Joint Random Access and Power Control Game in Ad Hoc Networks with Noncooperative Users

Chengnian Long and Xinping Guan

Department of Automation
School of Electronic, Information and Electrical Engineering
Shanghai Jiaotong University, Shanghai, China
longcn@sjtu.edu.cn

Abstract. We consider a distributed joint random access and power control scheme for interference management in wireless ad hoc networks. To derive decentralized solutions that do not require any cooperation among the users, we formulate this problem as non-cooperative joint random access and power control game, in which each user minimizes its average transmission cost with a given rate constraint. Using supermodular game theory, the existence and uniqueness of Nash equilibrium are established. Furthermore, we present an asynchronous distributed algorithm to compute the solution of the game based on myopic best response updates, which converges to Nash equilibrium globally.

Keywords: wireless ad hoc networks, random access, power control, supermodular game, Nash equilibrium.

1 Introduction

Since wireless ad hoc networks use a common transmission medium, collision may occur in the presence of simultaneous transmissions by two or more wireless links lying in the interference range of each other. Thus, mitigating interference is a fundamental problem for increasing spectral efficiency in wireless ad hoc networks. Important mechanisms for interference management in wireless networks are medium access control (MAC) and power control. This is a complex and intriguing problem since the selection of active link and its power level fundamentally affects many aspects of the operation of the network and its resulting performance; for instance the quality of the signal received at the receiver, the interference it creates for the other receivers and energy consumption at each node.

Traditionally, interference management for ad hoc networks is primarily implemented at the lower layers independently. In scheduling-based MAC protocols (TDMA), the interference management is implemented by a distributed power control for simultaneous active links in a slot. The scheduling problem is to decide in each time slot which source-destination pairs communicate according to some performance index, e.g., maximizing the number of simultaneous transmissions. The power control and scheduling design is separated. In the contention-based

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MAC protocols, to avoid collision a node accesses the wireless channel with a persistence probability a or waits for a random amount of time bounded by the contention window CW before a transmission, after it has sensed an idle channel. IEEE 802.11 Distributed Coordination Function (DCF) is a standard contention-based MAC designed for wireless local area networks (WLANs). It should be pointed out that a simple interference/collision model is used where a receiving nodes sees interference from another transmitter if and only if it is within some nominal range or hops both in scheduling-based and contention-based MAC. However, this simple model may mislead designs in practice. In reality, simultaneous transmissions within the nominal range do not necessarily collide if the signal to interference plus noise ratios (SINR) at the corresponding receivers are sufficiently high; and, at the other extreme, aggregate interference from multiple transmitters that well beyond the nominal range can be high enough to cause collisions. This observation motivates some recent studies on joint scheduling and power control design using a realistic SINR-based interference model in wireless networks. However, there is little work on joint random access and power control design [11].

In this paper, we study the energy efficient joint random access and power control problem. We focus on distributed algorithms with no centralized control using a game-theory framework. Our work is motivated by the following observations.

- Contention-based MAC is easy to be implemented asynchronously. However, scheduling-based MAC should be required to strict time synchronization. Thus, contention-based MAC is generally less signaling overheard than scheduling-based MAC;
- It is expect that a joint optimization design between contention-based MAC and power control will further improve the network performance;
- The game theoretic formulation is a useful approach to devise totally distributed algorithms. Furthermore, It is amenable to rigorous proofs of convergence for the cross-layer distributed algorithm.

To formulate the joint random access and power control design, we derive an analytical lower bound for *average* link capacity, which is related with the links' persistence probability and power allocation. Taking explicitly into account the rate constraints, we propose a strategic non-cooperative game, where each link is a player that competes against the others by choosing the persistence probability and power allocation over a common channel with the minimum transit cost. We will refer to this game as *joint random access and power control game (JRPG)*. Based on the supermodular property [15], the existence of a Nash equilibrium (NE) solution for the noncooperative JRPG is established. Furthermore, we present an analysis for each user's myopic best response (MBR) dynamics, which shows the uniqueness and stability of the NE. Interestingly, the MBR dynamics shows that the joint optimization design can be separated under mild condition, which is a guarantee of the architectural modularity for the cross-layer design.

An outline of the remainder of the paper is as follows. Section II discusses the related work. Section III gives the system model and formulates the noncooperative JRPG. Section IV provides a proof for the existence of NE of the JRPG. Section V contains the description of the distributed algorithm along with the global convergence. Finally, Section VI draws the conclusions and future work.

2 Related Work

There has been a rich literature to study interference management in wireless networks. In this section, we focus on summarizing related work on power control and contention-based MAC based on game-theoretical formulation. We also illustrate the recent work on joint link scheduling and power control with realistic SINR model.

In scheduling-based MAC, power control is a basic technique for mitigating interference. A variety of game-theoretic approaches have been applied to power control design [1], [2], [5]. Supermodular game theory, in particular, has been used to study power control in [3], [6], [10], [13]. Huang et al. [6] proposed a price-based power control framework for wireless ad hoc networks. Assuming that the users voluntarily cooperate by exchanging interference information, users announce prices to reflect their sensitivities to the current interference levels, and then adjust their power to maximize their surplus. They introduced fictitious non-cooperative games with supermodular property as the convergence proof technique.

In random access schemes, MacKenzie and Wicker [7] discuss the stability of slotted-Aloha with selfish user behaviors and perfect network information. In [4] Cagalj et al. propose an incentive mechanism borrowed from dynamic game theory to guide multiple selfish nodes with contention window control to a Pareto optimal Nash equilibrium (NE) in CSMA/CA ad hoc networks. But the convergence of their incentive algorithm is a bit slow. Recently, Tang et al. [14] reveal that the exponential backoff algorithm does implicitly participate in a noncooperative game. Then they apply supermodular game theory to study the existence of NE..

3 System Model and Problem Formulation

In this section, we clarify the assumptions and the constraints underlying the system model. First, we derive a lower bound of link capacity analytically, which is related with the corresponding persistence probability and power allocation. Second, we formulate the joint random access and power control game explicitly.

3.1 Random Access, Power Control and Link Capacity

Consider a static wireless ad hoc network shown in Fig. 1, modeled by a set $\mathcal{L} \triangleq \{1, \ldots, L\}$ of directed links. It is assumed that the system is time slotted and uses CDMA at the physical layer. Each link l's transmitter wants to communicate

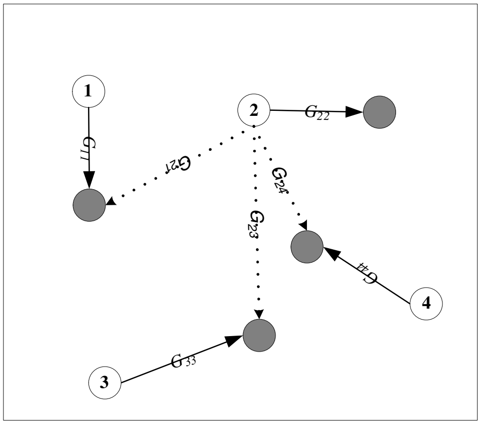

Fig. 1. A wireless ad hoc network diagram with four users

with its corresponding receiver over a shared channel. The channel over each link is assumed to be an additive white Guassian noise (AWGN) channel, with noise power spectral density N_0 over the bandwidth of operation B. The channel gain between link l's transmitter and link k's receiver is denoted by G_{lk}. We assume the channel gains of each link are fixed. Let X_k be the random variable that denotes the interference value from link k to link l, we have

$$X_{k,l} = \begin{cases} G_{kl}p_k & \text{with probability } a_k \\ 0 & \text{with probability } 1 - a_k \end{cases} \tag{1}$$

where a_k, and p_k are link k's persistence probability and transmission power respectively. Due to the random access property of the links whose transmission cause interference to the receiver of link l, the total interference and noise power at the receiver of link l is stochastic, which is denoted by $\mathcal{I}_l$ and is given by

$$\mathcal{I}_l = \sum_{k \neq l} X_{k,l} + N_0 B. \tag{2}$$

Therefore, the the *instantaneous* signal to interference plus noise ratio (SINR) of the lth link is

$$\gamma_l = \frac{G_{ll}p_l}{\mathcal{I}_l}.$$

Let C_l be the random variable that denotes link capacity that can be supported over link l

$$C_l = \begin{cases} B \log(1 + K\gamma_l) & \text{with probability } a_l, \\ 0 & \text{with probability } 1 - a_l, \end{cases} \tag{3}$$

where K is a constant depending on the modulation and required bit error rate of the link. Therefore the *average* link capacity can be expressed as follows:

$$\mathbf{E}[C_l] = a_l \mathbf{E}\left[B \log\left(1 + K\frac{G_{ll}p_l}{\mathcal{I}_l}\right)\right].$$

Based on Jensen's inequality and the fact that $f(\mathcal{I}_l) = B\log\left(1 + K\frac{G_{ll}p_l}{\mathcal{I}_l}\right)$ is convex with respect to $\mathcal{I}_l$, we have

$$\mathbf{E}[C_l] \geq a_l B\log\left(1 + K\frac{G_{ll}p_l}{\mathbf{E}[\mathcal{I}_l]}\right),$$

where $\mathbf{E}[\mathcal{I}_l] = \sum_{k\neq l} G_{lk}p_k a_k + N_0 B$.

3.2 Joint Random Access and Power Control Game

We formulate the system design within the framework of game theory. Specifically, we consider a strategic noncooperative game, in which the players are the links and the payoff functions are the transmit costs of the links. We use the terms "link", "player" and "user" interchangeably in the following. Each player competes against others by choosing the persistence probability and transmission power (i.e., its strategy) that minimizes its own transmit cost, given a constraint on the minimum achievable information rate on the link. The two-dimension strategy for user l is denoted by $x_l = (a_l, p_l)$ and the strategy profile of user l's opponents is defined to be $\mathbf{x}_{-l} = (\mathbf{a}_{-l}, \mathbf{p}_{-l})$, where $\mathbf{a}_{-l} = (a_1, \ldots, a_{l-1}, a_{l+1}, \ldots, a_L)$, $\mathbf{p}_{-l} = (p_1, \ldots, p_{l-1}, p_{l+1}, \ldots, p_L)$. Stated in mathematical terms, the game has the following structure:

$$\mathcal{G} = \left\{\mathcal{L}, \{\mathcal{X}_l(\mathbf{x}_{-l})\}_{l\in\mathcal{L}}, \{u_l(x_l)\}_{l\in\mathcal{L}}\right\},$$

where $\mathcal{L} \triangleq \{1, 2, \ldots L\}$ denotes the set of links, $\mathcal{X}_l(\mathbf{x}_{-l})$ is the joint set of admissible random access strategy a_l and power allocation strategy p_l of user l, defined as

$$\mathcal{X}_l(\mathbf{x}_{-l}) \triangleq \{x_l \in \mathcal{A}_l \times \mathcal{P}_l : R_l(x_l, \mathbf{x}_{-l}) \geq R_l^*\}, \tag{4}$$

where $R_l(x_l, \mathbf{x}_{-l}) = a_l B\log\left(1 + K\frac{G_{ll}p_l}{\mathbf{E}[\mathcal{I}_l]}\right)$, and R_l^* denotes the minimum transmission rate required by each user, which we assume $R_l^* \geq B$ for all users. $\mathcal{P}_l$ and $\mathcal{A}_l$ denote the action set for the player l, which are defined by

$$\mathcal{A}_l = [a_{l,\min}, a_{l,\max}], \mathcal{P}_l = [0, p_{l,\max}].$$

In the sequel we will make reference to the vector $\mathbf{R}^* \triangleq (R_l^*)_{l\in\mathcal{L}}$ as to the rate profile. The cost function of the l-th user is its own average power consumption, i.e., $u_l(x_l) = p_l a_l$. Observe that, because of the rate constraints, the set of feasible strategies $\mathcal{X}_l(\mathbf{p}_{-l}, \mathbf{a}_{-l})$ of each user l depends on the random access $\mathbf{a}_{-l}$ and power allocation $\mathbf{p}_{-l}$ of all the other users.

The optimal strategy for the l-th user, given the power allocation and random access of the others, is then the solution of the following minimization problem

$$\begin{array}{ll} \min_{x_l} & u_l(x_l) \\ \text{subject to} & x_l \in \mathcal{X}_l(\mathbf{x}_{-l}) \end{array}, l \in \mathcal{L} \tag{5}$$

where $\mathcal{X}_l(\mathbf{a}_{-l}, \mathbf{p}_{-l})$ is given in (4). In this paper, we define the noncooperative game $\mathcal{G}$ with minimization problem (5) as joint random access and power control game (JRPG). Such a JRPG has decoulped cost functions but coupled constraints (action spaces $\mathcal{X}_l$ are decoupled). Let $\Omega_l = \mathcal{P}_l \times \mathcal{A}_l$, $\Omega = \Omega_1 \times \cdots \times \Omega_L$. First, we give a feasible concept for the JRPG $\mathcal{G}$ with rate profile $\mathbf{R}^* \triangleq (R_l^* \geq B)_{l\in\mathcal{L}}$.

Definition 1 (Feasibility). *Given a rate profile $\mathbf{R}^* \triangleq (R_l^* \geq B)_{l\in\mathcal{L}}$, the JRPG $\mathcal{G}$ is feasible if the coupled equations*

$$R_l\left(x_l, \mathbf{x}_{-l}\right) = R_l^*, l \in \mathcal{L} \tag{6}$$

admit a nonempty and bounded solution $\mathbf{x} \in \Omega$.

An Nash equilibrium (NE) solution of the game $\mathcal{G}$ is defined as follows.

Definition 2 (NE). *Consider an L-player game, each player minimizing the individual cost function $u_l : \Omega_l \to \mathbb{R}_+$, subject to coupled inequality constraint $R_l^* - R_l\left(x_l, \mathbf{x}_{-l}\right) \leq 0$. Assume the JRPG $\mathcal{G}$ is feasible with rate profile $\mathbf{R}^* \triangleq (R_l^* \geq B)_{l\in\mathcal{L}}$, a vector $\mathbf{x}^* = \left(x_l^*, \mathbf{x}_{-l}^*\right)$ is called an NE solution of JRPG $\mathcal{G}$ if every given $\mathbf{x}_{-l}^*$,*

$$u_l\left(x_l^*\right) \leq u_l\left(x_l\right), \forall x_l \in \mathcal{X}_l\left(\mathbf{x}_{-l}^*\right), \forall l \in \mathcal{L}.$$

Given the feasible condition (6), the fundamental questions we want answers to are: *i) Does an NE solution exist for a given feasible users' rate profile? ii) If a NE solution exists, is it unique? iii) How can such a solution be reached in a totally distributed way?*

4 Existence of Nash Equilibrium

In this section, we give a positive result of the existence of NE by establishing the supermodular property for an equivalent game.

With $b_l = 1/a_l$, the JRPG $\mathcal{G}$ can be rewritten as follows.

$$\hat{\mathcal{G}} = \left\{\mathcal{L}, \left\{\mathcal{Y}_l\left(\mathbf{y}_{-l}\right)\right\}_{l\in\mathcal{L}}, \left\{u_l\left(y_l\right)\right\}_{l\in\mathcal{L}}\right\}, \tag{7}$$

where $u_l\left(y_l\right) = -p_l/b_l$, $y_l = (p_l, b_l) \in \mathcal{Y}_l\left(\mathbf{p}_{-l}, \mathbf{b}_{-l}\right)$, and

$$\mathcal{Y}_l\left(\mathbf{p}_{-l}, \mathbf{b}_{-l}\right) \triangleq \left\{y_l \in \mathcal{B}_l \times \mathcal{P}_l : R_l\left(y_l, \mathbf{y}_{-l}\right) \geq R_l^*\right\}, \tag{8}$$

$\mathcal{B}_l = \left[\frac{1}{a_{l,\max}}, \frac{1}{a_{l,\min}}\right]$. Thus the optimization problem (5) can be rewritten as follows.

$$\begin{array}{ll} \max_{y_l} & -p_l/b_l \\ \text{subject to} & y_l \in \mathcal{Y}_l\left(\mathbf{y}_{-l}\right) \end{array}. \tag{9}$$

Theorem 1. *The game $\hat{\mathcal{G}}$ defined by (7)-(9) is supermodular and admits a compact sublattice NE solution, then JRPG $\mathcal{G}$ admits a NE solution.*

Proof. First, we can determine that the cost function $u_l(y_l) = -p_l/b_l$ for user l is supermodular in user l's own strategy by

$$\frac{\partial^2 u_l(y_l)}{\partial p_l \partial b_l} = \frac{1}{b_l^2} > 0$$

for all $y_l \in \mathcal{Y}_l(\mathbf{y}_{-l})$. Second, for any feasible $y_l = (p_l, b_l)$ and $\hat{y}_l = \left(\hat{p}_l, \hat{b}_l\right)$, we have

$$\frac{\partial^2 u_l(y_l)}{\partial p_l \partial \hat{p}_l} = \frac{\partial^2 u_l(y_l)}{\partial p_l \partial \hat{b}_l} = \frac{\partial^2 u_l(y_l)}{\partial b_l \partial \hat{p}_l} = \frac{\partial^2 u_l(y_l)}{\partial b_l \partial \hat{b}_l} = 0$$

since the utility function $u_l(y_l)$ depends only on the own strategy of user l. Finally, we show that the strategy space $\mathcal{Y}_l$ of user l is indeed a compact sublattice for any given $\mathbf{y}_{-l}$. The strategy set $\mathcal{Y}_l(\mathbf{y}_{-l})$ in (8) can be rewritten as follows:

$$\mathcal{Y}_l(\mathbf{p}_{-l}, \mathbf{b}_{-l}) \triangleq \left\{ y_l \in \mathcal{B}_l \times \mathcal{P}_l : B \log\left(1 + \frac{KG_{ll}p_l}{\mathbf{I}_l(\mathbf{y}_{-l})}\right) - R_l^* b_l \geq 0 \right\},$$

where $\mathbf{I}_l(\mathbf{y}_{-l}) = \sum_{k \neq l} G_{lk} p_k / b_k + N_0 B$. For any two strategies $y_l = (b_l, p_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ and $\hat{y}_l = \left(\hat{b}_l, \hat{p}_l\right) \in \mathcal{Y}_l(\mathbf{y}_{-l})$, we can check $\min(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ and $\max(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ as follows. If $p_l \geq \hat{p}_l, b_l \geq \hat{b}_l$, it is easy to conclude $\min(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ and $\max(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ because of $\min(y_l, \hat{y}_l) = \hat{y}_l$, and $\max(y_l, \hat{y}_l) = y_l$. When $p_l < \hat{p}_l, b_l < \hat{b}_l$, it holds too. If $p_l \geq \hat{p}_l, b_l < \hat{b}_l$, we have $\min(y_l, \hat{y}_l) = (b_l, \hat{p}_l), \max(y_l, \hat{y}_l) = \left(\hat{b}_l, p_l\right)$. Then $\min(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ and $\max(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ can be checked as follows.

$$B \log\left(1 + \frac{KG_{ll}\hat{p}_l}{\mathbf{I}_l(\mathbf{y}_{-l})}\right) - R_l^* b_l \geq B \log\left(1 + \frac{KG_{ll}\hat{p}_l}{\mathbf{I}_l(\mathbf{y}_{-l})}\right) - R_l^* \hat{b}_l \geq 0,$$
$$B \log\left(1 + \frac{KG_{ll}p_l}{\mathbf{I}_l(\mathbf{y}_{-l})}\right) - R_l^* \hat{b}_l \geq B \log\left(1 + \frac{KG_{ll}\hat{p}_l}{\mathbf{I}_l(\mathbf{y}_{-l})}\right) - R_l^* \hat{b}_l \geq 0.$$

If $p_l < \hat{p}_l, b_l \geq \hat{b}$, $\min(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ and $\max(y_l, \hat{y}_l) \in \mathcal{Y}_l(\mathbf{y}_{-l})$ can be derived similarly as above. Thus, the game $\hat{\mathcal{G}}$ is supermodular. Based on Theorem 1 in [6], the set of NEs of game $\hat{\mathcal{G}}$ is a nonempty and compact sublattice. Because JRPG $\mathcal{G}$ is equivalent to the game $\hat{\mathcal{G}}$, we can conclude that the JRPG $\mathcal{G}$ admits a NE solution.

5 Distributed Algorithm

The JRPG $\mathcal{G}$ was shown to admit a NE. In this section, we focus on distributed algorithms to compute the NE solutions. More specifically, we employ asynchronous *myopic best response (MBR)* updates, i.e., the users update their strategies according their best response assuming the other player's strategies are fixed. Given $\mathbf{x}_{-l}$ at the user l's update time epoch, we can express the MBR updates as follows.

$$\bar{x}_l = \arg \min_{x_l \in \mathcal{X}_l(\mathbf{x}_{-l})} u_l(x_l). \tag{10}$$

Theorem 2. *Assuming the other player's strategies are fixed, the users' MBR are single-valued. Moreover, the user's MBR dynamics can be given as follows.*

$$\begin{cases} \text{if} \quad R_l(a_{l,\max}, p_{l,\max}) \geq R_l^*, \\ \qquad \bar{a}_l^{(t+1)} = a_{l,\max}, \\ \qquad \bar{p}_l^{(t+1)} = \frac{e^{R_l^*/\left(Ba_{l,\max}\right)}-1}{KG_{ll}} \mathbf{I}_l\left(\mathbf{x}_{-l}^{(t)}\right), \\ \text{otherwise} \\ \qquad \bar{a}_l^{(t+1)} = a_{l,\min}, \quad \bar{p}_l^{(t+1)} = 0. \end{cases} \tag{11}$$

where $R_l\left(a_{l,\max}, p_{l,\max}\right) = a_{l,\max} B \log\left(1 + K\frac{G_{ll}p_{l,\max}}{\mathbf{I}_l\left(\mathbf{x}_{-l}^{(t)}\right)}\right)$.

Proof. Let $z_l = p_l/b_l$, $w_l = (z_l, b_l)$, the MBR updates (10) can be equivalent to the following linear programming problem

$$\begin{array}{l} \min_{w_l \in \mathcal{W}_l} \ z_l \\ \text{subject to } g\left(w_l\right) \leq 0 \end{array} \tag{12}$$

where $\mathcal{W}_l = [0, p_{l,\max}a_{l,\max}] \times \mathcal{B}_l$, $g\left(w_l\right) = R_l^* b_l - B\log\left(1 + K\frac{G_{ll}z_l b_l}{\mathbf{I}_l(\mathbf{z}_{-l})}\right)$, $\mathbf{I}_l\left(\mathbf{z}_{-l}\right) = \sum_{k \neq l} G_{lk} z_k + N_0 B$. We can show the linear programming (12) is convex as follows. The Hessian of $g\left(w_l\right)$ is

$$\nabla^2 g\left(w_l\right) = \begin{bmatrix} \frac{\partial^2 g(w_l)}{\partial z_l^2} & \frac{\partial^2 g(w_l)}{\partial z_l \partial b_l} \\ \frac{\partial^2 g(w_l)}{\partial b_l \partial z_l} & \frac{\partial^2 g(w_l)}{\partial b_l^2} \end{bmatrix}$$

where

$$\frac{\partial^2 g\left(w_l\right)}{\partial z_l^2} = B\left(\frac{1}{1 + K\frac{G_{ll}z_l b_l}{\mathbf{I}_l(\mathbf{z}_{-l})}} K\frac{G_{ll}b_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right)^2 > 0,$$

$$\frac{\partial^2 g\left(w_l\right)}{\partial b_l^2} = B\left(\frac{1}{1 + K\frac{G_{ll}z_l b_l}{\mathbf{I}_l(\mathbf{z}_{-l})}} K\frac{G_{ll}z_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right)^2 > 0,$$

$$\frac{\partial^2 g\left(w_l\right)}{\partial z_l \partial b_l} = \frac{\partial^2 g\left(w_l\right)}{\partial b_l \partial z_l} = -B\frac{1}{\left(1 + K\frac{G_{ll}z_l b_l}{\mathbf{I}_l(\mathbf{z}_{-l})}\right)^2} K\frac{G_{ll}}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}$$

Then, if $R_l^* \geq B$ and $g\left(w_l\right) \leq 0$ holds, the Hessian matrix $\nabla^2 g\left(w_l\right)$ is positive by

$$\left\|\nabla^2 g\left(w_l\right)\right\| = \left(B\frac{1}{\left(1 + K\frac{G_{ll}z_l b_l}{\mathbf{I}_l(\mathbf{z}_{-l})}\right)^2} K\frac{G_{ll}}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right)^2 \left(\left(K\frac{G_{ll}z_l p_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right)^2 - 1\right) > 0.$$

Thus we can conclude $g\left(w_l\right)$ is convex on w_l. The Lagrangian function associated with problem (12) can be given as follows:

$$\Psi\left(\lambda_l, w_l\right) = z_l + \lambda_l\left(R_l^* b_l - B\log\left(1 + K\frac{G_{ll}z_l b_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right)\right)$$

where λ_l is the Lagrange multiplier on link l with an interpretation 'contention price'. Given $\mathbf{I}_l\left(\mathbf{z}_{-l}\right)$, The KKT conditions for user l are given by:

$$\bar{\lambda}_l \geq 0, \bar{\lambda}_l\left(R_l^* \bar{b}_l - B\log\left(1 + K\frac{G_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right)\right) = 0.$$

Consider the following two cases:

(i) $\bar{\lambda}_l > 0$, $R_l^* \bar{b}_l = B\log\left(1 + K\frac{G_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l(\mathbf{z}_{-l})}\right)$. In this case,

$$\begin{aligned}
&\left.\frac{\partial\Psi\left(\lambda_l, w_l\right)}{\partial b_l}\right|_{\lambda_l=\bar{\lambda}_l, b_l=\bar{b}_l, z_l=\bar{z}_l} \\
&= \bar{\lambda}_l\left(R_l^* - B\frac{1}{1 + K\frac{G_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l(\mathbf{z}_{-l})}}\frac{KG_{ll}\bar{z}_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right) \\
&= \frac{\bar{\lambda}_l B}{\bar{b}_l}\left(\log\left(1 + K\frac{G_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right) - \frac{1}{1 + K\frac{G_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l(\mathbf{z}_{-l})}}\frac{KG_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)}\right) \\
&> 0
\end{aligned}$$

The last inequality can be derived by $\log\left(1 + K\frac{G_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l(\mathbf{z}_{-l})}\right) = \frac{R_l^*}{B}\bar{b}_l > 1$. Therefore, the Lagrangian function $\Psi\left(\lambda_l, w_l\right)$ is monotone increasing with b_l. The MBR dynamics is given by

$$\begin{cases}
\bar{a}_l^{(t+1)} = a_{l,\max} \\
\bar{p}_l^{(t+1)} = \left[\frac{e^{R_l^*/\left(Ba_{l,\max}\right)} - 1}{KG_{ll}}\mathbf{I}_l\left(\mathbf{x}_{-l}^{(t)}\right)\right]_0^{p_{l,\max}}
\end{cases} \tag{13}$$

if $\bar{\lambda}_l^{(t)} > 0$.

(ii) $\bar{\lambda}_l = 0$.

$$\begin{aligned}
&\left.\frac{\partial\Psi\left(\lambda_l, w_l\right)}{\partial z_l}\right|_{\lambda_l=\bar{\lambda}_l, b_l=\bar{b}_l, z_l=\bar{z}_l} \\
&= 1 - \bar{\lambda}_l B\frac{1}{1 + K\frac{G_{ll}\bar{z}_l\bar{b}_l}{\mathbf{I}_l(\mathbf{z}_{-l})}}\frac{KG_{ll}\bar{b}_l}{\mathbf{I}_l\left(\mathbf{z}_{-l}\right)} \\
&= 1.
\end{aligned}$$

Then the Lagrangian function $\Psi\left(\lambda_l, w_l\right)$ is monotone increasing with $\bar{z}_l$. To minimize the Lagrangian function $\Psi\left(\lambda_l, w_l\right)$, we have the following MBR dynamics

$$\bar{a}_l^{(t+1)} = a_{l,\min},\ \bar{p}_l^{(t+1)} = 0, \text{ if } \bar{\lambda}_l^{(t)} = 0. \tag{14}$$

Based on the above analysis, the MBR dynamics can be determined by identifying the value of contention price $\bar{\lambda}_l^{(t)}$. In fact, the difference between case (i) and case (ii) is the feasible condition $R_l\left(a_{l,\max}, p_{l,\max}\right) \geq R_l^*$. The MBR dynamics is then simplified as Eqn. (11), thus the proof is completed.

Remark 1. The MBR dynamics in (11) shows that the joint optimal decision for minimal transmit cost is to access the spectrum as much as possible and compute the corresponding power allocation to support the user's rate requirement under feasible case. Apparently, the theoretical result is consistent with an intuition for the constraint condition $R_l\left(x_l, \mathbf{x}_{-l}\right) \geq R_l^*$. Because of the log operation from power allocation to link throughput, it prefers increasing the persistence probability to power allocation for supporting the rate requirement when the objective is to minimize the average power consumption.

Remark 2. The MBR dynamics in (11) shows that the joint random access and power control can be separated naturally, which maintains the architectural modularity among layers. It only needs the maximum persistence probability for computing the power allocation.

Remark 3. To update the user's power allocation, each link needs to compute the average interference plus noise $\mathbf{E}\left[\mathcal{I}_l\right]$. A simple exponential filter can achieve it as follows.

$$\mathbf{E}\left[\mathcal{I}_l\right]^{(t)} = \alpha \mathbf{E}\left[\mathcal{I}_l\right]^{(t-1)} + (1-\alpha) I_l^{(t)}, \tag{15}$$

where $I_l^{(t)}$ is the current measurement value of link l's total interference plus noise, $0 < \alpha < 1$ is a filter parameter.

Based on the Theorem 2, the detailed implementation of the asynchronous distributed random access and power control (ADRP) algorithm can be shown in `Algorithm 1`.

`Algorithm 1` ADRP Algorithm

1. INITIALIZATION: For each user $l \in \mathcal{N}$ choose some persistence probability $a_l \in \mathcal{A}_l$ and power $p_l \in \mathcal{P}_l$.
2. PERSISTENCE PROBABILITY AND POWER UPDATE: At each $t \in \mathcal{T}_l^J, \forall l \in \mathcal{L}$
   (a) each user l measures the total interference plus noise $I_l^{(t)}$ and update its average value according to (15).
   (b) each user l updates its persistence probability a_l and power p_l according to (11).

Remark 4. Only the local measurement information needed in (11), the ADRP algorithm implemented in `Algorithm 1` is then a totally decentralized algorithm. This merit is from the game theoretical design.

Based on Theorem 1 and Theorem 2, we can show the convergence of `Algorithm 1`.

Theorem 3. *If the JRPG $\mathcal{G}$ is feasible with a rate profile $\mathbf{R}^* \triangleq \left(R_l^* \geq B\right)_{l \in \mathcal{L}}$, then the Nash equilibrium of JRPG $\mathcal{G}$ is unique and ADRP Algorithm shown in `Algorithm 1` globally converges to the unique Nash equilibrium for any initial strategies.*

Proof. It is easy to show that the power update in 11 is a standard power control. Thus, based on supermodular property of JRPG $\mathcal{G}$ and scalability property of best response dynamics, the uniqueness and global convergence follow from [6, Theorem 1] and [3, Theorem 2].

6 Conclusion and Future Work

We use game theory to address the issue of joint random access and power control design in wireless ad hoc networks. Based on the realistic SINR model, we formulate the joint medium access and power control problem as a noncooperative game, where each user minimizes its average transmission cost to support a given rate requirement. The existence and uniqueness of NE is established. An asynchronous algorithm based on MBS updates is given. With some mild conditions, we show that the proposed asynchronous algorithm converges to the unique NE. In this paper, we only report the result of theoretical part. In the future, we will conduct simulations to test the performance of proposed algorithm. We will consider the delay and fairness performance to enhance the design. Furthermore, we intend to extend the game-theoretical cross layer design approach to upper layers.

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References

1. Alpcan, T., Basar, T., Srikant, R., Altman, E.: CDMA uplink power control as a noncooperative game. In: IEEE Conference on Decision and Control, Orlando, Florida, U.S.A. (2001)
2. Alpcan, T., Basar, T., Dey, S.: A power control game based on outage probabilities for multicell wireless data networks. IEEE Trans. Wireless Commun. 5(4), 890–899 (2006)
3. Altman, E., Altman, Z.: S-modular games and power control in wireless networks. IEEE Trans. Autom. Control 48(5), 839–842 (2003)
4. Cagalj, M., Ganeriwal, S., Aad, I., Hubaux, J.-P.: On Selfish Behavior in CSMA/CA Networks. In: Proceedings IEEE INFOCOM, Miami, Florida, USA, vol. 4, pp. 2513–2524 (2005)
5. Heikkinen, T.: A potential game approach to distributed power control and scheduling. Comput. Networks 50(13), 2295–2311 (2006)
6. Huang, J., Berry, R., Honig, M.L.: Distributed interference compensation for wireless networks. IEEE J. Sel. Areas Commun. 24(5), 1074–1084 (2006)
7. MacKenzie, A.B., Wicker, S.B.: Stability of multipacket slotted Aloha with selfish users and perfect information. In: Proceedings of IEEE INFOCOM, San Francisco, CA, vol. 3, pp. 1583–1590 (2003)
8. Moscibroda, T., Wattenhofer, R., Zollinger, A.: The complexity of connectivity in wireless networks. In: Proc. IEEE INFOCOM, Barcelona, Spain (2006)
9. Moscibroda, T., Wattenhofer, R., Zollinger, A.: Topology control meets SINR: the scheduling complexity of arbitrary topologies. In: 7th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MOBIHOC), Florence, Italy (2006)

10. Long, C., Zhang, Q., Li, B., Yang, H., Guan, X.: Non-cooperative power control for wireless ad hoc networks with repeated games. IEEE J. Sel. Areas Commun., special issue on Non-cooperative Behavior in Networking 25(6), 1101–1112 (2007)
11. Long, C., Li, B., Zhang, Q., et al.: The end-to-end rate control in multiple-hop wireless networks: cross-layer formulation and optimal allocation. IEEE J. Sel. Areas Commun., special issue on Control and Communications 26(5), 719–731 (2008)
12. Lu, G., Krishnamachari, B.: Energy efficient joint scheduling and power control for wireless sensor networks. In: Second IEEE International Conference on Sensor and Ad hoc Communications and Networks (SECON), Santa Clara, CA (2005)
13. Saraydar, C.U., Mandayam, N.B., Goodman, D.J.: Efficient power control via pricing in wireless data networks. IEEE Trans. Commun. 50(2), 291–303 (2002)
14. Tang, A., Lee, J.-W., Huang, J., Chiang, M., Calderbank, R.: Reverse engineering MAC. In: Proceedings of WiOpt, pp. 1–11 (2006)
15. Topkis, D.M.: Supermodularity and complementarity. Princeton Univ. Press, Princeton (1998)
16. Yates, R.D.: A framework for uplink power control in cellular radio systems. IEEE J. Select. Areas Commun. 13, 1341–1347 (1995)