

# Outage Probability for ARQ Decode-and-Forward Relaying under Packet-Rate Fading\*

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**Abstract.** In this paper, a new analytical approach is developed for the evaluation of the outage probability of decode-and-forward (DF) automatic-repeat-request (ARQ) relaying under packet-rate fading (fast fading) channels. Based on this approach, a closed-form asymptotically tight (as  $\text{SNR} \rightarrow \infty$ ) approximation of the outage probability is derived, and the diversity order of the DF cooperative ARQ relay scheme is shown to be equal to  $2L - 1$ , where  $L$  is the maximum number of ARQ retransmissions. The closed-form expression clearly shows that the achieved diversity is partially due to the DF cooperative relaying and partially due to the fast fading nature of the channels (temporal diversity). Numerical and simulation studies illustrate the theoretical developments.

**Keywords:** Automatic-repeat-request (ARQ) protocol, cooperative decode-and-forward relaying, outage probability.

## 1 Introduction

Conventional wireless networks involve point-to-point communication links and for that reason do not guarantee reliable transmissions over severe fading channels. On the other hand, cooperative wireless networks exhibit increased network reliability due to the fact that information can be delivered with the cooperation of other users in networks [1]–[5]. In particular, in cooperative systems each user utilizes other cooperative users to create a virtual antenna array and exploit spatial diversity that minimizes the effects of fading and improves overall system performance.

Automatic-repeat-request (ARQ) protocols for wireless communications have been studied extensively in the past and proved themselves as efficient control mechanisms for reliable packet data transmission [6], [7]. The basic idea

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of ARQ protocols is that the receiver requests retransmission when a packet is not correctly received. Recently, in an effort to increase network reliability over poor quality channels, ARQ protocols were studied in the context of cooperative relay networks [8]–[10]. In particular, [8] was among the first such studies to present a general framework of cooperative ARQ relay networks; [9] provided upper bounds for the diversity order of the decode-and-forward (DF) cooperative ARQ relay scheme for both slow and fast fading channels as a means to study the diversity-multiplexing-delay tradeoff; while [10] evaluated a closed-form expression of the outage probability of the DF cooperative ARQ relay scheme for slow fading channels (i.e. channels that are fixed over all retransmission rounds), but unfortunately the introduced approach cannot be extended to fast fading channels (that may change independently from one retransmission to another).

Outage probability is arguably a fundamental performance metric for wireless ARQ relay schemes and so is the diversity order. In this work, we develop a new analytical methodology for the treatment of DF cooperative ARQ relay networks in fast fading (packet-rate fading) channels, which leads, for the first time, to a closed-form asymptotically tight (as  $\text{SNR} \rightarrow \infty$ ) approximation of the outage probability. The closed-form expression shows that the overall diversity order of the DF cooperative ARQ relay scheme is equal to  $2L - 1$ , where  $L$  is the maximum number of ARQ retransmissions. The achieved diversity is partially due to the DF cooperative relaying and partially due to the fast fading nature of the channels (temporal diversity due to retransmissions over independent fading channels). We note that the diversity of the direct ARQ scheme (with no relaying) is only  $L$  and it is due to fast fading. It is important to note that the derived closed-form expression can be further used for optimization of system parameters (resource allocation).

The paper is organized as follows. In Section 2, we present the cooperative ARQ relay scheme. In Section 3, we develop the asymptotic outage probability of the cooperative ARQ relay scheme. Simulation studies are presented in Section 4 and, finally, some conclusions are drawn in Section 5.

## 2 System Model

For simplicity in presentation, we consider the cooperative relay model in Fig. 1. The cooperative ARQ relay scheme can be described as follows [8], [9].  $b$  information bits are encoded into a codeword of length  $LT$ , where  $L$  is the maximum number of ARQ retransmission rounds and  $T$  is the number of channel uses in a single ARQ retransmission round. Then, the codeword is divided into  $L$  different blocks each of length  $T$ . In each ARQ retransmission round a block of the message is sent, so the transmission rate is  $R = b/T$ . First, the source transmits a block of the message to the destination which is also received by the relay. The destination indicates success or failure of receiving the message by feeding back a single bit of acknowledgement (ACK) or non-acknowledgement (NACK). The feedback is assumed to be detected reliably at the source and the relay. If ACK is received or the retransmission reaches the maximum number of rounds,

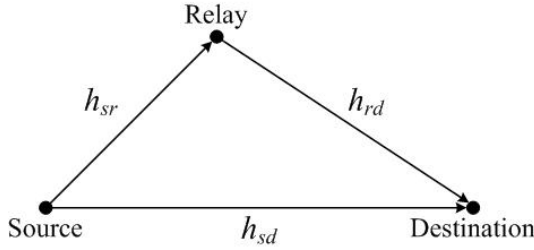


Fig. 1. Cooperative ARQ relay model

the source stops transmitting the current message and starts transmitting a new message. If NACK is received and the retransmission has not reached the maximum number of rounds, the source sends another block of the same message. If the relay decodes successfully before the destination is able to, the relay starts cooperating with the source by transmitting corresponding blocks of the message to the destination by using a space-time transmission [9], for example, the Alamouti scheme [11]. It is assumed that the relay knows the codeword of the message. After  $L$  ARQ retransmission rounds, if the destination still cannot decode the message, an outage is declared which means that the mutual information of the cooperative ARQ relay channel is below the transmission rate.

The cooperative ARQ relay scheme can be modeled as follows. The received signal  $y_{r,m}$  at the relay at the  $m$ -th ( $1 \leq m \leq L$ ) ARQ retransmission round can be modeled as

$$y_{r,m} = \sqrt{P_s} h_{sr,m} x_s + \eta_{r,m}, \tag{1}$$

where  $P_s$  is the transmitted power of the source signal  $x_s$ ,  $h_{sr,m}$  is the coefficient of the source-relay channel at the  $m$ -th ARQ retransmission round, and  $\eta_{r,m}$  is additive noise. If the relay is not involved in forwarding, the received signal  $y_{d,m}$  at the destination at the  $m$ -th ARQ retransmission round is

$$y_{d,m} = \sqrt{P_s} h_{sd,m} x_s + \eta_{d,m}, \tag{2}$$

where  $h_{sd,m}$  is the source-destination channel coefficient at the  $m$ -th ARQ retransmission round. If the relay receives the message from the source successfully, it helps in forwarding that to the destination using the Alamouti scheme. Specifically, if a block of the message is partitioned into two parts as  $x_s = [x_{s,1} \ x_{s,2}]$ , then the relay forwards a corresponding block  $x_r = [-x_{s,2}^* \ x_{s,1}^*]$ . The received signal  $y_{d,m}$  at the destination at the  $m$ -th ARQ retransmission round can be written as

$$y_{d,m} = \sqrt{P_s} h_{sd,m} x_s + \sqrt{P_r} h_{rd,m} x_r + \eta_{d,m}, \tag{3}$$

where  $P_r$  is the transmitted power at the relay and  $h_{rd,m}$  is the channel coefficient from the relay to the destination at the  $m$ -th ARQ retransmission round. At the destination, the message block  $x_s$  can be recovered based on the orthogonal structure of the Alamouti code. The channel coefficients  $h_{sd,m}$ ,  $h_{sr,m}$  and  $h_{rd,m}$  are modeled as independent, zero-mean complex Gaussian random

variables with variances  $\sigma_{sd}^2$ ,  $\sigma_{sr}^2$  and  $\sigma_{rd}^2$ , respectively. We consider a fast fading scenario, i.e. the channels remain fixed only within one ARQ retransmission round, but change independently from one round to another (packet-rate fading). The channel state information is assumed to be known at the receiver and unknown at the transmitter. The noise  $\eta_{r,m}$  and  $\eta_{d,m}$  are modeled as zero-mean complex Gaussian random variables with variance  $N_0$ .

### 3 Outage Performance Analysis

In this section, we derive the outage probability of the cooperative ARQ relay scheme under packet-rate fading conditions. For comparison purposes, we also derive the outage probability of the direct ARQ transmission scheme.

To calculate the outage probabilities of the cooperative ARQ transmission scheme, we develop the following two lemmas.

**Lemma 1.** *If  $u_{s_1, \dots, s_M}$  and  $v_{s_1, \dots, s_M}$  are two independent random variables satisfying the following properties*

$$\lim_{s_i \rightarrow \infty} \prod_{1 \leq i \leq M} s_i^{d_1} \cdot Pr[u_{s_1, \dots, s_M} < t] = a \cdot f(t),$$

$$\lim_{s_i \rightarrow \infty} \prod_{1 \leq i \leq M} s_i^{d_2} \cdot Pr[v_{s_1, \dots, s_M} < t] = b \cdot g(t),$$

where  $d_1, d_2, a$  and  $b$  are constants,  $f(t)$  and  $g(t)$  are monotonically increasing functions, and  $f'(t)$  is integrable, then

$$\lim_{s_i \rightarrow \infty} \prod_{1 \leq i \leq M} s_i^{d_1+d_2} \cdot Pr[u_{s_1, \dots, s_M} + v_{s_1, \dots, s_M} < t] = ab \cdot \int_0^t g(x)f'(t-x)dx. \quad (4)$$

**Lemma 2.** *For non-zero constants  $\beta_1, \beta_2, \dots, \beta_n$ , denote*

$$F_n(\beta_1, \dots, \beta_n; t) \triangleq \int_0^t \int_0^{x_1} \dots \int_0^{x_{n-1}} 2^{\beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n} dx_1 dx_2 \dots dx_n.$$

The function  $F_n(\beta_1, \dots, \beta_n; t)$  can be calculated as follows

$$F_n(\beta_1, \dots, \beta_n; t) = \sum_{\substack{\delta_1, \dots, \delta_{n-1} \\ \in \{0,1\}}} \frac{(-1)^{n+\delta_1+\dots+\delta_{n-1}} (\ln 2)^{-n}}{\prod_{m=1}^n [\sum_{l=1}^m i_{m,l}(\delta)\beta_l]} \left( 2^{[\sum_{l=1}^n i_{n,l}(\delta)\beta_l]t} - 1 \right),$$

where the variables  $\delta_1, \delta_2, \dots, \delta_{n-1} \in \{0, 1\}$ ,  $\delta \triangleq \{\delta_1, \delta_2, \dots, \delta_{n-1}\}$ , and the coefficients  $\{i_{m,l}(\delta) : 1 \leq m \leq n, 1 \leq l \leq m\}$  are specified as follows

$$i_{1,1}(\delta) = i_{2,2}(\delta) = \dots = i_{n,n}(\delta) = 1,$$

and, for any  $m = 2, 3, \dots, n$ ,

$$i_{m,l}(\delta) = \delta_{m-1} \cdot i_{m-1,l}(\delta), \quad l = 1, 2, \dots, m-1.$$

The proofs of Lemmas 1 and 2 are omitted here due to lack of space. We note that the special case  $M = 1$  of Lemma 1 was presented in [12].

### 3.1 Direct ARQ Transmission Scheme

In the direct ARQ transmission scheme, the destination receives information from the source directly, without involving the relay. The mutual information between the source and the destination in the  $m$ -th round of the direct ARQ transmission scheme is

$$I_{sd,m} = \log_2 \left( 1 + \frac{P_s}{\mathcal{N}_0} |h_{sd,m}|^2 \right). \tag{5}$$

The total mutual information after  $L$  ARQ rounds is  $I_{sd}^{tot} = \sum_{m=1}^L I_{sd,m}$ . Thus, the outage probability of the direct ARQ scheme after  $L$  ARQ rounds can be obtained as

$$P^{out,L} = \Pr [I_{sd}^{tot} < R]. \tag{6}$$

A closed-form expression of (6) is not tractable. However, by applying Lemma 1 with  $M = 1$ , an approximation of the outage probability in (6) can be obtained for high-SNR scenario as follows

$$P^{out,L} \approx g_L(R) \left( \frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s} \right)^L, \tag{7}$$

where  $g_L(\cdot)$  is defined as

$$g_n(t) = \int_0^t g_{n-1}(x) f'(t-x) dx, \quad n \geq 1, \tag{8}$$

with  $g_0(t) = 1$  and  $f(t) = 2^t - 1$ . We note that (7) and (8) were first developed in [12]. We can further calculate here  $g_n(t)$  explicitly as

$$g_n(t) = 2^t \sum_{m=1}^n \frac{(-1)^{n-m}}{(m-1)!} (t \cdot \ln 2)^{m-1} + (-1)^n. \tag{9}$$

### 3.2 DF Cooperative ARQ Relay Scheme

When the DF cooperative ARQ relay scheme is employed, the relay decodes the message from the source, say, at the  $k$ -th round. Then, at the  $(k + 1)$ -th round, the relay starts forwarding appropriate ARQ blocks. Let  $\{T_r = k\}$  denote the event of successful message decoding by the relay at the  $k$ -th round and subsequent ARQ block forwarding at the  $(k + 1)$ -th round. Let  $P_{T_r=k}^{out}$  denote the probability that the destination decodes the message unsuccessfully after  $L$  ARQ retransmission rounds if the event  $\{T_r = k\}$  occurs. Then, the outage

probability for the cooperative ARQ relay scheme after  $L$  ARQ retransmission rounds can be written as follows

$$P^{out,L} = \sum_{k=1}^L P_{T_r=k}^{out} \cdot \Pr [T_r = k]. \tag{10}$$

We will first calculate the probability  $\Pr [T_r = k]$ . We note that the mutual information between the source and the relay in the  $m$ -th ARQ round is

$$I_{sr,m} = \log_2 \left( 1 + \frac{P_s}{\mathcal{N}_0} |h_{sr,m}|^2 \right). \tag{11}$$

The probability that the relay decodes the message successfully at the first round ( $T_r = 1$ ) is

$$\Pr [T_r = 1] = \Pr [I_{sr,1} \geq R] = \exp \left( -\frac{2^R - 1}{\sigma_{sr}^2} \cdot \frac{\mathcal{N}_0}{P_s} \right). \tag{12}$$

For any  $T_r = k$ ,  $k = 2, 3, \dots, L - 1$ , we have

$$\begin{aligned} \Pr [T_r = k] &= \Pr \left[ \sum_{m=1}^{k-1} I_{sr,m} < R, \sum_{m=1}^k I_{sr,m} \geq R \right] \\ &= \Pr \left[ \sum_{m=1}^{k-1} I_{sr,m} < R \right] - \Pr \left[ \sum_{m=1}^k I_{sr,m} < R \right] \\ &\approx g_{k-1}(R) \left( \frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s} \right)^{k-1} - g_k(R) \left( \frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s} \right)^k, \end{aligned} \tag{13}$$

where  $g_{k-1}(\cdot)$  and  $g_k(\cdot)$  are given by (9). The approximation in (13) is obtained by applying Lemma 1 with  $M = 1$ . Finally, if  $T_r = L$ , we have

$$\Pr [T_r = L] = \Pr \left[ \sum_{m=1}^{L-1} I_{sr,m} < R \right] \approx g_{L-1}(R) \left( \frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s} \right)^{L-1}. \tag{14}$$

Next, we calculate the conditional outage probability  $P_{T_r=k}^{out}$ . When the relay cooperates with the source by jointly sending a message block via the Alamouti scheme, the mutual information of the cooperative channels in the  $m$ -th ARQ round is given by [9]

$$I_{srd,m} = \log_2 \left( 1 + \frac{P_s}{\mathcal{N}_0} |h_{sd,m}|^2 + \frac{P_r}{\mathcal{N}_0} |h_{rd,m}|^2 \right). \tag{15}$$

After  $L$  ARQ rounds, the total mutual information is given by

$$I_{d,T_r=k}^{tot} = \begin{cases} \sum_{m=1}^k I_{sd,m} + \sum_{m=k+1}^L I_{srd,m}, & 1 \leq k < L; \\ \sum_{m=1}^L I_{sd,m}, & k = L. \end{cases} \tag{16}$$

We note that if  $T_r = L$ , the relay has no chance to cooperate since the source starts sending a new packet. The conditional outage probability can be evaluated as

$$P_{T_r=k}^{out} = \Pr [I_{d,T_r=k}^{tot} < R]. \tag{17}$$

When  $T_r = L$ , the conditional outage probability is reduced to the direct ARQ scenario and it is given by

$$P_{T_r=L}^{out} = \Pr [I_{d,T_r=L}^{tot} < R] \approx g_L(R) \left( \frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s} \right)^L. \tag{18}$$

Next, we calculate the conditional outage probability (17) for any  $T_r = k$ ,  $k = 1, 2, \dots, L - 1$ . For simplicity in presentation, we introduce the following notation

$$u_m = \begin{cases} \log_2 (1 + s_1 |h_{sd,m}|^2), & 1 \leq m \leq k; \\ \log_2 (1 + s_1 |h_{sd,m}|^2 + s_2 |h_{rd,m}|^2), & k + 1 \leq m \leq L, \end{cases} \tag{19}$$

where  $s_1 = P_s/\mathcal{N}_0$  and  $s_2 = P_r/\mathcal{N}_0$ . Then, the total mutual information can be written as

$$I_{d,T_r=k}^{tot} = \sum_{m=1}^L u_m. \tag{20}$$

To calculate the conditional outage probability, we observe that for  $1 \leq m \leq k$ ,  $|h_{sd,m}|^2$  is an exponential random variable with parameter  $\sigma_{sd}^{-2}$ . Then, recursive application of Lemma 1 with  $M = 1$  leads to

$$\lim_{s_1 \rightarrow \infty} s_1^k \cdot \Pr \left[ \sum_{m=1}^k u_m < t \right] = \left( \frac{1}{\sigma_{sd}^2} \right)^k g_k(t). \tag{21}$$

For  $k + 1 \leq m \leq L$ ,  $u_m$  includes the sum of two independent exponential random variables, namely,  $|h_{sd,m}|^2$  and  $|h_{rd,m}|^2$  with parameters  $\sigma_{sd}^{-2}$  and  $\sigma_{rd}^{-2}$ , respectively. For each  $m = k + 1, \dots, L$ , we have

$$\begin{aligned} \Pr [u_m < t] &= \Pr [s_1 |h_{sd,m}|^2 + s_2 |h_{rd,m}|^2 < 2^t - 1] \\ &= \begin{cases} 1 - \left( 1 + \frac{1}{\sigma_{sd}^2} \frac{2^t - 1}{s_1} \right) \exp \left( -\frac{1}{\sigma_{sd}^2} \frac{2^t - 1}{s_1} \right), & \text{if } \frac{s_1}{\sigma_{rd}^2} = \frac{s_2}{\sigma_{sd}^2}; \\ 1 - \frac{s_1 \sigma_{sd}^2}{s_1 \sigma_{sd}^2 - s_2 \sigma_{rd}^2} \exp \left( -\frac{1}{\sigma_{sd}^2} \frac{2^t - 1}{s_1} \right) \\ \quad - \frac{s_2 \sigma_{rd}^2}{s_2 \sigma_{rd}^2 - s_1 \sigma_{sd}^2} \exp \left( -\frac{1}{\sigma_{rd}^2} \frac{2^t - 1}{s_2} \right), & \text{if } \frac{s_1}{\sigma_{rd}^2} \neq \frac{s_2}{\sigma_{sd}^2}. \end{cases} \end{aligned} \tag{22}$$

We can see that for any  $m = k + 1, \dots, L$ ,

$$\lim_{\substack{s_i \rightarrow \infty \\ 1 \leq i \leq 2}} s_1 s_2 \cdot \Pr [u_m < t] = \frac{1}{2\sigma_{sd}^2 \sigma_{rd}^2} (2^t - 1)^2. \tag{23}$$

Let  $q_0(t) = 1$  and  $p(t) = (2^t - 1)^2$ , then  $p'(t) = 2(2^{2t} - 2^t)\ln 2$ . Applying Lemma 1 with  $M = 2$  recursively, we obtain

$$\lim_{\substack{s_i \rightarrow \infty \\ 1 \leq i \leq 2}} (s_1 s_2)^n \cdot \Pr \left[ \sum_{m=k+1}^{k+n} u_m < t \right] = \left( \frac{1}{2\sigma_{sd}^2 \sigma_{rd}^2} \right)^n q_n(t), \tag{24}$$

where  $q_n(t) = \int_0^t q_{n-1}(x)p'(t-x)dx$  for any  $n = 1, 2, \dots, L - k$ . Then, using Lemma 2, we can obtain a closed-form expression for  $q_n(t)$  as follows

$$\begin{aligned}
 q_n(t) &= \int_0^t \int_0^{x_{n-1}} \cdots \int_0^{x_2} q_1(x_1)p'(x_2-x_1)p'(x_3-x_2) \cdots \\
 &\quad \times p'(x_{n-1}-x_{n-2})p'(t-x_{n-1})dx_1dx_2 \cdots dx_{n-1} \\
 &= \int_0^t \int_0^{x_{n-1}} \cdots \int_0^{x_2} (2^{x_1}-1)^2(2\ln 2)^{n-1} \prod_{m=1}^{n-2} (2^{x_{m+1}-x_m}-1) \\
 &\quad \times (2^{t-x_{n-1}}-1)2^{t-x_1}dx_1dx_2 \cdots dx_{n-1} \\
 &= (-2\ln 2)^{n-1} \sum_{\substack{\alpha_1, \dots, \alpha_{n-1} \\ \in \{0,1\}}} (-1)^{\alpha_1+\dots+\alpha_{n-1}} 2^{(1+\alpha_{n-1})t} \\
 &\quad \times \int_0^t \int_0^{x_{n-1}} \cdots \int_0^{x_2} (2^{x_1}-1)^2 \cdot 2^{-\alpha_1 x_1} \\
 &\quad \times \prod_{m=2}^{n-1} 2^{(\alpha_{m-1}-\alpha_m)x_m} dx_1 dx_2 \cdots dx_{n-1} \\
 &= (-2\ln 2)^{n-1} \sum_{\substack{\alpha_1, \dots, \alpha_{n-1} \\ \in \{0,1\}}} (-1)^{\alpha_1+\dots+\alpha_{n-1}} 2^{(1+\alpha_{n-1})t} \\
 &\quad \times \{F_{n-1}(1-\alpha_1, \beta_2, \dots, \beta_{n-1}; t) - 2F_{n-1}(-\alpha_1, \beta_2, \dots, \beta_{n-1}; t) \\
 &\quad + F_{n-1}(-1-\alpha_1, \beta_2, \dots, \beta_{n-1}; t)\}, \tag{25}
 \end{aligned}$$

where the function  $F_{n-1}(\cdot; t)$  is specified in Lemma 2, and  $\beta_2 = \alpha_1 - \alpha_2, \beta_3 = \alpha_2 - \alpha_3, \dots, \beta_{n-1} = \alpha_{n-2} - \alpha_{n-1}$ . In case where  $\beta_i$  in  $F_{n-1}(\cdot; t)$  is zero, for some  $i$ , we apply Lemma 2 with  $\beta_i = \varepsilon_i$  where  $\varepsilon_i$  is sufficiently small ( $\varepsilon_i \rightarrow 0$ ). Then, (21), (24), and Lemma 1 imply that for any  $T_r = k, k = 1, 2, \dots, L - 1$ , the conditional probability (17) can be approximated as follows

$$\begin{aligned}
 P_{T_r=k}^{out} &= \Pr \left[ \sum_{m=1}^k I_{sd,m} + \sum_{m=k+1}^L I_{srd,m} < R \right] \\
 &\approx \frac{b_k(R)}{2^{L-k}} \left( \frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s} \right)^L \left( \frac{\mathcal{N}_0}{\sigma_{rd}^2 P_r} \right)^{L-k}, \tag{26}
 \end{aligned}$$

where  $b_k(t) = \int_0^t g_k(x)q'_{L-k}(t-x)dx$ .

Finally, the outage probability for the DF cooperative ARQ relay scheme can be obtained as follows

$$\begin{aligned}
 P^{out,L} &\approx \sum_{k=1}^{L-1} \frac{b_k(R)}{2^{L-k}} \left[ g_{k-1}(R) - g_k(R) \frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s} \right] \\
 &\quad \times \left( \frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s} \right)^L \left( \frac{\mathcal{N}_0}{\sigma_{rd}^2 P_r} \right)^{L-k} \left( \frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s} \right)^{k-1} \\
 &\quad + g_L(R)g_{L-1}(R) \left( \frac{\mathcal{N}_0}{\sigma_{sd}^2 P_s} \right)^L \left( \frac{\mathcal{N}_0}{\sigma_{sr}^2 P_s} \right)^{L-1}. \tag{27}
 \end{aligned}$$



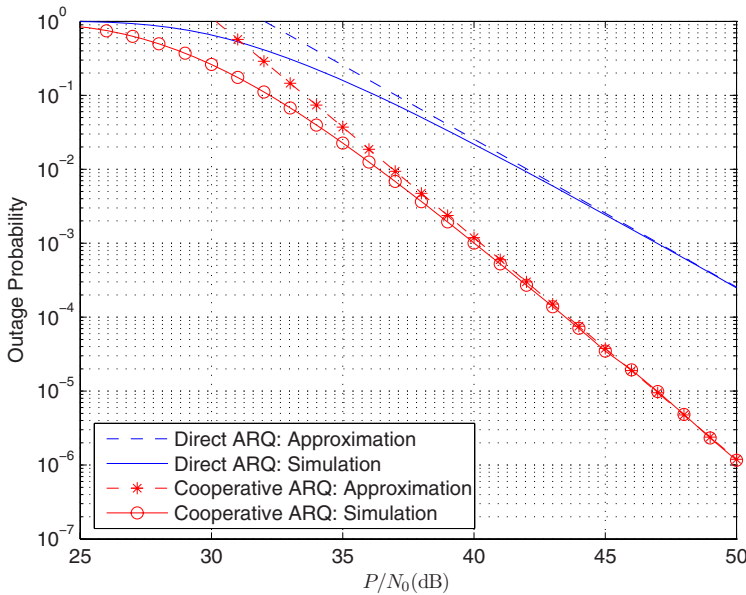
Furthermore, since  $g_k(R)\frac{N_0}{\sigma_{sr}^2 P_s}$  is much smaller than  $g_{k-1}(R)$  at high SNR, so the asymptotic outage probability in (27) can be further simplified as

$$P^{out,L} \approx \sum_{k=1}^L \frac{b_k(R)g_{k-1}(R)}{2^{L-k}} \left(\frac{N_0}{\sigma_{sd}^2 P_s}\right)^L \left(\frac{N_0}{\sigma_{rd}^2 P_r}\right)^{L-k} \left(\frac{N_0}{\sigma_{sr}^2 P_s}\right)^{k-1}. \quad (28)$$

Based on the above asymptotic outage analysis, we observe that the term  $\left(\frac{N_0}{\sigma_{sd}^2 P_s}\right)^L$  in (28) contributes a diversity order  $L$  which is due to the fast fading nature of the channels, while the term  $\left(\frac{N_0}{\sigma_{rd}^2 P_r}\right)^{L-k} \left(\frac{N_0}{\sigma_{sr}^2 P_s}\right)^{k-1}$  contributes an overall diversity order  $(L - k) + (k - 1) = L - 1$  which is due to the cooperative relaying. So, the DF cooperative ARQ relay scheme has a total diversity order  $2L - 1$ . We recall that the diversity order of the direct ARQ transmission scheme is  $L$ .

### 4 Simulation Results

In this section, we present numerical and simulation studies for both the direct and the cooperative ARQ schemes. In all studies, the variance of the channel  $h_{ij}$  is assumed to be  $\sigma_{ij}^2 = d_{ij}^{-\mu}$ , where  $d_{ij}$  is the distance between two nodes. The path loss exponent is  $\mu = 3$ . We assume that the relay is located in the midpoint between the source and the destination and the source-destination distance  $d_{sd} = 10$  m. We assume equal power allocation at the source and at the relay, i.e.,  $P_s = P_r = P$ , and transmission rate equal to  $R = 2$  bits/s/Hz.



**Fig. 2.** Outage probability of the direct and cooperative ARQ schemes ( $L=2$ )

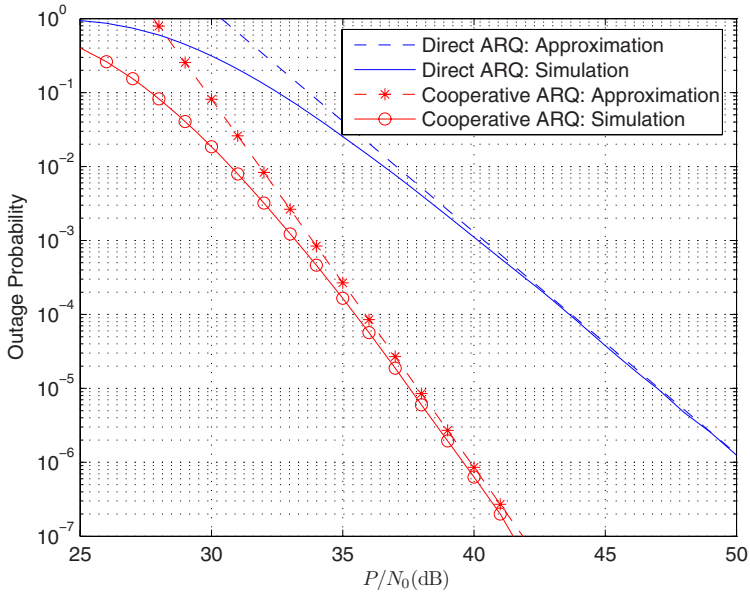


Fig. 3. Outage probability of the direct and cooperative ARQ schemes ( $L=3$ )

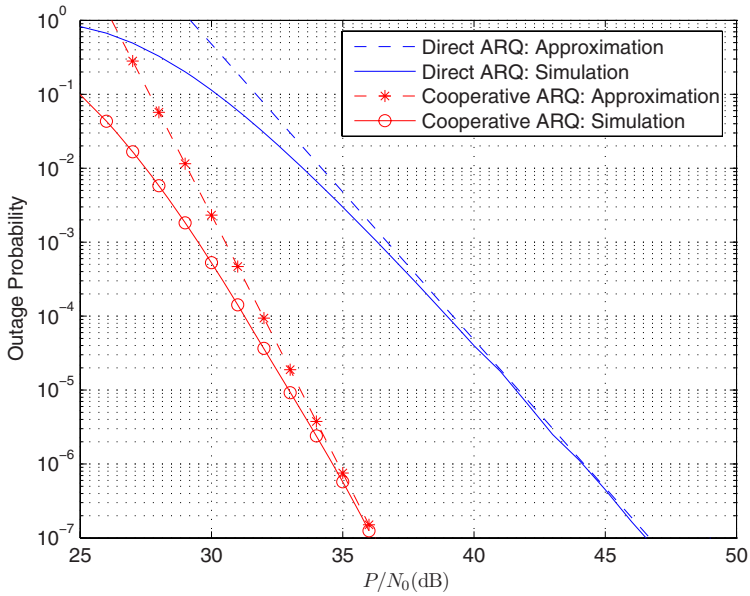


Fig. 4. Outage probability of the direct and cooperative ARQ schemes ( $L=4$ )

Figs. 2, 3 and 4 present the simulation studies for the cooperative ARQ relay scheme when the maximum number of ARQ retransmission rounds is  $L = 2, 3$  and 4, respectively. The studies show that the outage probability approximation of the cooperative ARQ relay scheme is asymptotically tight at high SNR. For example, in case of  $L = 2$  in Fig. 2, the analytical approximation merges with the simulated curve at an outage performance of about  $10^{-3}$ . Moreover, the larger the number of ARQ retransmission rounds, the higher the diversity order of the cooperative ARQ relay scheme.

For comparison purposes, Figs. 2–4 also include the performance of the direct ARQ scheme. Our theoretical developments show that the cooperative ARQ relay scheme has diversity order  $2L - 1$ , while the direct ARQ scheme has diversity order  $L$ . Our simulation studies validate the theoretical developments and indicate that the cooperative ARQ relay scheme significantly outperforms the direct ARQ scheme, while the diversity improvement of the cooperative over the direct ARQ scheme increases with the number of ARQ retransmission rounds.

## 5 Conclusions

In this paper, we developed, for the first time, a closed-form asymptotically tight (as  $\text{SNR} \rightarrow \infty$ ) approximation of the outage probability of the DF cooperative ARQ relay scheme under fast fading conditions. The approximation is based on Lemma 1 and Lemma 2 that we established in this work. The closed-form expression provides significant insight on the benefits of the DF cooperative ARQ relaying relative to direct ARQ schemes in fast fading scenarios and shows that the cooperative scheme achieves diversity order equal to  $2L - 1$  while the diversity order of the direct scheme is only  $L$ . Moreover, the closed-form expression developed in this work can be further used for optimization of system parameters (resource allocation).

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