithium Niobate (PPLN) waveguide (HC Photonics). PPLN waveguides have emerged as the preferred photon pair source at telecommunication wavelengths because of their extremely high spectral brightness [19,20]. The narrowband pump (Sacher Lasertechnik, 20s-linewidth $\approx$ 2 MHz, $\lambda_p = 776.1$ nm, $P_p \approx 2$ mW) is removed with a drop filter F insuring more than 30 dB isolation. The pairs, distributed around $\lambda_0 = 1552.2$ nm $= 2\pi c/\omega_0$, are sent through a Polarization Controller (PC) and then separated with a polarization maintaining 3dB-coupler. Interesting cases occur when the photon pair is split: one photon is sent to Alice (A) and the other
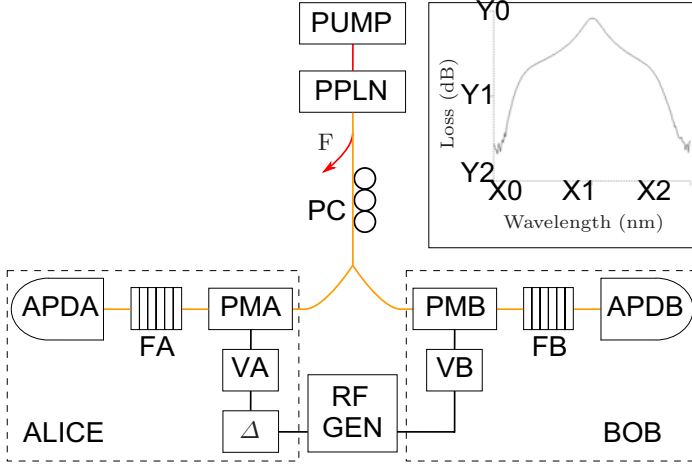
Fig. 1. Experimental setup. The quasi-monochromatic pump laser (PUMP) creates photon pairs in the periodically poled lithium niobate waveguide (PPLN) and is then removed by a filter (F). A fiber polarization controller (PC) is used to align the polarization of the photons with the active axis of electro-optic phase modulators (PM$_{A,B}$). The frequency bins are then selected by narrowband filters (F$_{A,B}$), whose typical transmission spectrum is shown in inset. The photons are detected by avalanche photo-diodes (APD$_{A,B}$). The phase modulators are driven by a 18 GHz radio frequency generator (RF GEN) whose output is controlled by variable attenuators (V$_{A,B}$) and a phase shifter (Δ).

to Bob (B). At the output of the coupler, the photons pass through electro-optic Phase Modulators (PM$_{A,B}$) (EOSPACE, 25 GHz bandwidth, 2.5 dB loss) to which are applied sinusoidally varying voltages at frequency $\Omega/2\pi = 18$ GHz (chosen to provide 20 dB isolation between each frequency bin), with amplitudes V$_{A,B}$ and phases α, β whose difference $\Delta = \alpha - \beta$ can be controlled. The induced time dependent optical phases $\phi_A(t) = a\cos(\Omega t - \alpha)$ and $\phi_B(t) = b\cos(\Omega t - \beta)$, where $a = \pi V_A/V_\pi$, $b = \pi V_B/V_\pi$, and V_π is the half-wave voltage of the modulators, lead to the unitary transformations

$$|\omega\rangle \rightarrow \sum_p |\omega + p\Omega\rangle U_p(c, \theta), \tag{1}$$

where $U_p(c, \theta) = J_p(c)e^{ip(\theta - \pi/2)}$, $c = a$ or b, $\theta = \alpha$ or β, and J_p is the pth-order Bessel function of the first kind. The photons are then sent through narrowband tunable filters (F$_{A,B}$) (AOS GmbH, FWHM ≈ 18 pm, 20 (40) dB isolation at 9 (18) GHz, tuning range ≈ 400 pm) centered on angular frequencies $\omega_{A,B}$ – in our case taken near the degeneracy point ω_0. The use of such narrowband filters together with a spectrally bright PPLN source of entangled photons has been reported previously in the context of four-photon experiments [21,20]. Finally the photons are detected by two Avalanche Photo-Diodes (APD$_{A,B}$) (id Quantique)

and a time-to-amplitude converter performs a coincidence measurement. The maximum coincidence rate was approximately 0.5 Hz.

3 Theoretical Modelisation and Experimental Results

A parametric down conversion source pumped by a monochromatic beam produces an entangled state (subscripts A and B refer to Alice and Bob's photon)

$$|\Phi\rangle = \int d\omega |\omega_0 + \omega\rangle_A |\omega_0 - \omega\rangle_B, \qquad (2)$$

in which the total energy of the photon pair is well defined, but the energy of each photon is uncertain. The narrowband filters select the photons belonging to a small frequency interval. We view all the photons selected by a given filter as belonging to a specific *frequency bin*. The effect of the filters can be idealized by the state

$$|\Psi\rangle = \sum_k |\omega_0 + k\Omega\rangle_A |\omega_0 - k\Omega\rangle_B, \qquad (3)$$

which would correspond to the case of infinitely narrow filters equally spaced in frequency.

Note that the amount of entanglement in Eq. (2) would appear to be infinite. In practice it is approximately given by the ratio of the linewidth of the pairs to the linewidth of the pump laser and can in principle reach very high values, for instance 4 THz / 2 MHz = $2 \ 10^6$ in our case. Note also that for simplicity of notation we have not normalized Eqs. (2, 3). This does not affect our predictions as we are in fact interested in the *ratios* of the probabilities of finding photon A at one frequency and photon B at another frequency. Note finally that taking the Fourier transform of Eqs. (2, 3) would yield a description of the state in terms of time entanglement: the time of arrival of each photon is uncertain, but the difference between the time of arrival of Alice and Bob's photon is well defined.

According to Eq. (1), the phase modulators realize interferences between frequency bins separated by integer multiples of Ω. They thus play the same conceptual role as the Mach-Zehnder interferometers in Franson's scheme which realize interferences between different time bins. Using Eqs. (1, 3), one can readily compute the entangled state after the phase modulation

$$|\Psi\rangle \rightarrow \sum_{n,d} |\omega_0 + (n+d)\Omega\rangle_A |\omega_0 - n\Omega\rangle_B \ c_{nd}(a,b,\alpha,\beta), \qquad (4)$$

with $c_{nd}(a,b,\alpha,\beta) = \sum_p U_p(a,\alpha)U_{d-p}(b,\beta)$. Because of the symmetries of Eqs. (1, 3), the probability $P(n+d, -n|a,b,\alpha,\beta)$ of A and B detecting photons at angular frequencies $\omega_A = \omega_0 + (n+d)\Omega$ and $\omega_B = \omega_0 - n\Omega$ depends only on the sum of the frequencies $\omega_A + \omega_B = 2\omega_0 + d\Omega$ (but not on n) and on the phase difference Δ (but not on $\alpha + \beta$):

$$P(d|a,b,\Delta) = P(n+d, -n|a,b,\alpha,\beta) = |c_{nd}(a,b,\alpha,\beta)|^2. \qquad (5)$$
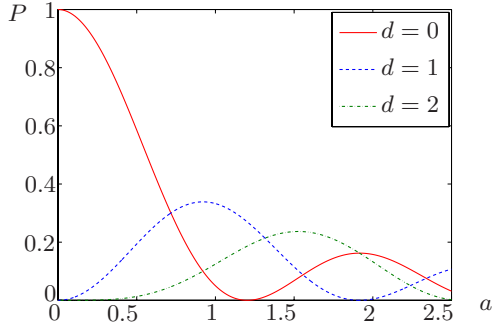
Fig. 2. Theoretical predictions for the probability $P(d|a, a, 0)$ of detecting photons with the sum of the frequencies given by $2\omega_0 + d\Omega$ for $d = 0, 1, 2$, when $a = b$ and $\Delta = 0$

As illustration, the predictions of Eq. (5) are plotted in Fig. 2 when $\Delta = 0$ as a function of the modulation amplitudes $a = b$ (taken to be equal) for $d = 0, 1, 2$. The amount of entanglement that is manipulated by the phase modulators, i.e. the number of frequency bins that interfere together, is approximately given by the number of values of d for which Eq. (5) takes a significant value, and increases when a, b increase.

In Fig. 3, we compare the prediction of Eq. 5 to our experimental results obtained when the phase Δ is scanned, with $a \approx b \approx 1.3$. This value is chosen so that $P(d = 0|a, a, 0)$ almost vanishes, leading to significant $d \neq 0$ contributions. The experimental visibility of interferences is approximately $V = 88\%$. We have also measured coincidence rates when $d = 1$ or 2. From these curves one can deduce that we are manipulating at least five-dimensional entanglement – otherwise the curve for $d = 2$ should be flat.

Two remarks about Fig. 3 are in order. First, we believe that noise and limited visibility are mainly due to drift of pump laser frequency and imperfect polarization control. Second, when changing Δ, because of reflections and interferences in the RF circuit, the amplitudes a and b also changed by up to 30%. This effect was carefully measured, and the curves in Fig. 3 take this into account. (This is why the theoretical curves are not perfectly symmetric with respect to $\Delta = \pi$).

4   Towards Bell Tests

We now discuss how our system can be used to realize quantum non locality experiments. Note that each measurement in principle produces an infinite number of possible outcomes, corresponding to the frequency bin in which the photon is found. (Of course in practice this is limited by the bandwidth of the photon pairs). For this reason it seems more appropriate to consider Bell inequalities adapted to high dimensional systems, such as the CGLMP inequality [22],
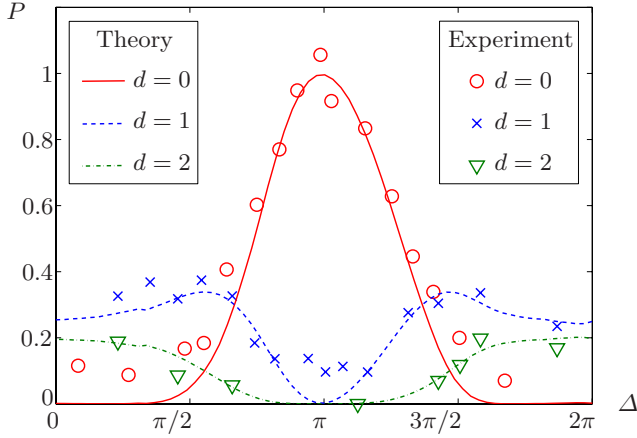
Fig. 3. Theoretical predictions (curves) and experimental measurements (symbols) of the coincidence probability $P(d|a, b, \Delta)$ when $a \approx b \approx 1.3$ and the phase Δ is scanned, for $d = 0, 1, 2$. Experimentally, P is the normalized coincidence rate, i.e. the ratio of true coincidences to accidental coincidences divided by this ratio for $d = 0$ when modulation is off. The acquisition time per point was constant, corresponding to a number of recorded coincidences approximately equal to $10^3 P$.

rather than Bell inequalities for two dimensional systems such as the Clauser-Horn-Shimony-Holt (CHSH) inequality [23]. We compare both cases below.

The three Bell expressions we will consider can all be written as a combination of four terms

$$S = C(A_1 B_1) + C(B_2 A_1) + C(B_1 A_2) - C(B_2 A_2),   \tag{6}$$

where $C(A_i B_j)$, $C(B_j A_i)$ are the *generalized correlators* when Alice chooses measurement setting A_i and Bob chooses measurement setting B_j. By measurement setting we mean a choice of amplitude a (or b) and phase α (or β). The correlators are chosen so that $-1 \leq C(AB), C(BA) \leq +1$, and so that local hidden variable (LHV) theories all satisfy $S_{LHV} \leq 2$.

Let us first consider how the outcomes can be grouped to yield the CHSH expression. We associate with frequency bin $\omega_A = \omega_0 + (n + d)\Omega$ the variable $x = (-1)^{n+d}$, i.e. $x = +1$ (-1) if $n + d$ is even (odd); and similarly we associate with frequency bin $\omega_B = \omega_0 - n\Omega$ the variable $y = (-1)^n$. The value of the correlator is taken to be the product $C_{CHSH} = xy$. In terms of the original variables n, d it can be written as $C_{CHSH}(AB) = P(d = 0 \bmod 2|AB) - P(d = 1 \bmod 2|AB) = C_{CHSH}(BA)$.

A second approach is to use the CGLMP inequality. The infinite dimensional limit of the inequality given in [22,24] corresponds to choosing $C_{CGLMP}(AB) = P(d \geq 0|AB) - P(d < 0|AB)$ and $C_{CGLMP}(BA) = P(d \leq 0|AB) - P(d > 0|AB)$.

A third approach is to define the correlators as

$$C_{EQ}(AB) = P(d = 0|AB) - P(d \neq 0|AB) = C_{EQ}(BA). \tag{7}$$

Inserting these different choices of correlators in Eq. (6) yields three inequivalent Bell expressions. To our knowledge the one based on C_{EQ} has not been considered before in the literature, but it arose naturally from the symmetries of the present problem.

In Fig. 4 we compare how much the correlations Eq. (5) can violate each of these inequalities. We choose for simplicity the modulation amplitudes a_1, a_2, b_1, b_2 to be equal and numerically optimize the phases α_1, α_2, β_1, β_2. Surprisingly we find that the CGLMP expression yields the smallest violation even though it was introduced specifically for high dimensional systems. We attribute this to the fact that the set of measurements explored in the present work is too restricted. On the other hand the largest violation is obtained for the new inequality based on the correlators C_{EQ}.

We have not computed how much the data given in Fig. 3 would violate the above Bell inequalities. The reason is that, as mentioned above, when changing the phase α of Alice's phase modulator, we simultaneously changed the amplitude and phase of Bob's phase modulator. This violates the no-signalling conditions necessary for investigating quantum non locality. Nevertheless, we expect that our setup can significantly violate the above Bell inequalities. Indeed the amount of noise reported in Fig. 3 should yield a Bell expression approximately equal to the maximum value times the visibility $V (= 0.88)$, which for $a = 1.3$ is well above the LHV bound of 2.
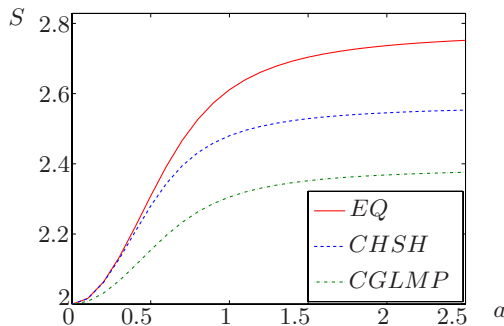
Fig. 4. Theoretical predictions for the violation of the CHSH, the CGLMP, and the EQ Bell inequalities as a function of the amplitude of the modulation a. The settings were chosen to be of the form $a_1 = a_2 = b_1 = b_2 = a$ and the phases $\alpha_{1,2}$ and $\beta_{1,2}$ were numerically optimized.

5   Conclusion

In summary we have demonstrated the manipulation of high dimensional frequency bin entanglement. In view of the proven success [12,13,14,15] of frequency encoding for long distance QKD, this seems a promising technique for quantum communication tasks. Future work will focus on studying the Bell inequalities discussed in the text, and demonstrating entanglement based QKD.

References

1. Aspect, A., Grangier, P., Roger, G.: Phys. Rev. Lett. 47, 460 (1981); 49, 91 (1982)
2. Kwiat, P.G., et al.: Phys. Rev. Lett. 75, 4337 (1995)
3. Rarity, J.G., Tapster, P.R.: Phys. Rev. Lett. 64, 2495 (1990)
4. Mair, A., Vaziri, A., Weihs, G., Zeilinger, A.: Nature 412, 313 (2001)
5. Kwiat, P.G., Steinberg, A.M., Chiao, R.Y.: Phys. Rev. A 47, R2472 (1993)
6. Brendel, J., Mohler, E., Martienssen, W.: Europhys. Lett. 20, 575 (1992)
7. Tittel, W., Brendel, J., Zbinden, H., Gisin, N.: Phys. Rev. Lett. 81, 3563 (1998)
8. Brendel, J., Gisin, N., Tittel, W., Zbinden, H.: Phys. Rev. Lett. 82, 2594 (1999)
9. Thew, R.T., Tanzilli, S., Tittel, W., Zbinden, H., Gisin, N.: Phys. Rev. A 66, 062304 (2002)
10. Franson, J.D.: Phys. Rev. Lett. 62, 2205 (1989)
11. Tittel, W., Brendel, J., Zbinden, H., Gisin, N.: Phys. Rev. Lett. 84, 4737 (2000)
12. Merolla, J.-M., Mazurenko, Y., Goedgebuer, J.-P., Porte, H., Rhodes, W.T.: Opt. Lett. 24, 104 (1999)
13. Merolla, J.-M., Mazurenko, Y., Goedgebuer, J.-P., Rhodes, W.T.: Phys. Rev. Lett. 82, 1656 (1999)
14. Bloch, M., McLaughlin, S.W., Merolla, J.-M., Patois, F.: Opt. Lett. 32, 301 (2007)
15. Cussey, J., Patois, F., Pelloquin, N., Merolla, J.-M.: Proceedings of the Optical Fiber Communication and National Fiber Optic Engineers Conference (2008)
16. Stefanov, A., Zbinden, H., Gisin, N., Suarez, A.: Phys. Rev. A. 67, 042115 (2003)
17. Huntington, E.H., Ralph, T.C.: Phys. Rev. A. 69, 042318 (2004)
18. Huntington, E.H., et al.: Phys. Rev. A. 71, 041802(R) (2005)
19. Tanzilli, S., et al.: Electron. Lett. 37, 26 (2001)
20. Halder, M., et al.: New J. Phys. 10, 023027 (2008)
21. Halder, M., et al.: Nat. Phys. 3, 692 (2007)
22. Collins, D., Gisin, N., Linden, N., Massar, S., Popescu, S.: Phys. Rev. Lett. 88, 040404 (2002)
23. Clauser, J.F., Horne, M.A., Shimony, A., Holt, R.A.: Phys. Rev. Lett. 23, 880 (1969)
24. Zohren, S., Gill, R.D.: Phys. Rev. Lett. 100, 120406 (2008)