The Cognitive Radio Channel: From Spectrum Sensing to Message Cribbing

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Abstract. Cognitive radios have been considered as an enabling technology toward resolving the spectrum scarcity through spectrum reuse facilitated by the cognitive capability of secondary users. Recent studies, however, have gone beyond spectrum sensing at the medium access control layer; it was conceived to have cognitive radios that are capable of extract information pertaining to the physical layer from the primary users to enable coexistence of multiple users. We take upon this newly proposed approach but refrain from imposing the unrealistic assumption of non-causal cooperation. Specifically, we study the so-called cognitive radio channels from the message cribbing perspective where the term 'cognitive' capability is strictly causal. Information theoretic performance bounds are obtained which shed lights on the impact of such causal cognitive capability.

Keywords: cognitive radio channel, message cribbing, interference channel, channel capacity.

1 Introduction

Cognitive radios have been proposed as a solution to the spectrum congestion problem and has attracted much interest from both academia and industry. The principal idea of cognitive radio systems is the capability of sensing the communication environment and changing transmission and reception parameters to better utilize the wireless spectrum and avoid interfering with licensed users. The research of cognitive radios is mainly concerned with spectrum sharing and interference management. Most of the previous literature explores the methods and protocols for the cognitive radio devices to sense the unoccupied spectrum and communicate efficiently with negligible interference to the primary users. However, those efforts are principally based on orthogonal transmission, in the sense that at a certain time slot with a certain frequency, only one user can occupy the spectrum. This orthogonalization approach, while increasing the spectral efficiency through dynamic spectrum access, is intrinsically limited by the available dimensions of the communication resource hypercube. Inevitably, exponentially increasing demand will soon put strain on this proposed paradigm. Recent researches in information theory goes beyond this spectrum sensing regime, and

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we present our current work in the context of this new development. Referred to as the cognitive radio channel [1], studies have been reported where primary and secondary users may simultaneously transmit without compromising the primary user's throughput [2, 3]. This is largely due to the enhanced cognitive capability of the secondary user. In particular, the cognitive user is capable of more than just sensing the spectrum; instead, it was assumed that it has a mechanism to extract information pertaining to the primary user's transmission, both its content and the relevant channel conditions.

The model assumed in [1] is in essence an interference channel (see, e.g., [4]) and the message for the primary user is assumed to be known non-causally at the secondary user, hence the term interference channel with degraded message sets (IC-DMS). Attempts in studying the capacity limits of cognitive radio channels based on this model include, among others, [1, 2, 3, 5, 6, 7, 8]. In [2] and [3], the authors derived the capacity rate region for Gaussian IC-DMS with weak interference. In [5], the capacity region for IC-DMS with strong interference was determined. Later, more general coding schemes were proposed in [6] and [7], which include the results in [2] and [3] as special cases. Most recently, the authors proposed in [8] an achievable rate region that generalizes those of [2, 3, 5, 6, 7] and includes Marton's broadcast channel region as its subset.

The fact that various tight capacity results can be obtained for IC-DMS can be primarily attributed to the assumption of non-causal cooperation, i.e., the primary user's message is known *a priori* at the secondary user's transmitter. In practice, however, such assumption is rarely valid except in perhaps the limiting case when the cooperation link is close to ideal. In the present paper, we report the latest wok in making the information theoretic study on cognitive radio channels a step closer to reality. Instead of the non-causal assumption of the degraded message set, we assume instead the message cribbing setting [9] where the cognitive transmitter receives causal channel feedback in deciding what to transmit. Initial attempt to study the causal cognitive radio channel was reported in [8]. The problem becomes nearly intractable for the general case - we note that the interference channel itself is still an open problem. Closed-form capacity bounds were obtained in [8] by imposing additional constraint such as physical degradedness. Here, we focus on the Z interference channel (ZIC) where the interference link is from the primary transmitter to the secondary receiver. This simplified model is largely motivated by many proposed cognitive radio schemes that require the so-called 'interference temperature' at the primary receiver to be sufficiently low. Thus the ZIC studied in this paper can be considered as an approximation to such cognitive radio channel. Preliminary studies of the cognitive Z channel has been reported in [10].

This cognitive Z channel is also a special case of the so-called interference channel with generalized feedback (IC-GF) [11, 12, 13, 8] where both transmitters receive channel feedback. While various capacity bounds were proposed in [11, 12, 13], the lack of concrete capacity results can be largely attributed to the complexity of the model itself. The cognitive Z channel, on the other hand, has some intrinsic structures not present in the general IC-GF. Exploitation of these structures may lead to more intuitive capacity bounds and is the focus of this paper.

The rest of this paper is organized as follows. In Section 2, we introduce the channel model of the cognitive Z channel. In Section 3, we analyze the lower bounds of the capacity region for different values of channel parameters. In Section 4, we propose an outer bound for the capacity region and discuss the case when the sum rate capacity can be obtained under certain channel conditions. Conclusion and discussions are given in Section 5.

2 Channel Model

The (causal) cognitive ZIC is illustrated in Fig.1. User 1 has message $W_1 \in \{1, 2, \cdots, 2^{nR_1}\}$ to be transmitted to receiver 1 ($Y_1^{'}$), and user 2 has message $W_2 \in \{1, 2, \cdots, 2^{nR_2}\}$ for receiver 2 ($Y_3^{'}$). In addition, user 2 can listen to the transmitted signal from user 1 through a noisy channel ($Y_2^{'}$). Thus, the channel model is given by

$$Y_1^{'} = h_{11}X_1^{'} + Z_1^{'} \tag{1}$$

$$Y_2^{'} = h_{12}X_1^{'} + Z_2^{'} \tag{2}$$

$$Y_3^{'} = h_{13}X_1^{'} + h_{23}X_2^{'} + Z_3^{'} \tag{3}$$

where h_{11}, h_{12}, h_{13} and h_{23} are fixed real positive numbers. $Z_1{'} \sim N(0, N_1)$, $Z_2{'} \sim N(0, N_2)$ and $Z_3{'} \sim N(0, N_3)$ are independent Gaussian random variables. The average power constraints of the input signals are

$$\frac{1}{n}\sum_{i=1}^{n}(x_{ti}^{'})^2 \le P_t^{'} \tag{4}$$

where $t = 1, 2$. From (2), we have assumed implicitly perfect echo cancellation.

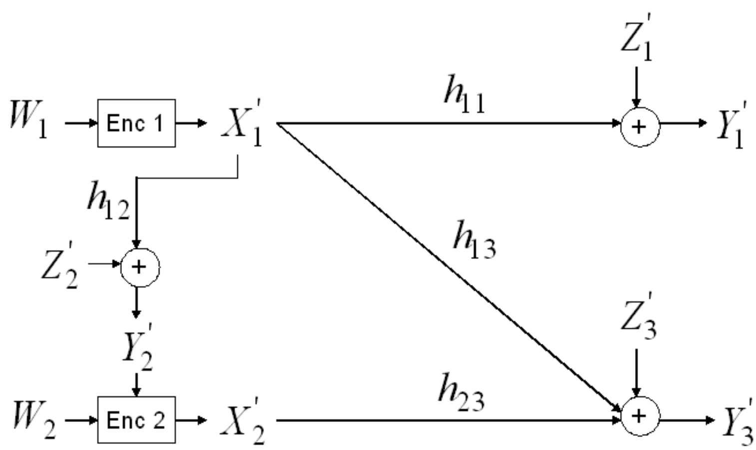

Fig. 1. The general model of the cognitive ZIC

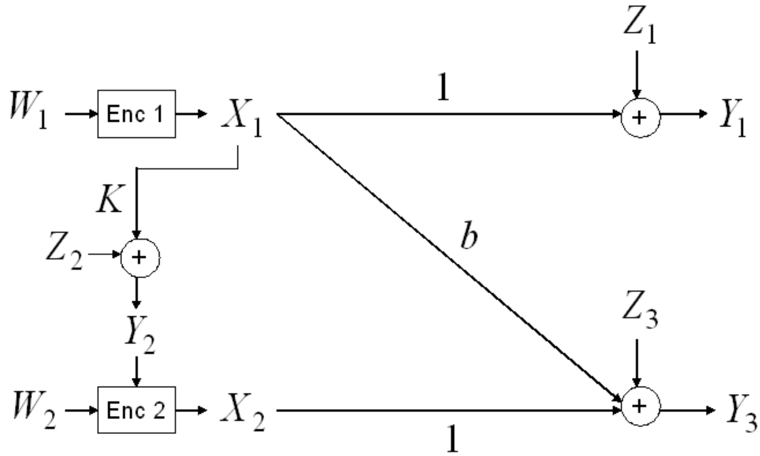

Fig. 2. The standard model of the cognitive ZIC

Lemma 1. *Any cognitive ZIC described by (1)-(3) is equivalent, in its capacity region, to the following cognitive ZIC in standard form*

$$Y_1 = X_1 + Z_1 \tag{5}$$
$$Y_2 = KX_1 + Z_2 \tag{6}$$
$$Y_3 = bX_1 + X_2 + Z_3 \tag{7}$$

where Z_1, Z_2 and Z_3 are independent zero mean, unit variance Gaussian variables, and X_1, X_2 are subject to respective power constraints P_1 and P_2. K and b are deterministic real numbers with $0 \leq K < \infty$, $0 \leq b < \infty$.

The proof is through a simple scaling transformation similar to that of [4], hence omitted.

The encoding functions f_1 and f_2 for users 1 and 2 are respectively:

$$\mathbf{x}_1 = f_1(W_1) \tag{8}$$
$$x_{2i} = f_2(W_2, y_{21}, \cdots, y_{2,i-1}) \tag{9}$$

for $i = 1, 2, \cdots, n$. We only consider deterministic encoders, as nondeterministic encoders do not enlarge the capacity region [9, Appendix D].

3 Capacity Lower Bounds

The cognitive ZIC includes the following two extreme cases: the classic ZIC (corresponding to $K = 0$) and the ZIC with degraded message sets ($K = \infty$). To simplify our notation, we define $\gamma(x) \triangleq \frac{1}{2}\log(1 + x)$.

For the classic ZIC, when $b \geq 1$, the capacity region, denoted by $\mathcal{R}_1$, is given below

$$R_1 \leq \gamma(P_1) \triangleq C_1 \tag{10}$$
$$R_2 \leq \gamma(P_2) \triangleq C_2 \tag{11}$$
$$R_1 + R_2 \leq \gamma(b^2 P_1 + P_2) \tag{12}$$

Apparently, $\mathcal{R}_1$ is an inner bound of the corresponding cognitive ZIC. When $b < 1$, we do not know the whole capacity region, but the sum rate capacity is known to be:

$$R_1 + R_2 \leq \gamma(P_1) + \gamma\left(\frac{P_2}{1 + b^2 P_1}\right) \tag{13}$$

which is achieved when user 1 is transmitting at its maximum rate, and user 2 is transmitting at a rate such that its message can be decoded at receiver 2 by treating user 1's signal as noise.

On the other extreme, for the ZIC with degraded message sets, where user 2 has *a priori* knowledge of user 1's message, the capacity region, denoted by $\mathcal{R}_2$, is the following rectangle, for all $b \geq 0$:

$$R_1 \leq C_1 \tag{14}$$

$$R_2 \leq C_2 \tag{15}$$

This is because for the Gaussian channel considered in this paper, user 2 can dirty paper code its own message treating user 1's signal as known interference [14]. Therefore, this is equivalent to two parallel interference free channels. $\mathcal{R}_2$ serves as a natural outer bound to the capacity region of the cognitive ZIC.

Little is known for the cognitive ZIC besides of the two extreme cases. The difficulty for the case with finite K is that it is not clear what is the optimal way to utilize the channel feedback at transmitter 2. Any information overheard through the cognitive link pertains only to message W_1, yet the absence of link from X_2 to Y_1 implies that existence of the cognitive link can not directly benefit the rate R_1 through cooperative transmission. On the other hand, since X_1 interferes receiver Y_3, encoding scheme should explore the potential of facilitating interference cancellation for the secondary user due to the existence of the cognitive link. In the following, we describe several cases where we can obtain closed form capacity bounds for the cognitive ZIC and discuss their implication in terms of the impact of the "cognitive capability" on the capacity.

3.1 $b^2 \geq 1 + P_2$

In the absence of the cognitive link, this reduces to the ZIC with very strong interference. The capacity region of such ZIC coincides with the outer bound $\mathcal{R}_2$ for the cognitive ZIC, suggesting that $\mathcal{R}_2$ is indeed the capacity region. Notice this is the case where there is no need to utilize the channel feedback Y_2 at user 2, as far as the capacity region is concerned.

3.2 $1 \leq b^2 < 1 + P_2$

This is a very interesting case. In the absence of the cognitive link, the capacity region is the pentagon described by $\mathcal{R}_1$. On the other hand, for ZIC with degraded message sets, the capacity is a rectangle, $\mathcal{R}_2$. The capacity region for the cognitive ZIC with $1 \leq b^2 < 1 + P_2$ should be between these two regions. We define another region $\mathcal{R}_3$ as the union of all nonnegative rate pairs (R_1, R_2) such that:

$$R_1 \leq \gamma(K^2 \alpha P_1) \tag{16}$$

$$R_1 \leq \gamma(P_1) \tag{17}$$

$$R_1 \leq \gamma(K^2(\alpha - \beta)P_1) + \gamma\left(\frac{b^2 \beta P_1}{1 + b^2(1-\beta)P_1 + P_2}\right) \tag{18}$$

$$R_1 \leq \gamma((1-\beta)P_1) + \gamma\left(\frac{b^2 \beta P_1}{1 + b^2(1-\beta)P_1 + P_2}\right) \tag{19}$$

$$R_2 \leq \gamma\left(\frac{P_2}{1 + b^2(\alpha - \beta)P_1}\right) \tag{20}$$

where $0 \leq \beta \leq \alpha \leq 1$.

Theorem 1. *The the convex hull of the union of $\mathcal{R}_1$ and $\mathcal{R}_3$ is achievable for the cognitive ZIC.*

Remark: Notice that while the encoding scheme mimics that for the relay channel [15], this is for a different purpose. The reason is that the new message in each block is required to be decoded by Y_2 with a maximum rate $\gamma(K^2 \alpha P_1)$. In the case of $K^2 \alpha > 1$, Y_1 will not be able to decode this new message; the cell index transmitted in the next block is to ensure reliable decoding of the message by Y_1. Thus decoding of W_1 at Y_1 is accomplished in two steps, much like the way decoding is done at the receiver for the classic three node relay channel: in block $i-1$, the received signal Y_1 allows the partition of message indices into $\gamma((\alpha - \beta)P_1)$ subsets, thereby allowing the construction of the ambiguity set; in block i the cell index can be reliably decoded and the message index is uniquely decided as the intersection of the ambiguity set and the cell index. In addition to facilitate the decoding of W_1 at Y_1, the cell index itself allows interference cancellation for the cognitive transceiver pair through dirty paper coding and decoding as it is known at the beginning of the each block for transmitter 2.

Some numerical examples are given in Fig.3. We see from Fig.3 that with different values of K, the subset relation between $\mathcal{R}_1$ and $\mathcal{R}_3$ varies. To find out their precise relation, we only need to consider the corner points of the two regions as $\mathcal{R}_1$ is a convex hull of its corner points whereas $\mathcal{R}_3$ is a convex region. For region $\mathcal{R}_1$, when $R_1 = C_1$,

$$R_2 = \frac{1}{2}\log\left(\frac{1 + b^2 P_1 + P_2}{1 + P_1}\right). \tag{21}$$

For region $\mathcal{R}_3$, when $R_1 = C_1$,

$$R_2 = \frac{1}{2}\log\left(1 + \frac{P_2}{1 + b^2 P_1 / K^2}\right). \tag{22}$$

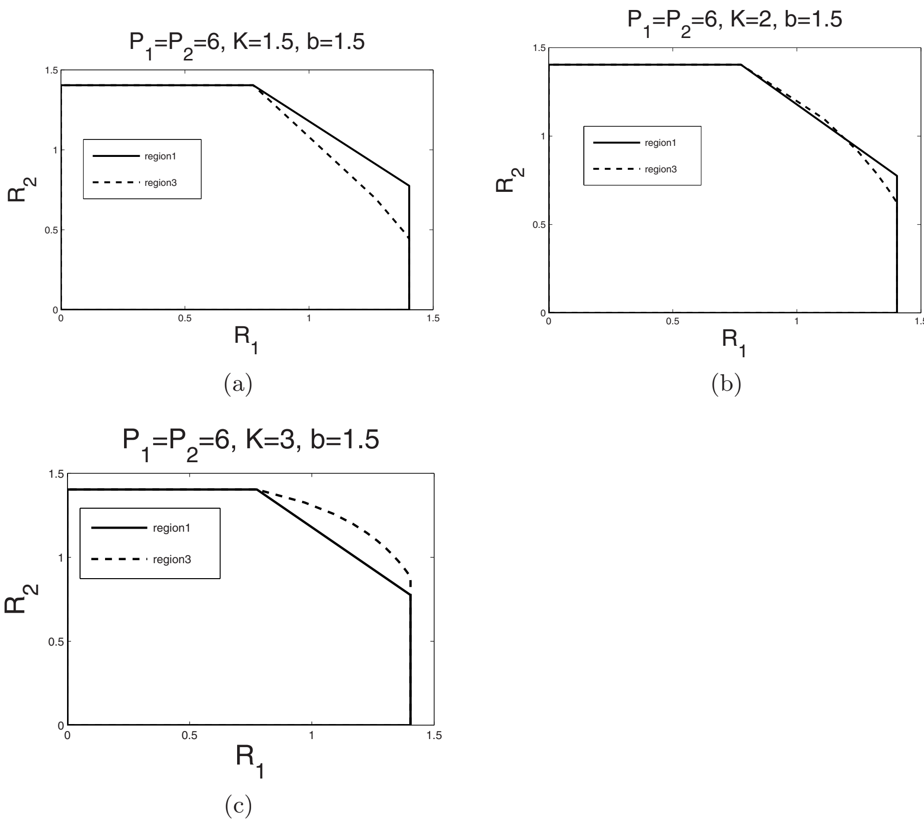

Fig. 3. (a) $P_1 = P_2 = 6, K = 1.5, b = 1.5$. (b) $P_1 = P_2 = 6, K = 2, b = 1.5$. (c) $P_1 = P_2 = 6, K = 3, b = 1.5$.

In order for $\mathcal{R}_1$ to be a subset of $\mathcal{R}_3$, we need

$$\frac{1 + b^2 P_1 + P_2}{1 + P_1} \leq 1 + \frac{P_2}{1 + b^2 P_1 / K^2} \tag{23}$$

which yields

$$K^2 \geq b^2 \cdot \frac{(b^2 - 1)P_1 + P_2}{1 + P_2 - b^2} \tag{24}$$

That is, when K is large enough, $\mathcal{R}_3$ dominates $\mathcal{R}_1$, i.e., the coding scheme where receiver 2 needs to decode both users' messages, which is capacity achieving for the classic ZIC with strong interference, is no longer optimal here. However, when K is small enough, especially when K is near 1, $\mathcal{R}_1$ will dominate $\mathcal{R}_3$, thus the cognitive link between the two users appears to be of no consequence for the proposed encoding scheme.

The reason why $\mathcal{R}_1$ can sometimes outperform $\mathcal{R}_3$ is that we apply sequential decoding at receiver 2, i.e., Y_3 decodes part of user 1's new message (W_{12}) first, then decodes W_2. To improve the rate region, we can apply simultaneous decoding for W_{12} and W_2 as in the multiple access channel, which leads to the following constraints at receiver 2:

$$R_{12} \leq I(W_{12}; Y_3 U_2) \tag{25}$$

$$R_2 \leq I(U_2; Y_3 W_{12}) - I(U_2; S_{11}) \tag{26}$$

$$R_{12} + R_2 \leq I(U_2 W_{12}; Y_3) - I(U_2; S_{11}) \tag{27}$$

where W_{12} is part of W_1 to be decoded by Y_3; S_{11} is the cell index of the message W_{11}, which is the other part of W_1; U_2 is an auxiliary variable for dirty paper coding W_2 against S_{11}. To evaluate (25)-(27), let $W_{12} \sim N(0, \beta P_1)$, $S_{11} \sim N(0, (1-\alpha)P_1)$, $U_2 = X_2 + \mu S_{11}$, where $X_2 \sim N(0, P_2)$ and μ is a deterministic real number. Thus, the right hand side of (25)-(27) can be evaluated as

$$\zeta_1 = \frac{1}{2}\log\left(\frac{(P_2 + \mu^2\bar{\alpha}P_1)(P_2 + b^2P_1 + 1) - (P_2 + \mu b\bar{\alpha}P_1)^2}{(P_2 + \mu^2\bar{\alpha}P_1)(P_2 + b^2\bar{\beta}P_1 + 1) - (P_2 + \mu b\bar{\alpha}P_1)^2}\right) \tag{28}$$

$$\zeta_2 = \frac{1}{2}\log\left(\frac{P_2(P_2 + b^2\bar{\beta}P_1 + 1)}{(P_2 + \mu^2\bar{\alpha}P_1)(P_2 + b^2\bar{\beta}P_1 + 1) - (P_2 + \mu b\bar{\alpha}P_1)^2}\right) \tag{29}$$

$$\zeta_3 = \frac{1}{2}\log\left(\frac{P_2(P_2 + b^2P_1 + 1)}{(P_2 + \mu^2\bar{\alpha}P_1)(P_2 + b^2\bar{\beta}P_1 + 1) - (P_2 + \mu b\bar{\alpha}P_1)^2}\right) \tag{30}$$

Plugging (28)-(30) into (25)-(27), we can define a new region, $\mathcal{R}_4$, based on the idea of simultaneous decoding at receiver 2:

$$R_1 \leq \gamma(K^2\alpha P_1) \tag{31}$$

$$R_1 \leq \gamma(P_1) \tag{32}$$

$$R_1 \leq \gamma(K^2(\alpha - \beta)P_1) + \gamma(\beta P_1) \tag{33}$$

$$R_1 \leq \gamma(K^2(\alpha - \beta)P_1) + \zeta_1 \tag{34}$$

$$R_1 \leq \gamma((1 - \beta)P_1) + \zeta_1 \tag{35}$$

$$R_2 \leq \zeta_2 \tag{36}$$

$$R_1 + R_2 \leq \gamma(K^2(\alpha - \beta)P_1) + \zeta_3 \tag{37}$$

$$R_1 + R_2 \leq \gamma((1 - \beta)P_1) + \zeta_3 \tag{38}$$

for all $0 \leq \beta \leq \alpha \leq 1$, $-\infty < \mu < \infty$, $\alpha + \bar{\alpha} = 1$ and $\beta + \bar{\beta} = 1$.

Theorem 2. *Region $\mathcal{R}_4$ is achievable for the cognitive ZIC.*

Comparison of $\mathcal{R}_1$, $\mathcal{R}_3$ and $\mathcal{R}_4$ is given in Fig.4. It can be seen that for all values of K, $\mathcal{R}_4$ is always the superset of both $\mathcal{R}_1$ and $\mathcal{R}_3$.

It is worth noting that the two achievable regions $\mathcal{R}_1$ and $\mathcal{R}_3$ have a common corner point

$$R_1 = \gamma\left(\frac{b^2P_1}{1 + P_2}\right), R_2 = C_2. \tag{39}$$

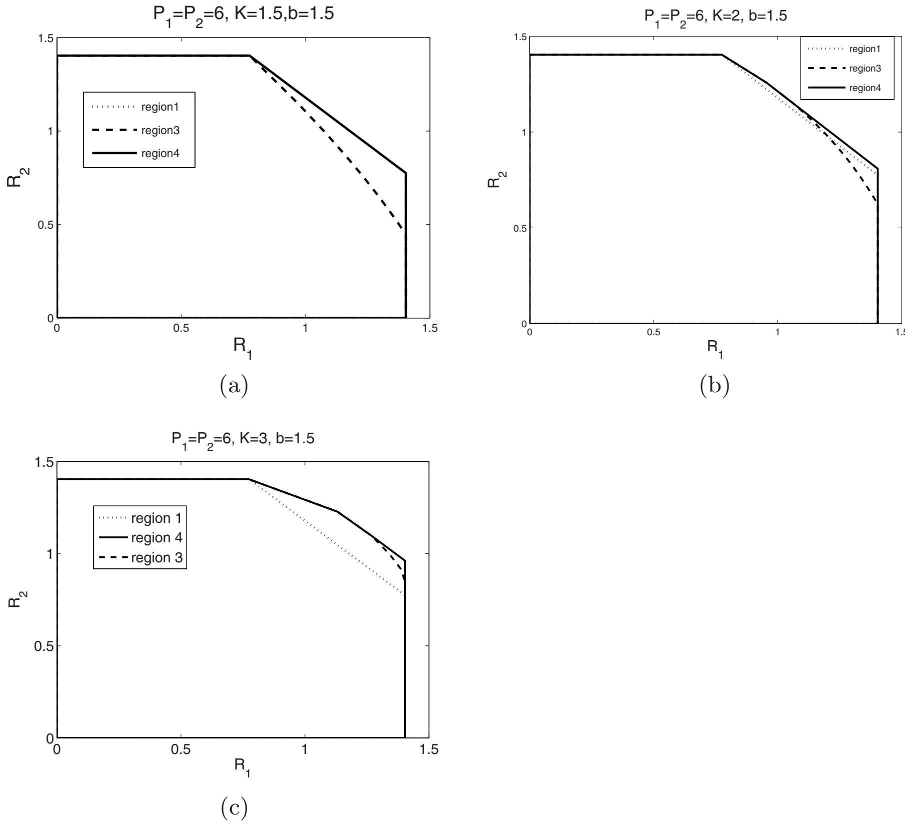

Fig. 4. Comparison of $\mathcal{R}_1$, $\mathcal{R}_3$ and $\mathcal{R}_4$. (a) $P_1 = P_2 = 6, K = 1.5, b = 1.5$. (b) $P_1 = P_2 = 6, K = 2, b = 1.5$. (c) $P_1 = P_2 = 6, K = 3, b = 1.5$.

It is conjectured that for the classic ZIC, this is indeed the corner point of the capacity region [16, 17]. It is not clear that with the help of the cognitive link, this corner point may be extended using some other coding schemes for finite K.

3.3 $b^2 < 1$

For the weak interference case, we only know the sum rate capacity of the classic ZIC achieved at the corner point

$$R_1 = C_1, R_2 = \gamma\left(\frac{P_2}{1 + b^2 P_1}\right). \tag{40}$$

The other corner point of the known achievable region for the ZIC is

$$R_1 = \gamma\left(\frac{b^2 P_1}{1 + P_2}\right), R_2 = C_2. \tag{41}$$

Let us define $\mathcal{R}_5$ as the Han-Kobayashi region [18] with $Q = \phi$ and Gaussian inputs for the classical ZIC. Then, after Fourier-Motzkin elimination, and removing redundant inequalities due to $b < 1$, $\mathcal{R}_5$ can be expressed by

$$R_1 \leq \gamma(\alpha P_1) + \gamma\left(\frac{b^2(1-\alpha)P_1}{1 + b^2\alpha P_1 + P_2}\right) \tag{42}$$

$$R_2 \leq \gamma\left(\frac{P_2}{1 + b^2\alpha P_1}\right) \tag{43}$$

for $\alpha \in [0, 1]$. For the cognitive ZIC with weak interference, the regions $\mathcal{R}_3$ and $\mathcal{R}_4$ are still achievable. We can compare these three regions for different values of K, as plotted in Fig.5.

When $K > 1$, $\mathcal{R}_3$ and $\mathcal{R}_4$ are the same and they outperform the HK region.

When $K = 1$, regions $\mathcal{R}_3$ and $\mathcal{R}_4$ are indistinguishable from $\mathcal{R}_5$. In fact, we can prove that $\mathcal{R}_3$ is indeed inequivalent to $\mathcal{R}_5$ for $b < 1$ and $K = 1$.

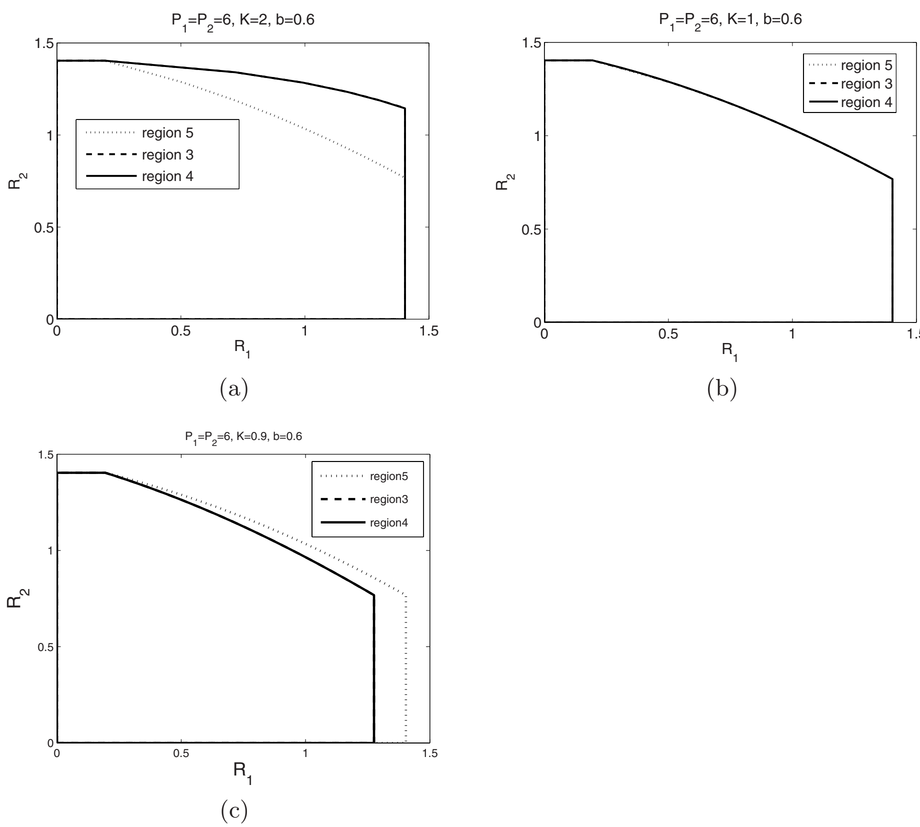

Fig. 5. Comparison of $\mathcal{R}_3$, $\mathcal{R}_5$ and $\mathcal{R}_5$. (a) $P_1 = P_2 = 6, K = 2, b = 0.6$. (b) $P_1 = P_2 = 6, K = 1, b = 0.6$. (c) $P_1 = P_2 = 6, K = 0.9, b = 0.6$.

Lemma 2. *$\mathcal{R}_3$ is equivalent to $\mathcal{R}_5$ for $b < 1$ and $K = 1$.*

When $K < 1$, $\mathcal{R}_3$ and $\mathcal{R}_4$ are still the same, and they are outperformed by HK region. It is clear that, when $K \leq 1$, the idea of utilizing the cognitive link and applying dirty paper coding to boost R_2 is strictly suboptimal for the proposed coding scheme. Again, it is not clear if there is any coding scheme that can improve the rate region by utilizing the cooperating link when $K \leq 1$.

4 Capacity Outer Bounds

Besides the trivial outer bound (14)-(15) mentioned above, in this section, we propose a nontrivial outer bound to the capacity region of the cognitive ZIC when $b \leq 1$, and discuss some interesting implications from this result. Before introducing the outer bound, let us consider the following lemma first.

Lemma 3. *If $b \leq 1$, the capacity region of the cognitive ZIC (5)-(7) is the same as the cognitive ZIC given below:*

$$Y_1^{'} = X_1 + Z_1 \tag{44}$$

$$Y_2^{'} = KX_1 + Z_2 \tag{45}$$

$$Y_3^{'} = bY_1^{'} + X_2 + \tilde{Z}_3 \tag{46}$$

where $\tilde{Z}_3 \sim N(0, 1 - b^2)$ is independent of all the other random variables. Thus, given X_2,

$$X_1 \Longrightarrow Y_1^{'} \Longrightarrow Y_3^{'} \tag{47}$$

forms a Markov chain.

With the lemma above, we are ready to derive our outer bound given in the following theorem.

Theorem 3. *The capacity region of the cognitive ZIC is upper bounded by the union of all nonnegative rate pairs (R_1, R_2), denoted by $\mathcal{R}_o$, satisfying*

$$R_1 \leq I(X_1; Y_1Y_2|UX_2) \tag{48}$$

$$R_2 \leq I(UX_2; Y_3) \tag{49}$$

for all joint distributions that factor as

$$p(u)p(x_1x_2|u)p(y_1y_2y_3|x_1x_2). \tag{50}$$

Proof. Define $U_i = (Y_1^{i-1}, Y_2^{i-1}, W_2, X_2^{i-1})$, and we have

$$nR_1 = H(W_1|W_2) \tag{51}$$

$$= I(W_1; Y_1^n|W_2) + H(W_1|Y_1^n, W_2) \tag{52}$$

$$\leq I(W_1; Y_1^n|W_2) + n\epsilon_1 \tag{53}$$

$$\leq I(W_1; Y_1^n Y_2^n | W_2) + n\epsilon_1 \quad (54)$$

$$= \sum_{i=1}^{n} I(W_1; Y_{1i} Y_{2i} | Y_1^{i-1} Y_2^{i-1} W_2) + n\epsilon_1 \quad (55)$$

$$= \sum_{i=1}^{n} I(W_1; Y_{1i} Y_{2i} | Y_1^{i-1} Y_2^{i-1} W_2 X_2^i) + n\epsilon_1 \quad (56)$$

$$= \sum_{i=1}^{n} I(X_{1i}; Y_{1i} Y_{2i} | U_i X_{2i}) + n\epsilon_1 \quad (57)$$

(53) is due to Fano's inequality; (56) is because the codeword X_{2i} is a function of W_2 and Y_2^{i-1} as stated in (9); (57) is due to the memoryless channel model.

$$nR_2 = H(W_2) = I(W_2; Y_3^n) + H(W_2 | Y_3^n) \quad (58)$$

$$\leq I(W_2; Y_3^n) + n\epsilon_2 \quad (59)$$

$$= \sum_{i=1}^{n} I(W_2; Y_{3i} | Y_3^{i-1}) + n\epsilon_2 \quad (60)$$

$$= \sum_{i=1}^{n} \left\{ h(Y_{3i} | Y_3^{i-1}) - h(Y_{3i} | Y_3^{i-1} W_2) \right\} + n\epsilon_2 \quad (61)$$

$$\leq \sum_{i=1}^{n} \left\{ h(Y_{3i}) - h(Y_{3i} | Y_3^{i-1} W_2 X_2^i) \right\} + n\epsilon_2 \quad (62)$$

$$\leq \sum_{i=1}^{n} \left\{ h(Y_{3i}) - h(Y_{3i} | Y_1^{i-1} W_2 X_2^i) \right\} + n\epsilon_2 \quad (63)$$

$$\leq \sum_{i=1}^{n} \{ h(Y_{3i}) - h(Y_{3i} | U_i X_{2i}) \} + n\epsilon_2 \quad (64)$$

$$= \sum_{i=1}^{n} I(U_i X_{2i}; Y_{3i}) + n\epsilon_2 \quad (65)$$

(63) follows from Lemma 3. When $b \leq 1$, the capacity region of the cognitive ZIC is equivalent to the cognitive ZIC such that given X_2,

$$X_1 \Longrightarrow Y_1 \Longrightarrow Y_3. \quad (66)$$

So it is enough to prove the outer bound for the cognitive ZIC satisfying (66). Due to (66), conditioning on (X_2^{i-1}, Y_1^{i-1}), the random vector Y_3^{i-1} is independent of all other variables, including Y_{3i}. Thus, given X_2^i, W_2,

$$Y_{3i} \Longrightarrow Y_1^{i-1} \Longrightarrow Y_3^{i-1}. \quad (67)$$

So,

$$h(Y_{3i} | Y_3^{i-1} W_2 X_2^i) \geq h(Y_{3i} | Y_1^{i-1} W_2 X_2^i). \quad (68)$$

Thus, (63) follows.

Now we introduce the time sharing random variable I to be independent of all other variables and uniformly distributed over $\{1, 2, \cdots, n\}$. Define $U = (U_I, I)$, $X_1 = X_{1I}$, $X_2 = X_{2I}$, $Y_1 = Y_{1I}$, $Y_2 = Y_{2I}$ and $Y_3 = Y_{3I}$. Then, we have

$$nR_1 \leq \sum_{i=1}^{n} I(X_{1i}; Y_{1i}Y_{2i}|U_iX_{2i}) + n\epsilon_1 \tag{69}$$
$$= nI(X_{1I}; Y_{1I}Y_{2I}|U_IX_{2I}I) + n\epsilon_1 \tag{70}$$
$$= nI(X_1; Y_1Y_2|UX_2) + n\epsilon_1 \tag{71}$$
$$nR_2 \leq \sum_{i=1}^{n} I(U_iX_{2i}; Y_{3i}) + n\epsilon_2 \tag{72}$$
$$= nI(U_IX_{2I}; Y_{3I}|I) + n\epsilon_2 \tag{73}$$
$$\leq nI(IU_IX_{2I}; Y_{3I}) + n\epsilon_2 \tag{74}$$
$$= nI(UX_2; Y_3) + n\epsilon_2 \tag{75}$$

Remarks:

1) Although the outer bound (48)-(49) requires Y_3 satisfying Markov chain (66), i.e., the noise $Z_3 = bZ_1 + \tilde{Z}_3$ as in Lemma 3, due to the special form of (48)-(49), we can relax Z_3 to be independent of Z_1 as in the original channel model. According to the channel model and the Markov chain (50), (U, X_1, X_2) are independent of all the noises Z_1, Z_2 and Z_3. Therefore, to compute (49),

$$I(UX_2; Y_3)$$
$$= h(bX_1 + X_2 + Z_3) - h(bX_1 + Z_3|UX_2) \tag{76}$$
$$= h(bX_1 + X_2 + Z_3) - h(b(X_1|UX_2) + Z_3) \tag{77}$$

Thus, any value that $I(UX_2; Y_3)$ can achieve with $Z_3 = bZ_1 + \tilde{Z}_3$ can also be achieved with Z_3 independent of Z_1. Therefore, The relaxation of Z_3 to its original model will yield the same outer bound in Theorem 3.

2) Theorem 3 is valid for all cognitive ZIC as long as $b \leq 1$. This outer bound is analogous to the capacity region of a degraded broadcast channel where receiver Y_3 sees a degraded channel compared with that of (Y_1, Y_2).

Next, we derive another outer bound for the specific case where $b \leq 1$ and $K \geq 1$. Before that, we introduce another lemma.

Lemma 4. *If $K \geq 1$, the capacity region of the cognitive ZIC (5)-(7) is the same as the cognitive ZIC given below:*

$$\tilde{Y_1} = \frac{1}{K}\tilde{Y_2} + \tilde{Z_1} \tag{78}$$
$$\tilde{Y_2} = KX_1 + Z_2 \tag{79}$$
$$\tilde{Y_3} = bX_1 + X_2 + Z_3 \tag{80}$$

where $\tilde{Z}_1 \sim N(0, 1 - \frac{1}{K^2})$ is independent of all the other random variables. Thus,

$$X_1 \Longrightarrow \tilde{Y_2} \Longrightarrow \tilde{Y_1}. \tag{81}$$

Further, for all random variable U such that $U \Longrightarrow (X_1, X_2) \Longrightarrow (\tilde{Y_1}, \tilde{Y_2}, \tilde{Y_3})$,

$$X_1 \Longrightarrow (U, X_2, \tilde{Y_2}) \Longrightarrow \tilde{Y_1}. \tag{82}$$

When $K \geq 1$, according to Lemma 4, we only need to consider the cognitive ZIC such that (82) is satisfied. For all U under condition (50), due to (82),

$$I(X_1; Y_1|UX_2Y_2) = 0. \tag{83}$$

Thus, we can rewrite the outer bound $\mathcal{R}_o$ as

$$R_1 \leq I(X_1; Y_2|UX_2) \tag{84}$$
$$R_2 \leq I(UX_2; Y_3). \tag{85}$$

Next, we give the second outer bound in the theorem below.

Theorem 4. *If $b \leq 1$ and $K \geq 1$, the capacity region of the cognitive ZIC is upper bounded by the union of all nonnegative rate pairs (R_1, R_2) satisfying*

$$R_1 \leq C_1 \tag{86}$$
$$R_2 \leq C_2 \tag{87}$$
$$R_1 \leq \gamma(K^2 \alpha P_1) \tag{88}$$
$$R_2 \leq \gamma\left(\frac{b^2 \bar{\alpha} P_1 + P_2 + 2b\sqrt{\bar{\alpha} P_1 P_2}}{1 + b^2 \alpha P_1}\right) \tag{89}$$

for $0 \leq \alpha \leq 1$ and $\alpha + \bar{\alpha} = 1$.

Remark: If $K = 1$, $b \leq 1$, when $R_1 = C_1$, according to Theorem 4, $R_2 \leq \gamma(\frac{P_2}{1+b^2 P_1})$. This is the corner point of the classic ZIC, which can be indeed achieved. Thus, This corner point is on the boundary of the capacity region of

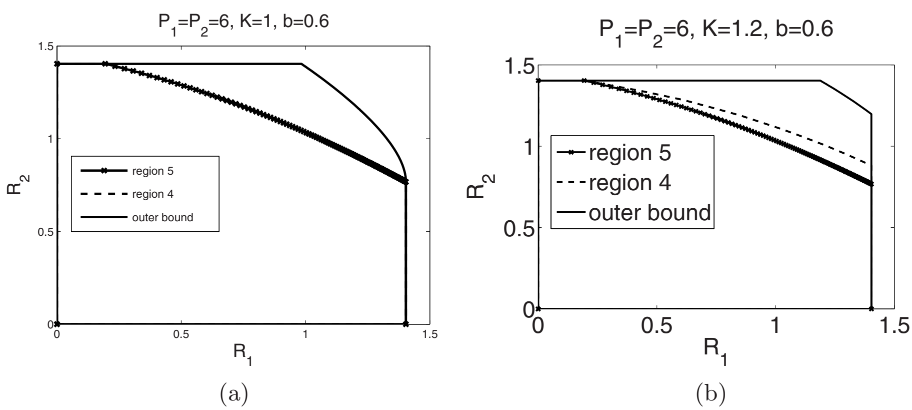

Fig. 6. Comparison of $\mathcal{R}_5$, $\mathcal{R}_4$ and $\mathcal{R}_o$. (a) $P_1 = P_2 = 6, K = 1, b = 0.6$. (b) $P_1 = P_2 = 6, K = 1.2, b = 0.6$.

the cognitive ZIC when $K = 1$. Since the capacity outer bound for the case with $K = 1$ is also an outer bound for the case with $K < 1$ if all other parameters remain the same, that is to say, when the conferencing link is weak, $K \leq 1$, it becomes useless when user 1 is transmitting at its maximum rate.

The comparison of the outer bound $\mathcal{R}_o$ and the inner bounds are given in Fig. 6.

5 Conclusion and Discussions

In this paper, we studied the fundamental performance limits of a cognitive radio channel where the cognitive capability goes beyond the traditionally defined spectrum sensing. Instead, we assume that the cognitive transmitter has the ability to obtain physical layer information through channel feedback. By Imposing a strictly causal setting, we obtained various capacity bounds on the so-called cognitive Z channel where the interference link between the cognitive transmitter and the primary receiver receiver is assumed absent. This simplifying assumption arises from the fact that in existing cognitive radio systems, it is often required that the interference temperature at the primary receiver needs to be below the noise level.

The obtained capacity bounds shed light on how the cognitive capability helps with improving the capacity. In particular, it was demonstrated that while in general the cognitive capability improves the capacity, there exist certain parameter regimes that the presence of cognitive link does not have any effect on the capacity region.

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