Interferometric Technique for Density Matrix Reconstruction by On/Off Detectors

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Abstract. The density matrix provides the most complete description of a quantum optical state and a scheme addressed to its reconstruction for arbitrary sources can be fundamental for several applications, ranging from quantum information to the foundations of quantum mechanics and quantum optics. We demonstrate an innovative state reconstruction technique, which provides the density matrix of a field mode and requires only avalanche photodetectors (coupled with phase modulation with respect to a local oscillator), without any phase or amplitude discrimination power. It represents an alternative of simpler implementation to quantum homodyne tomography.

Keywords: Nanomechanical quantum systems and their interaction with nonclassical light.

1 Introduction

Gaining knowledge on the density matrix of an arbitrary quantum state is fundamental for several applications, ranging from quantum information [1] to the foundations of quantum mechanics [2] and quantum optics [3, 4]. Quantum tomography [5, 6, 7], which requires the measurement of a continuous set of field quadrature, can deliver such kind of information, but since it is based on homodyne detection, it turns out to be rather a complicate technique when applied to short pulses in photo-counting regime. Since also reliable photo-counting detectors are not available at the moment[8, 9, 10, 11, 12, 13] (in the sense that photo-detectors that can operate as photon counters are rather rare and affected either by a low quantum efficiency or require cryogenic conditions), various theoretical studies [14, 15, 16, 17] have been addressed to obtain a reconstruction of the (diagonal) elements of the density matrix by using the information that one can achieve by means of realistic detectors, more precisely, the frequencies of the *off* events at different quantum efficiencies of the detector. The technique

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found a favourable experimental test in [18, 19, 20, 21, 22, 23], where a very satisfactory reconstruction of the statistics of mono-partite and bi-partite quantum optical states was obtained by using on/off detectors following the method of Ref.[16, 17].

Furthermore, full state reconstruction is possible by a suitable processing of the data obtained with on/off detectors, such as single-photon avalanche photodiodes coupled with some phase modulation. This modulation of the signal, corresponds to the application of a coherent displacement_by mixing the state under investigation with a known coherent reference in a Mach-Zehnder interferometer [24], and measuring or reconstructing the photon distribution for different values of the modulation one has enough information for a full state reconstruction of the original state[25].

In this paper, we present an experimental application of this method addressed to the full reconstruction of the density matrix for two possible optical states (coherent or pseudo-thermal) by using on/off detection coupled to phase modulation. The work is structured as follows: the first section reviews the method to reconstruct the density matrix and gives a simple description of the experiment. In the second section we explain in details our setup. In the third section we present the results concerning a coherent and a pseudo-thermal signal. The fourth section closes the paper with some concluding remarks.

2 Reconstruction Method

In this section we briefly introduce the density matrix reconstruction method proposed in Ref. [25]. Let ρ be the density matrix of the state we want to reconstruct. The reconstruction is based on the measurement of the photon distribution of the displaced state $\rho(\beta) \equiv D(\beta)\,\rho\,D^{\dagger}(\beta)$, $\beta = |\beta|\,e^{i\varphi}$, for fixed $|\beta|$ and different values of φ, i.e., $p_n(\varphi) = \langle n|\rho(\beta)|n\rangle$. The density matrix elements of the reconstructed state ϱ are then given by [25]:

$$\langle m+s|\varrho|m\rangle = \frac{1}{\mathcal{N}_\varphi}\sum_{l=1}^{\mathcal{N}_\varphi}\sum_{n=0}^{\overline{n}} F_{m,n}^{(s)}\, e^{is\varphi_l}\, p_n(\varphi_l), \tag{1}$$

where $\mathcal{N}_\varphi$ is the number of phase φ_l considered, $l = 1, \ldots, \mathcal{N}_\varphi$, $\overline{n}$ is such that $\varrho_{kh} \equiv \langle h|\varrho|k\rangle$ can be neglected if $h, k > \overline{n}$, and $F_{m,n}^{(s)}$ are functions of $|\beta|$ whose analytic expression is reported in Ref. [25].

The challenging task is now the measurement of the photon statistics (*i. e.* the amplitudes of the diagonal elements in the density matrix expressed in Fock basis), since this for a direct measurement requires number-resolving photodetectors. An alternative method has been given so far in Ref. [16, 17], where it was shown that the photon distribution p_n for a quantum optical state can be retrieved via on/off detectors. Assuming negligible dark counts, the *off* probability is related to the p_n's thanks to the relation:

$$P_0(\eta) = \sum_n (1-\eta)^n p_n, \tag{2}$$

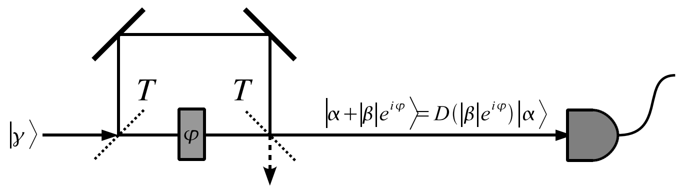

Fig. 1. Simplified sketch of the experiment aimed to reconstruct the density matrix of a coherent state $|\alpha\rangle$. A signal excited to the coherent state $|\gamma\rangle$, γ real, enters a Mach-Zehnder interferometer with two beam splitters with transmissivity T. The output field can be written as $D(|\beta|e^{i\varphi})|\alpha\rangle$, where $|\beta| = T\gamma$, $\alpha = R\gamma$ and φ is the phase difference between the signals added by the interferometer. The other output is discharged.

η being the quantum efficiency of the detector. Measuring a given signal for different quantum efficiencies η_ν, $\nu = 1, \ldots, N$, starting form Eq. 2 we obtain a statistical model for the positive parameters p_n, which can be solved by *Expectation Maximization Algorithm* (EM) [16, 17].

The additional phase information needed by Eq. (1) can be obtained following the interferometric scheme proposed in [25]. As a first step toward a test of that, here we analyze the reconstruction of the density matrix of a coherent state $|\alpha\rangle$ and, without loss of generality, we assume the amplitude α as real. Fig. 1 shows a simplified version of the scheme used in our experiment. The amplitude of the coherent state and the one of the displacement can be also modified by reducing the power along one of the optical paths. This scheme allows to vary φ by suitably tuning the interferometer and, then, the $p_n(\varphi)$ in Eq. (1) are finally retrieved by on/off measurements onto the output field and EM algorithm.

3 Experimental Setup

The setup for the implementation of our experiment for the coherent state is depicted in Fig. 2.

The output of a He-Ne laser ($\lambda = 632.8$ nm) is lowered to single photon regime by neutral filters. The spatial profile of the beam is purified by a spatial filter realized by two converging lenses and a 100 μm diameter wide pinhole. The laser cavity is also preserved by backreflections, which may cause instability by means of an optical isolator consisting in a Faraday rotator between two orthogonal polarizers.

After a beam-splitter, part of the beam is addressed to a control detector in order to monitor the laser amplitude fluctuations, while the remaining part, which is the signal to be reconstructed, is sent to the interferometer, its main structure consisting in a single invar block custom designed and developed at INRIM. Fig. 3 shows the interferometer itself and Fig. 4 reports on the expected stability of the system against thermal noise as simulated in Ansys software environment. The interferometer also features a piezo-movement system, which

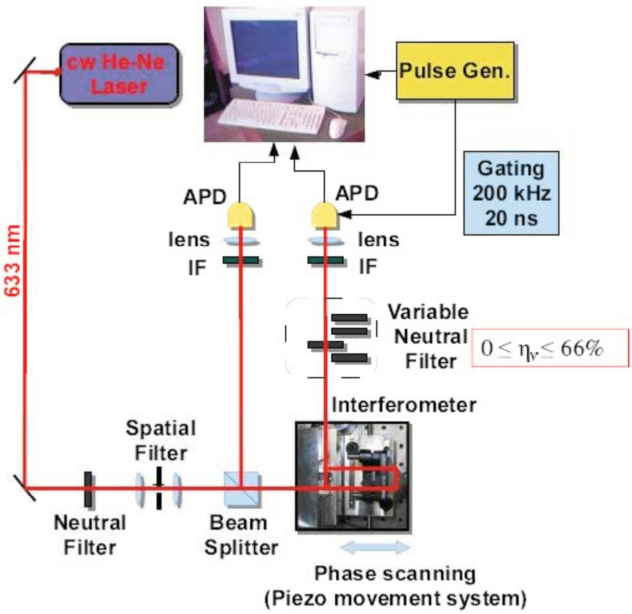

Fig. 2. Setup for the reconstruction of the density matrix for a coherent state. The emission of a He-Ne laser ($\lambda = 632.8$ nm) is lowered to single photon regime by neutral filters. A spatial filter realized by two converging lenses and a $100\mu m$ diameter-wide pinhole purifies the shape of the beam and allows to select a single spatial mode. A beam-splitter reflects part of the beam to a control detector used to monitor the laser amplitude fluctuations, while the remaining part, which is the signal to be reconstructed, is sent to the interferometer. The phase between the "short" and "long" paths in the interferometer can be changed by driving the position of the reflecting prism by means of a piezo-movement system. A set of variable neutral filters allows to collect photons for different values of the quantum efficiency. The detectors used are Perkin-Elmer Single Photon Avalanche Photodiode(SPCM-AQR) gated by a 20 ns wide time window with (repetition rate = 200 kHz). A single run consists of 5 repetitions of 4 seconds acquisitions and events are recorded by a NI-6602 PCI counting module.

allows to scan the phase between the "short" and "long" paths by driving the position of the reflecting prism with nanometric resolution and high stability.

For each position of the prism, the "no-click" probabilities are collected for different sets of neutral filters in front a Perkin-Elmer Single Photon Avalanche Photodiode (SPCM-AQR).

The detector is gated by a 20 ns wide time window with a repetition rate of 200 kHz. In order to obtain a reasonable statistics, a single run consists of 5 repetitions of 4 seconds acquisitions. Events are recorded by a NI-6602 PCI counting module.

In a second acquisition the signal to be reconstructed was a pseudo-thermal state simulated by inserting an Arecchi's rotating glass in the signal arm of the interferometer.

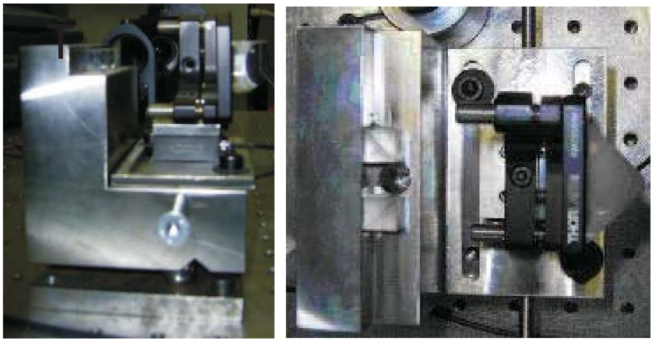

Fig. 3. Side view (left) and top view (right) of the realized interferometer

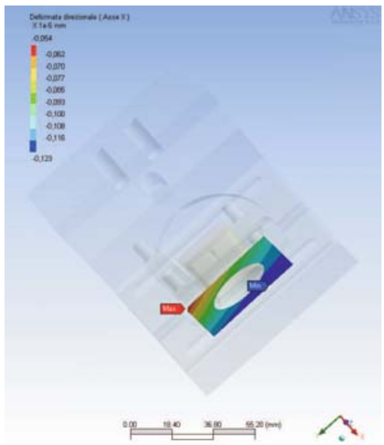

Fig. 4. Ansys simulation of the longitudinal stability of the systems for different external conditions and materials. The simulation shows an expected maximum expansion for the system below the order of the nanometer.

4 Experimental Results

In Fig. 5 the interference fringes at the output of the interferometer are shown. We have chosen $\mathcal{N}_\varphi = 11$ different phases and the average energy E has been obtained from the reconstructed $p_n(\varphi)$. Even if the visibility is not very high, for coherent signals this is not an issue, since this can be controlled by rescaling the experimental data without affecting the nature of the reconstructed density matrix.

In the bottom part of Fig. 6 we report the reconstructed density matrix in the Fock representation (diagonal and subdiagonal) for a coherent state with real amplitude $\alpha \simeq 1.8$ and a thermal state with average number of photons equal to $n_{th} \simeq 1.4$.

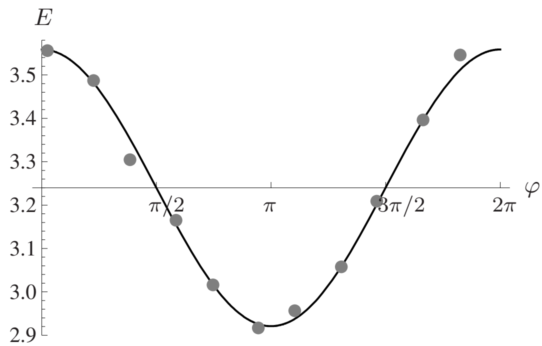

Fig. 5. Interference fringes: experimental data (gray disks) and fit (solid line)

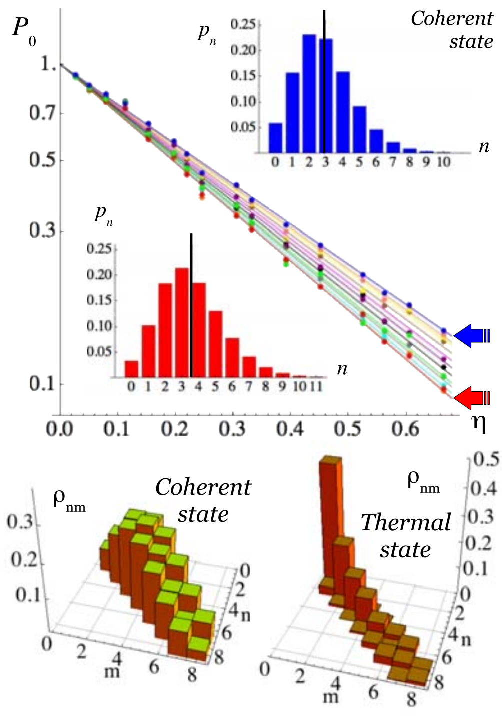

Fig. 6. (Color online) Upper plot: *off* frequencies as a function of the quantum efficiency when the signal is a coherent state and for different phase-shifts. The two insets show the reconstructed photon distributions for the two phase-modulated versions of the signal corresponding to maximum and minimum visibility at the output of the Mach-Zehnder interferometer. The vertical black bars denote the mean value of the photon number for the two distributions, $\langle a^\dagger a\rangle = 3.5$ and $\langle a^\dagger a\rangle = 2.9$. lower left plot: (left) the corresponding reconstructed density matrix in the Fock representation (diagonal and subdiagonal elements). lower right plot: density matrix for the signal excited in a thermal state.

As it is apparent from the plots the off-diagonal elements are correctly reproduced in both cases despite the limited visibility. Here the raw data are frequencies of the *off* event as a function of the detector efficiency, taken at different phase modulations, ϕ, whereas the intermediate step corresponds to the reconstruction of the photon distribution for the phase-modulated signals. In our experiments we used $|\alpha|^2 = 0.01$ for the coherent state and $|\alpha|^2 = 1.77$ for the thermal state. The use of a larger $\mathcal{N}_\varphi$ would allow the reliable reconstruction of far off-diagonal elements, which is not possible in the present configuration. In the insets of Fig. 6 we report the reconstructed distributions at the minimum and maximum of the interference fringes.

The evaluation of uncertainties on the reconstructed states involves the contributions of experimental fluctuations of on/off frequencies as well as the statistical fluctuations connected with photon-number reconstruction. For our purposes this implies that neither large displacement amplitudes may be employed, nor states with large field and/or energy may be reliably reconstructed, although the mean values of the fields measured here are definitely non-negligible. For the regime of weak field or low energy, the density matrix can be safely evaluated from experimental data (see Fig. 7).

Notice also that any uncertainty in the nominal efficiency η_{max} of the involved photodetectors does not substantially affect the reconstruction [16].

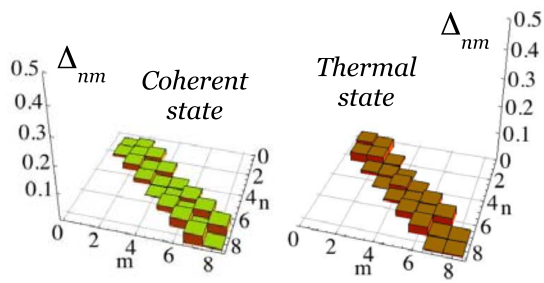

Fig. 7. (Color online) Difference $\Delta_{nm} = |\varrho_{nm}^{exp} - \varrho_{nm}^{th}|$ between reconstructed and theoretical values of the density matrix elements for the coherent (left) and thermal (right) states used in our experiments

We point out that our scheme allows to estimate the uncertainty in real time, that it is suitable both for pure and mixed states and that statistic information is obtained by sampling a discrete matrix rather than measuring a continuous distribution in phase space as homodyne quantum tomography schemes.

In order to improve the efficiency of the reconstruction scheme, the next efforts will be aimed in the directions of optimizing the visibility of the interference fringes and of extending the reconstruction to elements in density matrix farther from the diagonal by increasing the number of phase steps.

5 Concluding Remarks

We demonstrated the partial reconstruction of the density matrix both of a coherent state and a pseudo-thermal one following Ref. [25]. Our results show the validity of the method and prompt to further efforts in order to provide complete reconstruction of the density matrix for arbitrary quantum optical sources by using on/off detection coupled to phase measurements.

Acknowledgments

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