Local Transformation of Two EPR Photon Pairs into a Three-Photon W State Using a Polarization Dependent Beamsplitter

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Abstract. We have theoretically proposed and experimentally demonstrated that two EPR photon pairs can be transformed into a three-photon W state by local operation and classical communication (LOCC). The fidelity of the final state to the ideal W state was 0.778 ± 0.043. The obtained expectation value of the witness operator for distinguishing between the three-photon W state and bi-separable states was -0.111 ± 0.043.

Keywords: Multipartite entanglement, W state, LOCC.

1 Introduction

Entanglement has been used for key theoretical and experimental progresses in quantum information science. Unlike bipartite entanglement, where Einstein-Podolsky-Rosen (EPR) pairs of qubits act as a universal resource to prepare any bipartite state by local operation and classical communication (LOCC), there is no N-partite ($N \geq 3$) entanglement which can be used as a universal resource to prepare N-partite states due to the fact that there are distinct classes of multipartite entangled states which cannot be converted into each other by stochastic local operation and classical communication (SLOCC). Greenberger-Horne-Zeilinger (GHZ) and W states are well-known examples of such distinct classes [1].

It is of significance to study how different classes of multipartite entangled states can be prepared among distantly located parties sharing EPR pairs using only LOCC. Previously, preparation of GHZ states from two EPR pairs by LOCC was experimentally demonstrated [2]. Moreover, it was shown that a pseudo tripartite W state could be prepared from a tripartite GHZ state [3] although W and GHZ are distinct classes of entangled states. Such an approximate conversion

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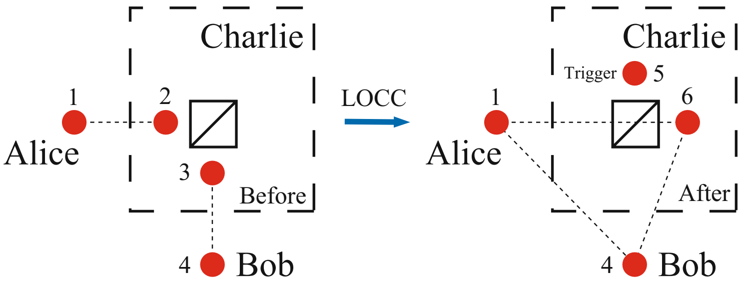

Fig. 1. Concept of local transformation from two EPR photon pairs to the W state using a polarization dependent beamsplitter. Two EPR photon pairs are shared by Alice-Charlie and Bob-Charlie.

has the drawback of trade-off between fidelity and success probbaility, i.e., unit fidelity cannot be achieved with a non-zero success probability.

Here we study a transformation of two EPR photon pairs into a three-photon W state by LOCC with unit fidelity and show an experimental demonstration [4]. When each of Alice-Charlie (modes 1 and 2) and Charlie-Bob (modes 3 and 4) share one EPR pair, the required transformation is performed by a local operation using the two photons on Charlie's side as in Fig. 1. The local operation uses a polarization dependent beamsplitter (PDBS) which has suitable transmission/reflection characteristics for horizontally and vertically polarized photons.

2 Theoretical Analysis

2.1 Optimal Method

We assume that four photons in state $|\,\mathrm{EPR}\rangle_{12}|\,\mathrm{EPR}\rangle_{34} = (|\,\mathrm{HHHH}\rangle_{1234} + |\,\mathrm{HHVV}\rangle_{1234} + |\,\mathrm{VVHH}\rangle_{1234} + |\,\mathrm{VVVV}\rangle_{1234})/2$ are distributed such that Alice has the photon in mode 1, Bob has mode 4, and Charlie has modes 2 and 3. Charlie sends his two photons to a PDBS, whose output modes are labeled as 5 and 6 in Fig. 1. The PDBS transforms the H- and V-polarized photons as

$$\hat{a}^{\dagger}_{2\mathrm{H}} = \sqrt{1-\mu}\,\hat{a}^{\dagger}_{5\mathrm{H}} - \sqrt{\mu}\,\hat{a}^{\dagger}_{6\mathrm{H}}, \quad \hat{a}^{\dagger}_{3\mathrm{H}} = \sqrt{\mu}\,\hat{a}^{\dagger}_{5\mathrm{H}} + \sqrt{1-\mu}\,\hat{a}^{\dagger}_{6\mathrm{H}}, \tag{1}$$

and

$$\hat{a}^{\dagger}_{2\mathrm{V}} = \sqrt{1-\nu}\,\hat{a}^{\dagger}_{5\mathrm{V}} - \sqrt{\nu}\,\hat{a}^{\dagger}_{6\mathrm{V}}, \quad \hat{a}^{\dagger}_{3\mathrm{V}} = \sqrt{\nu}\,\hat{a}^{\dagger}_{5\mathrm{V}} + \sqrt{1-\nu}\,\hat{a}^{\dagger}_{6\mathrm{V}} \tag{2}$$

where $\hat{a}^{\dagger}_{j\mathrm{H}}$ ($\hat{a}^{\dagger}_{j\mathrm{V}}$) denotes the creation operator of H (V)-polarized photon in the j-th mode of PDBS, and μ (ν) is the transmission coefficient for H (V)-polarization. Using the relations given in Eqs. (1) and (2), we find that the action of the PDBS on the four possible input states, $|\,1_{\mathrm{H}}\rangle_2|\,1_{\mathrm{H}}\rangle_3 = \hat{a}^{\dagger}_{2\mathrm{H}}\hat{a}^{\dagger}_{3\mathrm{H}}|\,vac\rangle_{23}$, $|\,1_{\mathrm{H}}\rangle_2|\,1_{\mathrm{V}}\rangle_3 = \hat{a}^{\dagger}_{2\mathrm{H}}\hat{a}^{\dagger}_{3\mathrm{V}}|\,vac\rangle_{23}$, $|\,1_{\mathrm{V}}\rangle_2|\,1_{\mathrm{H}}\rangle_3 = \hat{a}^{\dagger}_{2\mathrm{V}}\hat{a}^{\dagger}_{3\mathrm{H}}|\,vac\rangle_{23}$ and $|\,1_{\mathrm{V}}\rangle_2|\,1_{\mathrm{V}}\rangle_3 = \hat{a}^{\dagger}_{2\mathrm{V}}\hat{a}^{\dagger}_{3\mathrm{V}}|\,vac\rangle_{23}$ with $|\,vac\rangle$ denoting the vacuum state, transforms them into

$$\begin{aligned}
|1_H\rangle_2|1_H\rangle_3 \to{} & \sqrt{2\mu(1-\mu)}\;|2_H\rangle_5|0\rangle_6 \underline{+(1-2\mu)|1_H\rangle_5|1_H\rangle_6} \\
& -\sqrt{2\mu(1-\mu)}\;|0\rangle_5|2_H\rangle_6, \\
|1_H\rangle_2|1_V\rangle_3 \to{} & \sqrt{\nu(1-\mu)}\;|1_V1_H\rangle_5|0\rangle_6 - \underline{\sqrt{\nu\mu}\;|1_V\rangle_5|1_H\rangle_6} \\
& +\underline{\sqrt{(1-\mu)(1-\nu)}\;|1_H\rangle_5|1_V\rangle_6} - \sqrt{\mu(1-\nu)}\;|0\rangle_5|1_V1_H\rangle_6, \\
|1_V\rangle_2|1_H\rangle_3 \to{} & \sqrt{\mu(1-\nu)}\;|1_V1_H\rangle_5|0\rangle_6 + \underline{\sqrt{(1-\nu)(1-\mu)}\;|1_V\rangle_5|1_H\rangle_6}, \\
& -\underline{\sqrt{\mu\nu}\;|1_H\rangle_5|1_V\rangle_6} - \sqrt{\nu(1-\mu)}\;|0\rangle_5|1_V1_H\rangle_6, \\
|1_V\rangle_2|1_V\rangle_3 \to{} & \sqrt{2\nu(1-\nu)}\;|2_V\rangle_5|0\rangle_6 \underline{+(1-2\nu)|1_V\rangle_5|1_V\rangle_6} \\
& -\sqrt{2\nu(1-\nu)}\;|0\rangle_5|2_V\rangle_6.
\end{aligned} \tag{3}$$

Keeping only the cases leading to coincidence detection where a photon is present in each of the modes 1, 4, 5 and 6, we find that the state after the PDBS is given by

$$\begin{aligned}
& \frac{1}{2}\left[(1-2\mu)|HHH\rangle_{146} + \sqrt{(1-\mu)(1-\nu)}|HVV\rangle_{146} - \sqrt{\mu\nu}|VHV\rangle_{146}\right]|H\rangle_5 \\
& +\frac{1}{2}\left[(1-2\nu)|VVV\rangle_{146} - \sqrt{\mu\nu}|HVH\rangle_{146} + \sqrt{(1-\mu)(1-\nu)}|VHH\rangle_{146}\right]|V\rangle_5.
\end{aligned} \tag{4}$$

If Charlie has detected an H-polarized photon or a V-polarized photon in mode 5, he announces it and switches the polarization mode 6 as $|H\rangle_6 \leftrightarrow |V\rangle_6$. At this point, the three parties share the following states

$$\frac{1}{2}\left[(1-2\mu)|HHV\rangle_{146} + \sqrt{(1-\mu)(1-\nu)}|HVH\rangle_{146} - \sqrt{\mu\nu}|VHH\rangle_{146}\right] \tag{5}$$

and

$$\frac{1}{2}\left[(1-2\nu)|VVH\rangle_{146} - \sqrt{\mu\nu}|HVV\rangle_{146} + \sqrt{(1-\mu)(1-\nu)}|VHV\rangle_{146}\right], \tag{6}$$

respectively for H- and V-polarization detection in mode 5. With local filtering, the states in Eqs. (5) and (6) can be transformed into the ideal polarization encoded state $|W_3\rangle \equiv (|HHV\rangle + |HVH\rangle + |VHH\rangle)/\sqrt{3}$. The probabilities of obtaining these cases are

$$p_H \equiv \frac{3}{4}\min\{(2\mu-1)^2,\ (1-\mu)(1-\nu),\ \mu\nu\} \tag{7}$$

and

$$p_V \equiv \frac{3}{4}\min\{(2\nu-1)^2,\ (1-\mu)(1-\nu),\ \mu\nu\}. \tag{8}$$

In the following, we consider the optimization of μ and ν to obtain the highest success probability. For a fixed value of μ, $\min\{(1-\mu)(1-\nu),\ \mu\nu\}$ takes its maximum value $\mu(1-\mu)$ for $\nu = 1-\mu$. The maximum value of $\min\{(2\mu -$

$1)^2,\ \mu(1-\mu)\}$ is 1/5 for $\mu = (5 \pm \sqrt{5})/10$. Therefore, the maximum success probability is $p_{\mathrm{H}} = 3/20$ for $\mu = (5+\sqrt{5})/10$ and $\nu = (5-\sqrt{5})/10$ or vice versa. For this choice of μ and ν, p_{V} also takes its maximum value 3/20. It is noted that for these optimal choices of μ and ν, the states in Eqs. (5) and (6) are already W states and the local filtering is not necessary.

2.2 Experimental Method

In our experiment, we used a sub-optimal choice of the PDBS parameters, $\mu = (7+\sqrt{17})/16$ and $\nu = 1/2$. One of the reasons for this choice is that the two-photon interference for the V polarization is observed with a high visibility, which makes the alignment easier and gives us a clue about how well the two photons from different pairs are overlapped at the PDBS. Setting $\nu = 1/2$ in Eq. (4), we obtain

$$\frac{1}{2}\left[(2\mu-1)|\,\mathrm{HHH}\rangle_{146} + \sqrt{\frac{1-\mu}{2}}|\,\mathrm{HVV}\rangle_{146} - \sqrt{\frac{\mu}{2}}|\,\mathrm{VHV}\rangle_{146}\right]|\,\mathrm{H}\rangle_5$$
$$+\frac{1}{2}\left[-\sqrt{\frac{\mu}{2}}|\,\mathrm{HVH}\rangle_{146} + \sqrt{\frac{1-\mu}{2}}|\,\mathrm{VHH}\rangle_{146}\right]|\,\mathrm{V}\rangle_5, \quad (9)$$

where we see that if Charlie has detected a V-polarized photon in mode 5, three parties share a bi-separable state $(\sqrt{\mu}|\,\mathrm{HV}\rangle_{14} + \sqrt{1-\mu}|\,\mathrm{VH}\rangle_{14})|\,\mathrm{H}\rangle_6/2\sqrt{2}$. On the other hand, if Charlie has detected an H-polarized photon in mode 5 and Alice introduces a phase shift locally, they will end up with a W-like state from which $|\,\mathrm{W}_3\rangle$ can be prepared by equalizing the weights of the components with local filtering. From Fig. 2, we see that the success probability is optimized when $(\mu-1/2)^2 = (1-\mu)/8$ or $(\mu-1/2)^2 = \mu/8$. These equations, respectively, give

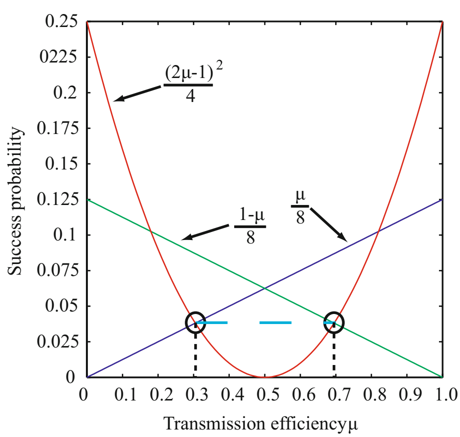

Fig. 2. Success probability of each terms by detecting H-polarized photon on Charlie's side. The circle gives $\mu = (9-\sqrt{17})/16$ and $\mu = (7+\sqrt{17})/16$.

$\mu = (7+\sqrt{17})/16$ and $\mu = (9-\sqrt{17})/16$, both of which lead to the same success probability. Substituting $\mu = (7+\sqrt{17})/16$ in Eq. (9), and keeping only the terms leading to coincidence detection triggered by an H-photon detection in mode 5, the post-selected state in modes 1, 4 and 6 becomes

$$\frac{\sqrt{9-\sqrt{17}}}{8\sqrt{2}}\left[|\,\mathrm{HHV}\rangle + |\,\mathrm{HVH}\rangle + \frac{\sqrt{7+\sqrt{17}}}{\sqrt{9-\sqrt{17}}}|\,\mathrm{VHH}\rangle\right]_{146}, \tag{10}$$

after Charlie compensates the phase shift locally and changes the polarization of his photon in mode 6. We see that the component $|\,\mathrm{VHH}\rangle$ is the only one with a V-photon in mode 1; thus its weight can be equalized to the others by introducing polarization dependent losses in mode 1. Then the final state becomes

$$\sqrt{3(9-\sqrt{17})/128}|\,\mathrm{W}_3\rangle_{146} \tag{11}$$

implying that local transformation of two EPR photon pairs into a $|\,\mathrm{W}_3\rangle$ is achieved with unit fidelity at a success probability of $3(9-\sqrt{17})/128 \sim 11.4\%$.

3 Experimental Demonstration

The experimental setup is shown in Fig. 3 (a). Two EPR photon pairs were prepared by using parametric down-conversion at a pair of type-I phase-matched β-barium borate (BBO) crystals stacked together with optical axes orthogonal to

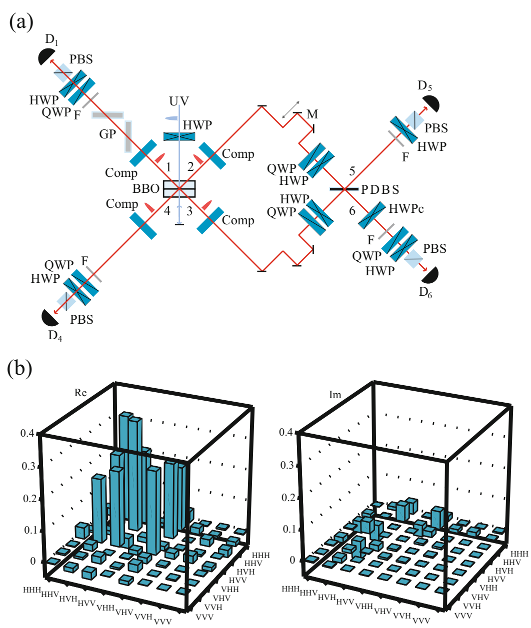

Fig. 3. (a) Proposed experimental setup. (b) Real and imaginary parts of the reconstructed density matrix of the experimentally obtained W state.

Table 1. Peres-Horodecki criterion, Concurrence and Entanglement of Formation (EOF) of the prepared marginal bipartite states

	Peres-Horodecki criterion	Concurrence	EOF
modes 1-4	-0.091 ± 0.026	0.322 ± 0.073	0.244 ± 0.066
modes 1-6	-0.143 ± 0.030	0.421 ± 0.066	0.263 ± 0.065
modes 4-6	-0.123 ± 0.027	0.415 ± 0.068	0.195 ± 0.065

each other. Then one photon from each of the EPR photon pairs was mixed at the PDBS. Whenever it was confirmed that an H-polarized photon was in mode 5, a three-photon W state was prepared in the rest of output modes (1, 4 and 6).

The density matrix of the three-photon state was estimated as in Fig. 3 (b) using quantum state tomography [5]. The fidelity of the converted state to the ideal three-photon W state was 0.778 ± 0.043. The obtained expectation value of the witness operator for distinguishing between the three-photon W state and biseparable states (including separable states)) [6] was -0.111 ± 0.043. This negative value proves that the prepared state is not a biseparable state. Since marginal bipartite states of a W state should be entangled, we have tested all pairwise combinations for the prepared W state. We evaluated the entanglement of the marginal bipartite states using the Peres-Horodecki criterion [7,8], concurrence [9] and entanglmement of formation (EOF) [10] which, respectively, have the values of -0.206, 0.667 and 0.55 for marginal entangled states of an ideal three-photon W state. The results of these analysis for the experimentally prepared W-state are given in Table. 1, which clearly shows the existence of entanglement in these marginal bipartite states.

4 Conclusion

We have theoretically proposed and experimentally demonstrated a scheme in which two EPR pairs are converted into a three-photon W-state using LOCC. The required transformation is provided by a polarization dependent beamsplitter, phase compensator and polarization dependent losses. Through these results, it is now possible to generate arbitrary three-qubit states of W and GHZ classes via LOCC starting from a single resource of two EPR pairs.

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