Constructing Minimum Relay Connected Sensor Cover in Heterogeneous Wireless Sensor Networks

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Abstract. Energy efficiency is an important problem in wireless sensor networks. In this paper, we consider the energy efficiency problem in a heterogeneous wireless sensor network which consists of energy-constrained sensor nodes and resource-rich relay nodes. We firstly formulate the minimum relay-connected sensor cover (MRCSC) problem for heterogeneous sensor networks. The purpose of this problem is to activate as small number of sensor nodes as possible while satisfying two requirements simultaneously: (1) all active sensor nodes must cover the task area completely; (2) all active sensor nodes must be relay connected to the backbone network formed by all relay nodes. Then we propose a distributed algorithm to construct the MRCSC of a randomly deployed heterogeneous sensor network. The basic idea of this algorithm is to construct the minimum sensor cover firstly and then ensure its relay connectivity afterwards. To construct the minimum sensor cover, we present a principle for selecting sensing nodes based on the triangle lattice. In order to guarantee the relay connectivity of selected sensing nodes, we propose verification and reinforcement procedures. Extensive simulations show that the proposed algorithm can achieve the coverage performance comparable to OGDC algorithm and effectively improve the relay connectivity of the sensor cover with small number of additional sensor nodes.

Keywords: heterogeneous wireless sensor network, minimum relay-connected sensor cover, coverage, relay connectivity.

1 Introduction

Due to advances in micro-sensors, wireless networking and embedded processing, wireless sensor networks (WSNs), which consist of a large number of tiny sensor nodes with limited computation, communication capabilities and constrained energy resource, are becoming increasingly applicable to civilian and military applications, such as environmental monitoring, chemical attack detection, and battlefield surveillance, etc [1,2].

By now, most applications and research work focus on homogeneous wireless sensor networks, where all sensor nodes are identical in terms of energy resource, computation and wireless communication capabilities. However, homogeneous sensor network lacks good support for network scalability, data aggregation, and is usually not energy efficient with the many-to-one communication pattern.

To overcome these problems, heterogeneous sensor networks consisting of two or more different types of nodes with different energy supplies are proposed. In this type

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of heterogeneous networks, nodes equipped with richer power supply, larger storage capacity, more powerful computation and wireless communication capability form a reliable backbone network and relay sensing data from resource-constrained sensor nodes to remote sink. In the following description, such powerful nodes are referred as “relay nodes”. Tiny sensor nodes are responsible for data collection and the sensing data are transmitted to relay nodes in multi-hop manner. That is, the remote sensor node will ask for multi-hop data forwarding to transmit data to relay nodes. A typical multi-hop heterogeneous wireless sensor network is illustrated in Fig. 1. This type of network organization and data communication paradigm can effectively save sensor nodes’ energy, extend network lifetime, enhance network scalability, and improve data delivery ratio and latency [13,3].

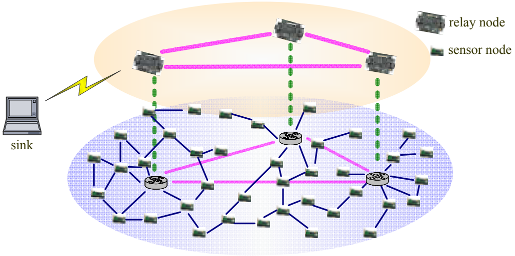


Fig. 1. Heterogeneous wireless sensor networks

This paper focuses on the energy efficiency problem related to coverage and connectivity requirements in the two-tiered heterogeneous sensor networks. Coverage depicts the quality of sensing physical world and is a fundamental problem in WSNs [4,5,6,7]. In general, there are three types of coverage in WSNs, i.e., point coverage, barrier coverage and area coverage [4]. In this paper, we concentrate on area coverage, which requires the entire area be covered by active sensor nodes so as to ensure the monitoring quality of the target field. Traditional connectivity constraint in WSNs requires sensor nodes form a connected communication network, which means any pair of node exists a communication path [8,9,10]. However, this constraint is often too rigorous for most applications of WSNs. The typical communication paradigm of most applications is as follows: sensor nodes report sensing data to and receive few instructions from the sink. Instead of establishing a communication path between two arbitrary sensors, it is sufficient for sensors to seek a path to sink in WSNs. In the two-tiered heterogeneous wireless sensor networks, since all relay nodes are reliably connected to sink via reliable backbone communication network, sensor nodes only need to maintain a communication path to an arbitrary relay node. Note this path usually still contains multi-hop wireless links. In sharp contrast to wired link, wireless link is notorious for quick path loss and fluctuation. As a result, wireless link disrupts frequently and is featured with high bit error rate. Usually, successful data delivery, defined as the probability that a sensor node correctly receives a packet from the peer packet sender of a wireless link, is used as a metric to measure the communication quality of a wireless communication link. The uncertain data delivery of wireless link leads to the uncertainty of a communication path consisting of multiple wireless links. Therefore, although a path exists in communication graph, it is still unavailable if its successful data delivery is smaller than a required threshold. A communication path is available only if its successful data delivery is above the requirement. A sensor node connected by an available communication path to any relay node is said to be relay-connected to the backbone network formed by all relay nodes.

Suppose that all sensors are relay-connected to relay nodes and cover the target area completely after initial deployment. This paper concentrates on the energy efficient organization of the two-tiered heterogeneous wireless sensor networks. Minimum connected sensor cover is an important energy efficient organization of wireless sensor networks, especially in data query applications [10]. In this paper, we aim to construct a minimum relay connected sensor cover, which consists of minimum active sensor nodes that satisfy both full coverage and relay connectivity requirements. The primary difference between the traditional minimum connected sensor cover is how “connectivity” is defined. Here we use relay-connectivity, not traditional graph connectivity.

The major contributions of this paper are as follows: (1) This paper formulates the minimum relay connected sensor cover problem in heterogeneous sensor networks. To the best of our knowledge, this is the first work addressing both the relay-connectivity and full coverage problems in heterogeneous wireless sensor networks. (2) This paper proposes a rule to select a minimum sensor cover based on triangular lattice and provide a method to enforce the relay connectivity of the sensor cover. (3) Finally, this paper designs a distributed algorithm to construct an approximate optimal minimum relay connected sensor cover and evaluates its performance extensively.

The rest of this paper is organized as follows. Section 2 reviews some related work in the literature. Section 3 formulates the MRCSC problem and section 4 presents some theoretical analysis. The distributed algorithm is described in section 5 and its performance is evaluated in section 6. Finally, we conclude this paper in section 7.

2 Related Work

A few research efforts have been made to exploit the inherent heterogeneity to prolong network lifetime, improve data delivery and extend network scalability. Cheng [3] proposes two approximate solutions to maintain connectivity by introducing relay sensors in a wireless sensor network and studies the topology improvement when relay sensors are introduced. Yarvis [11] analyzes the benefit of energy and link heterogeneity, focusing on the questions of where, how many, and what types of heterogeneous resources to deploy. It is shown that energy heterogeneity can provide more than 5-fold increase in network lifetime. Assuming that type 0 and type 1 nodes are distributed within the target area with the intensity of λ_0 and λ_1 respectively, Mhatre [12] determines the optimal node intensities (λ_0, λ_1) and node energy that can work at lest T units while ensuring connectivity and coverage with a high probability. Considering the different manufacturing costs of the hardware as well as the energy of two types of nodes, Mhatre [13] presents a cost based comparative study of homogeneous and heterogeneous clustered sensor networks. Under cost constraints, Lee [14] derives the optimal heterogeneous mixture of nodes to maximize lifetime in single-hop communication model, and investigates the impact of heterogeneity on lifetime sensing coverage and coverage aging.

I.Caidei [15] introduces heterogeneous connected sensor cover problem trying to find maximum number of set covers where each set cover monitors all discrete point targets and each sensor node is connected to at least one relay node. M.Caidei [16] addresses a target coverage problem under sensing heterogeneity where sensors are assigned suitable sensing ranges to form a maximum number of set covers. M.Caidei [17] addresses the k-degree anycast topology control problem in a heterogeneous wireless sensor network. This work aims to optimize sensors' communication radius such that each sensor is k-vertex relay node connected and minimize the maximum sensor transmission range. Assuming communication devices of the same type may have different maximal transmission powers, Ning [18] proposes two localized topology control algorithms (DRNG and DLMST) from the point of view of transmission range heterogeneity.

In this paper, we address the energy efficiency problem in heterogeneous sensor network by constructing an approximately minimum set of active sensors to respond the data query operation from the sink node. The most close work is [10], where the authors firstly propose to activate as small number of sensors as possible to reply the data query request in a homogeneous network. The active sensors must satisfy both connectivity and coverage constraints, that is, all active sensors must form a connected graph and cover the area of interest completely simultaneously. Compared to work in [10], this paper considers the heterogeneous network and requires active sensors to be "relay-connected", which is quite different from the connectivity requirement in paper [10].

3 Problem Formulation

Assume n sensors $S = \{s_1, s_2, ..., s_n\}$ and m relay nodes $RN = \{rn_1, rn_2, ..., rn_m\}$ are randomly deployed within target area R. The main tasks of sensor nodes are sensing, reporting and forwarding data. All sensor nodes are identical in terms of sensing range (r_s) and communication radius (r_c). S_i is the sensing area of node s_i. Using binary sensing model, S_i is a disk centered at s_i with radius r_s. Initially the task area R is covered completely by all sensors, that is, the union of all S_i is no less than R. Relay nodes form a backbone communication network to relay data from tiny sensor nodes to remote sink. Let R_c denote the wireless communication radius of a relay node. Since relay nodes have rich energy and powerful wireless communication capability, it is reasonable to assume that the wireless link between relay nodes is reliable. On the other hand, due to low data rate, multi-path effect and spreading fluctuation, the wireless link between two tiny sensor nodes is not deterministic. We represent this feature by associating a wireless link with a probability, **successful data delivery** (*sdd*) , which reflects the probability of receiving a packet from a sender over a wireless link successfully. Obviously, $sdd \leq 1$ for any wireless link. Note that links between relaying nodes are assumed to have $sdd = 1$.

Let $G(V, E)$ represent a heterogeneous WSN, where $V = S \cup RN$, $E = E_1 \cup E_2 \cup E_3$, and $E_1 = \{e_{ij} | d(s_i, s_j) \leq r_c, s_i \in S, s_j \in S\}$ consists of links between sensor nodes, $E_2 = \{e_{ij} | d(s_i, rn_j) \leq r_c, s_i \in S, rn_j \in RN\}$ consists of links between sensor node and relay node, $E_3 = \{e_{ij} | d(rn_i, rn_j) \leq R_c, rn_i \in RN, rn_j \in RN\}$ consists of links between relay nodes.

Definition 1. ***Graph Connectivity.*** *A heterogeneous sensor network $G(V, E)$ is graph connected if there always exists a path consisting of a sequence of edges in E between any two vertices in V.*

Graph connectivity is the traditional concept of network connectivity and is widely used in most of previous work. However, for data collecting WSN, the typical communication pattern is transmitting data from sensor nodes to the sink node. It is not necessary to maintain a path between any two sensor nodes in most cases. It is sufficient to maintain a path to one of the relay nodes for any sensor node. Furthermore, a path from a sensor node to a relay node may consist of multiple unreliable wireless links. Obviously, the sdd of such a path equals to the multiplication of sdd of all links in this path. Let $P_{ij} = \{e_{ik}, e_{kh}, ..., e_{lj}\}$ denote a path from sensor node s_i to relay node rn_j. Then $sdd_{P_{ij}} = \prod\limits_{e_{kh} \in P_{ij}} sdd_{e_{kh}}$.

Definition 2. ***Reachable Path.*** *Given a specific threshold value α, if $sdd_{P_{ij}} \geq \alpha$, path P_{ij} is a reachable path.*

Definition 3. ***Relay Connectivity.*** *Given a heterogeneous sensor network $G(V, E)$, for $\forall s_i \in S$, $\exists rn_j \in RN$, such that there exists at least one reachable path connecting s_i and rn_j, then sensor s_i is relay-connected. If every sensor node in S is relay-connected, sensor set S is relay-connected.*

Definition 4. ***Relay Connected Sensor Cover (RCSC).*** *Given a heterogeneous sensor network $G(V, E)$ deployed within target area R, a set of sensor nodes $M = \{s_{i_1}, s_{i_2}, ..., s_{i_m}\}$ is a relay connected sensor cover for R if the following two conditions hold:*

1. *$R \subseteq \{S_{i_1} \cup S_{i_2} \cup ... \cup S_{i_m}\}$. That is, area R is completely covered by sensor set M.*
2. *For $\forall s_{i_j} \in M$, s_{i_j} is relay connected through reachable paths only involving sensor nodes in M.*

Minimum Relay Connected Sensor Cover (MRCSC) Problem. Given a heterogeneous network $G(V, E)$ deployed within area R, the minimum relay connected sensor cover problem is to find the smallest relay connected sensor cover.

Suppose that there is only one relay node and all links are reliable, that is, the sdd value of each link is 1, then the MRCSC problem reduces to the minimum connected set cover problem (MCSC) in homogeneous WSN as formulated in [10]. Since the MCSC problem is NP-complete, the MRCSC problem is also NP-complete.

4 Theoretical Analysis

4.1 Full Coverage

It has been shown that triangular lattice (where the distance of two immediately adjacent sensors remains $\sqrt{3}r_s$) is the optimal deployment of sensor networks in terms of the number of needed sensors with sensing radius of r_s for the full coverage of the deployment area [8]. In a randomly deployed network, the locations of sensors follow random distribution. So it is usually hard (even impossible) to guarantee that sensor nodes locate at the exact location corresponding to triangular lattice. Therefore in most cases, it is rational to construct a near regular triangular lattice that covers the entire task area. For this purpose, a method should be proposed to restrict the spread of lattice irregularity and recover from location deviation of sensor nodes.

Theorem 1. *Let* l *denote the distance of two fixed sensors. If* $0 < l \leq \sqrt{3}r_s$, *the desired location of the third sensor should keep* $\sqrt{3}r_s$ *away from the two given sensors. If* $\sqrt{3}r_s < l \leq 2r_s$, *the desired sensor should keep* $\sqrt{2}r_s\sqrt{1+\sqrt{1-(l/2r_s)^2}}$ *away from both sensors.*

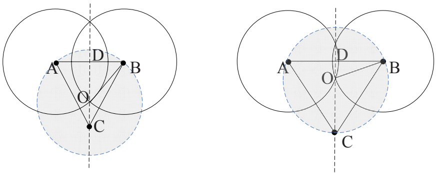
Proof. Suppose A and B represent two neighboring sensors (as shown in Fig. 2) and their distance is l. In Fig. 2(a), $0 < l \leq \sqrt{3}r_s$ and in Fig. 2(b) $\sqrt{3}r_s < l \leq 2r_s$. Let C and O denote the desired location of the third sensor and one of the crossing points of sensing disks of A and B respectively. The C is decided by two constraints: (1) the distance between C and A ($|AC|$), and the distance between C and B ($|BC|$) should be close to $\sqrt{3}r_s$, (2) the sensing disk centered at C should cover point O. Let x denote the distance, the above requirements can be formalized as the following constrained optimization problem:

$$\begin{aligned} \text{Minimize: } & |x - \sqrt{3}r_s| \\ \text{Subject to: } & |AC| = |BC| = x \qquad (1) \\ & |OC| \leq \sqrt{3}r_s \qquad (2) \end{aligned}$$

The first constraint requires that the desired location lie in the bisector between A and B. The second constraint requires it lie in the circle centered at point O with radius of r_s (the dotted circle as shown in Fig. 2). By solving this constrained optimization problem, if $0 < l \leq \sqrt{3}r_s$, the optimization solution is point C, which satisfies $|AC| = |BC| = \sqrt{3}r_s$(as shown in Fig. 2(a)). And if $\sqrt{3}r_s < l \leq 2r_s$, the optimization solution should be the intersection point between the dotted circle and the bisector line (as shown in Fig. 2(b)), which satisfies $|AC| = |BC| = \sqrt{2}r_s\sqrt{1+\sqrt{1-(l/2r_s)^2}}$. ■

According to theorem 1, in the first situation where $0 < l \leq \sqrt{3}r_s$, the new desired sensor, together with the initial two nodes, will form two new edges of length $\sqrt{3}r_s$. On the base of the new edges, a regular triangular lattice is hopeful to be created. On the other hand, if $\sqrt{3}r_s < l \leq 2r_s$, the distance between the new desired sensor and any of the initial two nodes will be smaller than $\sqrt{3}r_s$. Then this falls back to the first situation, and after another round of selection, the maximal edge length will also be limited to $\sqrt{3}r_s$. Therefore, theorem 1 can effectively restrict the spread of position irregularity during the process of constructing minimum cover set (MCS).

Theorem 1 shows how to select new sensor nodes into active sensor set. According to lemma 2 in paper [8], the set of sensors selected by theorem 1 can cover the deployment area completely. Considering the random distribution of sensor nodes, it is not guaranteed that there always exists a sensor node located at the desired location (C). Instead, we should select the sensor node that is mostly close to the desired location (i.e., the



(a) (b)

Fig. 2. Desired location of candidate sensor node

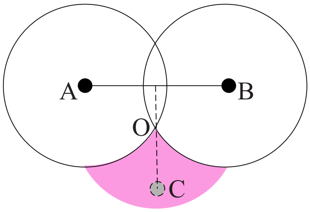


Fig. 3. Preferred region for full coverage

optimization solution). For this purpose, each sensor nodes is assigned a priority, which is reversely proportional to the distance to the desired location. Shorter the distance, higher the priority. On the other hand, to ensure complete coverage of the task area, the candidate sensors should lie within the colored area (i.e, the sensing disk centered at point O minus the union of sensing disks centered at point A and B as shown in Fig. 3). When selecting new sensor nodes into the active sensor set, candidate sensor node located within the colored area and with higher priority should be preferred.

4.2 Relay Connectivity

Given a heterogeneous sensor network $G(V, E)$ as defined in section 3. We add a virtual relay node s_v to V and accordingly add to E m virtual edges (denoted by E_v) connecting s_v and each relay node in RN. Assume these virtual edges are reliable, i.e, their sdd is 1. Let $V_e = V \cup s_v$ and $E_e = E_1 \cup E_2 \cup E_v$. Here we intentionally exclude edge set E_3 from E_e to focus on the relay connectivity of sensor nodes. Let $G(V_e, E_e)$ denote this new graph. If each vertex of V_e exists a reachable path to s_v, $G(V_e, E_e)$ is relay connected.

Theorem 2. *$G(V_e, E_e)$ is relay connected if and only if $G(V, E)$ is relay connected.*

Proof. (Sufficient) Because all virtual edges are reliable, each reachable path in $G(V, E)$ is also reachable after being extended with a virtual edge in $G(V_e, E_e)$.

(Necessary) Each vertex exists at least a reachable path connecting s_v. After omitting the virtual edge, the path also connects a real relay node and is reachable in $G(V, E)$. ■

We define $w(e_{ij}) = -\log(sdd_{ij})$ as the weight of edge $e_{ij} \in E_e$. For short, denote $w_{ij} = w(e_{ij})$. Then the weight of path P_{ij}, $w(P_{ij}) = \sum\limits_{e_{kh} \in P_{ij}} w_{kh}$. Therefore, path P_{ij} is reachable equals to $w(P_{ij}) < -\log \alpha = c$, where c is referred as *local critical weight*.

Assume $G(V_e, E_e)$ is relay connected, thus each sensor node has at least one reachable path to s_v. We can obtain a spanning tree rooted at s_v by choosing just one reachable path for each sensor node.

Theorem 3. *$G(V_e, E_e)$ is relay connected if and only if $G(V_e, E_e)$ contains a spanning tree that is also relay connected.*

Proof. (Sufficient) Suppose that $T(V_e, E')$ is a spanning tree of $G(V_e, E_e)$ and is relay connected. Due to $E' \subseteq E_e$, then $G(V_e, E_e)$ is also relay connected.

(Necessary) Suppose $T(V_e, E'')$ is constructed by choosing the shortest reachable path from E_e for each vertex in V_e. If a vertex has more than one shortest reachable path, arbitrary one is picked to break the tie. Now we prove that $T(V_e, E'')$ is a relay connected spanning tree: (1)relay connected. $G(V_e, E_e)$ is relay connected, therefore the shortest path of a vertex is reachable in $G(V_e, E_e)$ and belongs to $T(V_e, E'')$, thus $T(V_e, E'')$ is relay connected. (2) a spanning tree. If $T(V_e, E'')$ contains a cycle crossing node s_i, then s_i has more than one reachable path to s_v, which will result in contradiction to the construction of $T(V_e, E'')$. ■

Theorem 4. *$T(V_e, E')$, a spanning tree of $G(V_e, E_e)$, is relay connected if and only if all its leaf nodes are relay connected.*

Proof. (Sufficient) All vertices can be divided into two types in $T(V_e, E')$: 1) leaf vertex, which is relay connected from the hypothesis. 2) branch vertex, which is passed by at least one reachable path from a leaf vertex to s_v. Consequently, it remains relay connected.

(Necessary) If $T(V_e, E')$ is relay connected, obviously all vertices are still relay connected in $G(V_e, E_e)$. ■

5 Distributed Algorithm

In this section, we propose a distributed algorithm to find an approximate MRCSC. The algorithm runs in two steps: (1) finding a approximate minimum set cover (MSC) that minimize the number of active sensing nodes according to theorem 1. (2) verifying and reinforcing relay connectivity of selected sensing nodes based on theorems 2, 3 and 4. There are four possible states for each sensor node: STANDBY, SENSING, ENHANCING and SLEEP. Initially all sensors are in STANDBY state waiting for message from neighbors. When a sensor decides to join the set cover, it changes to SENSING state. If a node is required to enhance the relay connectivity of set cover, it changes to ENHANCING state. At the end of the algorithm, all SENSING and ENHANCING nodes keep active, while all other nodes enter into low-power SLEEP state.

5.1 MSC Construction

Triangular lattice achieves optimality in terms of the number of sensors needed. In order to find a regular triangular lattice, sensor selection follows two rules.

Rule 1: if disks of two sensing neighbors do not overlap, a stand-by node, whose distance to one of neighbors is close to $\sqrt{3}r_s$, is chosen.

Rule 2: if disks of two sensing neighbors overlap, a stand-by node is chosen according to theorem 1.

Clearly, rule 1 tries to find maximum amount of node pair whose distance equals to $\sqrt{3}r_s$, and rule 2 tries to keep the regularity of node lattice. The details of the construction process are described below.

Step 1. Sensor nodes adjacent to relay nodes report their residual energy to contend to be starting node, and then relay nodes choose the sensor node with maximum residual energy in order to keep load balance. The chosen sensor enters SENSING state and broadcasts SENSING message to declare that it will join the set cover.

Step 2. In case that a stand-by node receives an SENSING message from a neighbor,

Step 2.1. If only one sensing node exists in its neighbor list, or disks of sensing nodes do not overlap, set waiting timer $T_w = \delta T_0(\sqrt{3}r_s - d_\alpha)$, where T_0 is the duration for message transmitting, δ is a constant coefficient, d_α denotes the distance to the sensing node. Obviously, nodes with d_α closer to $\sqrt{3}r_s$ will have more chances to join set cover.

Algorithm 1. Constructing Minimum Set Cover (s_i)

```
initialize to be STANDBY state
if succeed in contending as the starting node then
    broadcast SENSING message
    return
else
    if receive SENSING message then
        set waiting timer $T = T_w$
    end if
    while $T$ not expires do
        if receive SENSING message then
            calculate all crossing points of sensing disks of neighbor nodes
            if no crossing points exist then
                Update timer $T$ using the distance $d_c$ that is mostly close to $\sqrt{3}r_s$
                continue
            end if
            if all crossing points are covered then
                return
            end if
            if find multiple uncovered crossing points then
                set $T = T_{cl}$
                continue
            end if
            if able to cover multiple uncovered crossing points then
                set $T = min(T_{cc}, T_{cl})$
                continue
            end if
        end if
    end while
    broadcast SENSING message
end if
```

Step 2.2. If at least one pair of sensing neighbors overlap and at least one crossing point remains uncovered, then cancels waiting timer T_w and starts a new delay timer T_{cl} initialized as

$$T_{cl} = \begin{cases} \beta T_0 \left(d_c^2 + (d_o - r_s)^2 + \phi \right) & : \quad d_o > r_s \\ \beta T_0 d_c^2 & : \quad \text{otherwise} \end{cases}$$

where β is a constant coefficient, d_c is the distance from the desired location, d_o is the distance to the crossing point O (as shown in Fig. 2(b)). From the expression of T_{cl}, smaller d_c (i.e, the distance to the desired location) leads to shorter delay, thus improves the chance to join the cover set. However, if there are several nodes with the same d_c, to ensure complete coverage of the target area, the sensor node with $d_o \leq r_s$ (i.e, in the colored area as shown in Fig. 3) should have higher priority to join the cover set. Therefore, in the above definition of T_{cl}, if $d_o > r_s$, the delay time of T_{cl} is longer.

Step 2.3. If multiple sensing disks of neighbor nodes overlap and form some crossing points within a node's sensing disk, there are two possibilities: (1) All crossing points are covered by those active neighbors, then cancels all timers and ignores all received message. (2) If there are more than one uncovered crossing point within the node's sensing disk, this node starts a delay timer $T = min(T_{cl}, T_{cc})$ where $T_{cc} = \gamma T_0 / f$, γ is a constant coefficient and f is the number of crossing points within the sensing disk.

Step 2.4. If no message from neighbors is received before timer expires, broadcasts an advertising message to declare its SENSING state, otherwise jumps to step 2.1.

The pseudo code description of the MSC constructing algorithm is presented in Algorithm 1.

5.2 Relay Connectivity Reinforcement

When Algorithm 1 terminates, sensor set S is divided into two subset: S_c containing all active sensors of the MSC, and the complementary set S/S_c. Based on the construction process of MSC, the subgraph induced by $S_c \cup RN \cup s_v$ is not always relay connected. Therefore, a method is needed to verify and reinforce the relay connectivity of MSC.

The reinforcement process involves the following two concepts:

Definition 5. ***Extremely Short Path (ESP).*** *A shortest path from sensor node* $s_i \in S_c$ *to* RN *only involving nodes in* $S_c \cup RN$ *is called an extremely shortest path .*

Definition 6. ***Path Separation Degree (PSD).*** *Suppose path* P_i *is a path from sensor node* s_i *to a relay node. The integer* $q = |P_i \cap (S/S_c)\,|$ *is called the path separation degree of* P_i*.*

The PSD represents the association degree of a path with set S/S_c. Obviously, the PSD value of an ESP path is 0.

The basic idea of verification and reinforcement is as follows: (1) broadcast probing messages to build a relay tree rooted at s_v with vertices in $S_c \cup RN$; (2) check whether all nodes in the relay tree are relay connected; (3) staring from leaf nodes, reinforce the relay connectivity hop by hop reversely. The probing message mainly includes the following three fields: the weight of the ESP (w_e), the weight of the shortest path (w_s) and PSD. Reinforcing request message contains critical acceptant weight (*caw*) information, which is the minimum path weight required by its transmitter to maintain reachable.

Step 1. A probing message ($w_e = w_s = 0, PSD = 0$) is originated by a relay node in RN

Step 2. Listen to neighbors,

Step 2.1. If a STANDBY sensor receives a probing message from the shortest path, it records the message and broadcasts it after increasing w_s by the weight of the edge where the message is received and increasing PSD by 1.

Step 2.2. If a SENSING sensor receives a probing message from the shortest path, it records the message and broadcasts it after increasing w_s by the weight of the edge where the the message is received. If the message is received from an ESP, it should respond an acknowledgement (ACK) to its father node besides updating w_e.

Step 2.3. If a SENSING node receives an ACK aiming at itself, it marks itself as a branch node. If no ACK is received during this period, marks itself as a leaf node.

Step 3. Verify and reinforce relay connectivity.

Step 3.1. Leaf node checks whether they are relay connected by comparing w_s with c, the local critical weight. If $w_s > c$, leaf node broadcasts a path reinforcement request (**PRR**) with $caw = c$ and removes leaf marks. The father node of this leaf node checks whether itself becomes a new leaf node.

Step 3.2. While a node receives a PRR, it calculates the foreign request weight (c_f) as the request message's caw minus the weight of edge where message is received. If $c_f > w_s$, it starts a timer $T_r = T_0 \eta q$ (η is a is a constant and q is the PSD of its shortest path). Note that if the $c_f > w_e$ for a SENSING or ENHANCING node, q is se to 0.

Step 3.3. If no PRR is received before timer T_r expires, it sets *flag=1* to indicate that it should be in ENHANCING state. Otherwise jumps to step 3.2.

After the verification and reinforcement process terminates, all SENSING and ENHANCING sensor nodes remain active, while other nodes go to SLEEP state to save energy.

The pseudo code description of algorithms for verifying and reinforcing the relay connectivity are presented in Algorithm 2 and Algorithm 3 respectively.

6 Performance Evaluation

We evaluate our algorithm via extensive simulations which are implemented on Windows XP platform in C++ programming language. Sensor nodes with sensing range of $10m$ are randomly scattered in a rectangular area. For each simulation scenario, fifty runs with different random node distributions are conducted and only the average is presented.

We use the energy model in [20] where the energy consumption ratio for transmitting, receiving, idle and sleep is $20 : 4 : 4 : 0.01$. Assume that the energy consumption of a node in idle state for 1 second is 1 unit. All sensor nodes have initial energy of 5000 units. The wireless capacity is 40Kbps, the average length of message is 20 bytes,

Algorithm 2. Verify Relay Connectivity (s_i)

```
while receive a probing message do
    if message from a neighbor in $S/S_c$ then
        increase PSD of the message by 1
    end if
    if $s_i \in S/S_c$ then
        compute the weight of the shortest path $w_s$
    else
        compute $w_s$ or $w_e$
        reply an ACK to the father node on the ESP
    end if
    broadcast a probing message with new $w_s$, $w_e$ and new PSD
    if $s_i \in S_c$ and no ACK received then
        mark itself as leaf node
    end if
end while
```

Algorithm 3. Reinforce Relay Connectivity (s_i)

```
if $s_i$ is a leaf node and $w_e > c$ then
    broadcast a PRR message and remove leaf mark
else
    flag = 0
    while receive a message from neighbor do
        if message is a PRR then
            compute $c_f$
            if $c_f \geq w_e$ then
                reply an ACK to the request
                return
            end if
            if $c_f > w_s$ then
                update or compute $T_r$
            end if
        end if
        if message is an ACK to neighbor's request then
            cancel $Tr$
            if $s_i \in S_c$ and all child nodes are relay-connected then
                set leaf mark
            end if
        end if
        if $T_r$ expires then
            flag = 1, reply an ACK to the request
        end if
        if ($s_i$ is a leaf node and $w_e > c$) or (flag = 1 and $c_f > w_e$) then
            broadcast a PRR and remove leaf mark
            return
        end if
    end while
end if
```

thus $T_0 = 4ms$. The link communication model in [21] is employed, where the sdd of a wireless link is defined as follows:

$$sdd\,(e_{ij}) = \begin{cases} 1 - 0.2d/r_{c0} & : \quad d \le r_{c0} \\ 2.4 - 1.6d/r_{c0} & : \quad r_{c0} \le d \le 1.5r_{c0} \\ 0 & : \quad d > 1.5r_{c0} \end{cases}$$

where e_{ij} is the link between node s_i and s_j, d denotes the Euclidean length of e_{ij}, r_{c0} is the nominal communication range of sensor with maximum value of $1.5r_{c0}$.

The key metrics of interest are (1)coverage performance, which includes (a) the number of active sensors in SENSING state required to cover the task area and (b) coverage ratio, i.e., the ratio of covered area to the total area; (2) relay-connectivity performance, i.e, (a) the ratio of the number of relay connected sensors after to before reinforcement, and (b) the ratio of set size after to before reinforcement;(3) networking lifetime, which is the duration that network works before network coverage falls below a pre-determined percentage. In particular, the percentage is set to 0.8 throughout our simulations.

To evaluate coverage quality, we compare our algorithm against two outstanding coverage algorithms: OGDC [8] and CSC [10]. Since the cover sets formed by the latter algorithms only remain graph connected, our comparison is limited to MSC without reinforcing for the purpose of equity. The area is bound to $50m \times 50m$, one relay node locates at the center of the task area, and sensor's nominal communication range is $13.3m$(since $1.5r_{c0} = 2r_s$). The simulation results on coverage performance are shown in Fig. 4 and Fig. 5. Fig. 4 shows how the number of active SENSING nodes varies with the number of total sensor nodes initially deployed. There is only a slight increase in the number of active SENSING nodes when network size varies from 100 to 1000. When network size increases, more sensor nodes will be likely deployed near the boundary of the target region. Boundary nodes will have fewer neighbors and the lack of neighbor nodes will keep these sensor nodes active. Fig. 5 shows how the coverage ratio provided by active SENSING nodes varies with the number of sensor nodes. It can be seen that our algorithm produces the minimal set of active sensor nodes, while providing the coverage ratio of at least 0.97, which is only slightly smaller than that of OGDC algorithm. This is reasonable since more active sensor nodes can certainly provide higher coverage ratio. However, fewer active nodes will be more efficient to save energy and prolong network lifetime.

The sdd of a path is impacted by both the path length and the quality of links in the path. When the sdd of a reachabel path is predefined to 0.4 and the task area expands to $100m \times 100m$ with 3 relay nodes, Fig. 6 shows the impact of reinforcement procedure on the relay connectivity of sensing nodes. We can see that reinforcement procedure effectively improves the relay connectivity of MSC, especially when the relay connectivity ratio is poor (e.g, < 0.85). Fig. 7 depicts the comparison of the size of MRCSC and MSC. The result shows that the size of MRCSC is within 1.1 times the size of MSC, which means that the reinforcement process can effectively improve the relay connectivity only through few additional nodes.

The total network lifetime is divided into slots with equal length of 100 seconds and networks run in round-robin manner. At the beginning of each round, MRCSC is built. The factors that affect network lifetime are node density, number of relay nodes

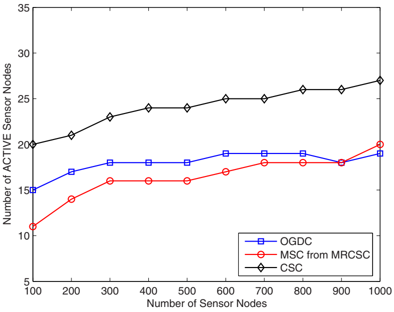


Fig. 4. Number of active sensor nodes

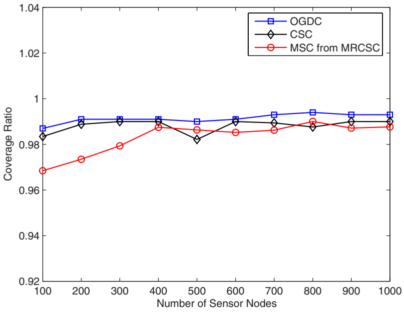


Fig. 5. Coverage ratio provided by active sensor nodes

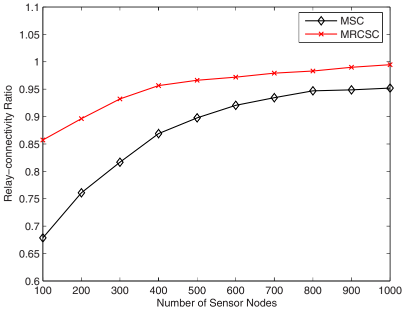


Fig. 6. Relay-connectivity in MRCSC and MSC

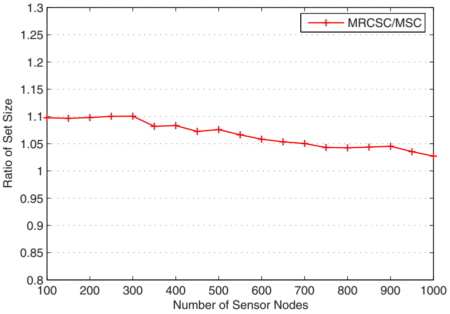


Fig. 7. Ratio of set size

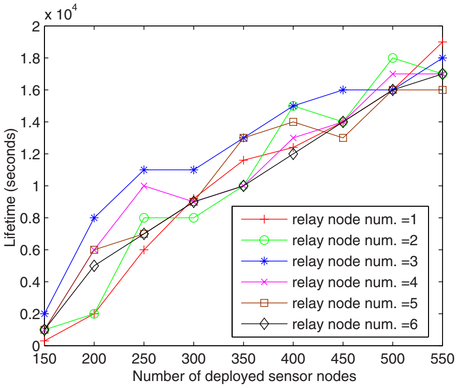


Fig. 8. Network lifetime

and size of MRCSC at each round. Network lifetime is measured by the time interval before network coverage falls below 0.8. Fig. 8 shows how the network lifetime varies with the number of deployed sensor nodes and relay nodes. Similar to our intuition, the network lifetime improves with the increase of the number of sensor nodes. Lifetime also increases with the number of relay nodes when sensor number is smaller than 450. However, more relay nodes result in more communication overheads, which is harmful to energy saving. As a result, the increasing rate of lifetime will slow down. Under our current setting, evenly deploying 3 relay nodes can achieve optimization in terms of network lifetime.

7 Conclusions

In this paper, we consider the energy efficiency problem in a heterogeneous wireless sensor network which consists of battery-powered sensor nodes and resource-rich relay nodes. The purpose of this paper is to construct a minimum relay-connected sensor cover (MRCSC) in order to improve the energy efficiency and prolong the lifetime of the whole network. The MRCSC consists of as small number of active sensor nodes as possible while maintaining both full coverage and relay connectivity simultaneously. A distributed, two-stage algorithm is proposed to construct the MRCSC of a randomly deployed heterogeneous sensor network. Inspired by triangular lattice, we present a principle for selecting sensing nodes to restrict the spread of irregularity of lattice. In order to guarantee the relay connectivity of selected sensing nodes, we propose verification and reinforcement procedures. Extensive simulations show that the proposed algorithm can achieve the coverage performance comparable to OGDC algorithm and effectively improve the relay connectivity of the sensor cover with small number of additional sensor nodes.

Currently this paper assumes the unit disk model for both sensing and communication ranges of the sensors. In the future work, we will consider the probabilistic sensing and communication model, and revise the proposed approach to the MRCSC problem under the realistic assumption. Another direction of our future work is to evaluate the energy efficiency, communication overhead of the proposed protocol more comprehensively.

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