Exact Models for the *k*-Connected Minimum Energy Problem

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Abstract. We consider the minimum energy problem for a mobile ad hoc network, where any node in the network may communicate with any other via intermediate nodes. To provide quality of service, the network must be connected, even if one or more nodes drop out. This motivates the notion of k-connectivity. The minimum energy problem aims to optimise the total energy that all nodes spend for transmission. Previous work in the literature includes exact mixed-integer programming formulations for a 1-connected network. We extend these models for when the network is k-connected, and compare the models for various network sizes. As expected, the combinatorial nature of the problem limits the size of the networks that we can solve to optimality in a timely manner. However, these exact models may be used for the future design of mobile ad hoc networks and provide useful benchmarks for heuristics in larger networks.

Keywords: Minimum transmission, exact model, k-connectivitiy, mixed-integer programming.

1 Introduction

We study the problem of a wireless ad hoc network. In such a network, mobile phones connect to each other without the intermediacy of a transmitting tower. Instead, each phone communicates directly with its peers. To ensure proper communication both phones must be able to reach each other before a link can be established. If a phone cannot reach its intended target directly, it may communicate through a chain of intermediary mobile phones. The underlying topology of the network is therefore an undirected graph; phones which communicate directly are linked in the graph.

We wish to find the optimal transmission range for each participant in the network such that the network is connected – the intention is to preserve battery life for each user in the network. This problem is known in the literature as the symmetric, unicast transmission problem ([14]). As a related problem, we can

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motivate the asymmetric, unicast transmission problem by data exchange such as e-mails and SMS. However, the asymmetric problem is beyond the scope of this paper.

In the problem studied here, each node within the network can operate at different transmission energies. Using power attenuation functions, we can define the range achievable with each energy ([4]). Within the network, a node's transmission range determines the number of nodes to which it can connect. As a result, we are interested in constructing range assignments for the network that satisfy connectivity requirements while minimising the energy expenditure of the network. However, if one node drops out in a connected graph (for example, if a phone is turned off), then the graph may no longer be connected. This may lead to the disconnection of calls. We wish to ensure quality of service by ensuring a greater level of connectivity. This motivates the notion of k-connectivity.

Definition 1. *A graph is k-connected if removing any set of $k-1$ nodes results in a connected graph.*

A k-connected network can absorb the loss of $k-1$ nodes without losing connectivity. Note that a 1-connected network is equivalent to a connected network.

There are two philosophical ways to solve the minimum energy problem – exact and heuristic solutions. While there have been heuristic solutions for the k-connected problem, as far as we know there have only been 1-connected exact models. This paper aims to address this hole in the literature. One of the uses of a k-connected exact model is that we will now be able to evaluate precisely the effectiveness of the heuristics for this problem.

From a practical standpoint, in real life each node will have only limited information on the status of the other nodes. Moreover, if we had full information, the likely size of the network would prevent a viable exact solution. This motivates the application of a heuristic so that each node can quickly determine a good transmission range. Shpungin and Segal ([15]) presented such a heuristic, which they based on an optimal algorithm for solving the same problem on a linear network. Berend, Segal and Shpungin later improved this heuristic in [2]. Kirousis *et al.* ([11]) showed that the k-connected solution for a one-dimensional network can be solved in polynomial time.

A possible approach for the 2-connected case is to solve the 2-connectivity augmentation problem ([9,14]). This begins with a connected network and adds the minimum number of edges such that the network is 2-connected, producing an approximate solution to the minimum energy problem. Another approach, used by Shpungin and Segal ([15]), is to find a Hamiltonian cycle in the network. Hamiltonian cycles are by definition 2-connected, so this provides an approximate solution for the 2-connected problem, which can also be extended to an approximate solution for the k-connected problem. Jia *et al.* ([10]) have provided approximation algorithms for the 3 and 4-connected cases by searching for inconnected subgraphs in the network, which are subgraphs which have only inward edges from the rest of the graph.

From a theoretical viewpoint, we would like to solve the problem to optimality, by constructing an exact model. Often these models take the form of integer or mixed-integer programs. Over the last couple of decades, advances in algorithms for solving large scale mixed-integer programs allow us to consider problems that were previously computationally unthinkable. Although, as we will see in Section 5, the computation time grows exponentially with k, we are able to obtain optimal solutions for networks of up to 18 nodes and 3-connectivity.

Mixed-integer programming models have been considered for both the homogeneous problem, where each node must have the same transmission range ([5]), and the heterogeneous problem, where this restriction does not apply ([8,12,16]). We concentrate on exact solutions in this paper and examine these models closely in Section 3. However, these models only cover the 1-connected case.

In this paper, we look at the minimum energy problem for both the connected and k-connected cases using the exact methodology of integer or mixed-integer programs. In Section 3, we discuss models for the connected minimum energy problem, including a new model and previous results from Montemanni and Gambardella ([12]) and Yuan *et al.* ([16]). In Section 4, we extend these models to k-connectivity. In Section 5, we implement all of the models, solve them for varying network sizes, and compare the results. We conclude the paper with a discussion in Section 6.

2 The Homogeneous Minimum Energy Problem

We begin by considering the special case of homogeneous transmission radius, where every node uses the same transmission radius regardless of the topology of the network. For this problem, there is an easy solution involving the minimum spanning tree, which is simple to find. The next theorem was stated in [5], but not proved. We provide a brief proof.

Theorem 1. *The minimum homogeneous transmission radius such that the network is connected is the length of the largest edge of the minimum spanning tree.*

Proof. Suppose that the theorem is false and there is a smaller transmission radius, r, which connects the network. Let the largest edge in the minimum spanning tree be e. If we now remove e from the minimum spanning tree, the result is a forest with two trees. Since the minimum transmission radius connects the network, there must exist two nodes, one from each tree in the forest, which are at most r distance apart. If we add the edge connecting these two nodes to the forest, we get a spanning tree which is smaller than the minimum spanning tree, a contradiction. Therefore the minimum transmission radius is the length of the largest edge of the minimum spanning tree.

As the minimum spanning tree can be found in polynomial time, this theorem may provide further insights into obtaining good but easily found upper bounds on the optimal solution for the heterogeneous problem. Although we will not investigate such bounds in this paper, we provide the theorem here for literature completion.

3 Exact 1-Connected Models

Now we consider the more complicated heterogeneous problem, where each node can have a different transmission radius to any other. In this section, we consider the minimum energy problem when the network need only be 1-connected. First, we look at existing models in the literature.

3.1 Literature Review

Das *et al.* ([8]) looked at the multicast/broadcast version of the problem and presented three integer programs to solve it exactly. In this paper, however, we will focus on formulations that address the symmetric unicast problem or can be easily translated to it.

Montemanni and Gambardella ([12]) presented two mixed-integer programs to solve this problem. Both of these models were based on a network flow model, where we imagine that a source node, s, emits $|V| - 1$ units of flow (where $|V|$ is the number of nodes in the network). Every other node will absorb exactly 1 unit of flow. If this is possible, then the edges with nonzero flow must span the graph, and so the network is connected. They represented the flow from node i to node j by $x_{i,j}$.

The transmission radius of each node is determined by the indicator variable, $y_{i,j}$, which is 1 if node i has enough transmission to reach node j, and 0 otherwise. To capture the ranges in the objective function, Montemanni and Gambardella used the idea of *ancestor nodes*: they defined a^i_j to be the farthest node from i that is closer than j, and $a^i_j = i$ if j is the nearest node to i. Then they defined $c_{i,j}$ to be the incremental cost involved in extending the range of node i from a^i_j to j. This is illustrated in Figure 1, where $a^i_m = k$, and $a^i_k = j$. An advantage of using ancestor nodes is that the transmission value is not a variable. Therefore it is a simple task to account for power attenuation in this model (without transforming it into a nonlinear model).

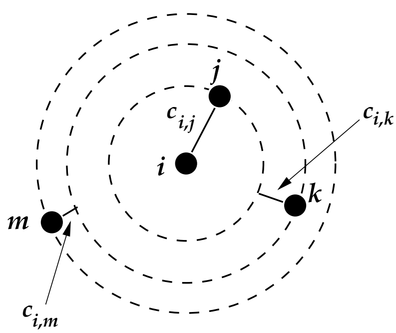

Fig. 1. Counting partial range costs using ancestor nodes

Using these ideas, and defining A to be the set of all possible edges, they produced the following model.

$$\begin{aligned}
\mathbf{MG1}: \quad \text{Minimise} \quad & \sum_{(i,j)\in A} c_{i,j} y_{i,j} \\
\text{subject to} \quad & y_{i,j} \le y_{i,a^i_j} && \forall\ (i,j) \in A, a^i_j \ne i, && (1)\\
& x_{i,j} \le (|V|-1) y_{i,j} && \forall\ (i,j) \in A, && (2)\\
& x_{i,j} \le (|V|-1) y_{j,i} && \forall\ (i,j) \in A, && (3)\\
\sum_{j:(i,j)\in A} x_{i,j} - \sum_{j:(j,i)\in A} & x_{j,i} = \begin{cases} |V|-1 & \text{if } i = s, \\ -1 & \text{otherwise,} \end{cases} && && (4)\\
& x_{i,j} \in \mathbb{R}, && && (5)\\
& y_{i,j} \in \{0,1\}. && && (6)
\end{aligned}$$

The objective function minimises the total energy requirements for all nodes. Constraint (1) ensures that node i cannot reach node j unless it can also reach its ancestor node, a^i_j, and constraints (2)–(4) are the flow constraints. The number of variables and constraints in this model is $\mathcal{O}(2n^2)$ and $\mathcal{O}(3n^2)$ respectively. In their original model, Montemanni and Gambardella defined the flow variable to be continuous. However, improved computation times may be achieved if we restrict this variable to be integer and solve the model with the faster methods available for pure integer programs (as opposed to mixed-integer programs). Althaus *et al.* ([1]) later presented a model which is structurally similar to MG1, so we will not repeat it here.

Montemanni and Gambardella recognised the difficulty in solving this model for large network sizes and devoted much time to improving the computational efficiency of their model. To do this, they developed a new model (**MG2** below), which defines $z_{i,j}$ to be 1 if node i communicates with node j. Using the same notion of ancestor nodes as in the previous model, they forced connectivity by ensuring that every possible subset of nodes has at least one edge to the complementary set of nodes (constraint (10)). In doing so, they considered $2^{|V|} - 2$ sets (the null and full sets need not be considered). This gave the following model:

$$\begin{aligned}
\mathbf{MG2}: \quad \text{Minimise} \quad & \sum_{(i,j)\in A} c_{i,j} y_{i,j} \\
\text{subject to} \quad & y_{i,j} \le y_{i,a^i_j} && \forall\ (i,j) \in A, a^i_j \ne i, && (7)\\
& z_{i,j} \le y_{i,j} && \forall\ (i,j) \in E, && (8)\\
& z_{i,j} \le y_{j,i} && \forall\ (i,j) \in E, && (9)\\
\sum_{\{i,j\}\in E, i\in W, j\in V\setminus W} & z_{i,j} \ge 1 && \forall\ W \subset V, && (10)\\
& z_{i,j} \in \{0,1\}, && && (11)\\
& y_{i,j} \in \{0,1\}. && && (12)
\end{aligned}$$

They also provided a set of valid inequalities to help quicken the computation time:

$$y_{i,j} = 1 \qquad \forall\ (i,j) \in A\ s.t.\ a^i_j = i, \tag{13}$$

$$y_{a^i_j,i} \geq y_{i,a^i_j} - y_{i,j} \qquad \forall\ (i,j) \in A\ s.t.\ a^i_j \neq i, \tag{14}$$

$$y_{j,i} \geq y_{i,j} \qquad \forall\ (i,j) \in A\ s.t.\ \nexists (i,k) \in A, a^i_k = j, \tag{15}$$

$$\sum_{(i,j)\in A} y_{i,j} \geq 2(|V| - 1), \tag{16}$$

$$\sum_{j\in V\ s.t.\ (i,j)\in A} y_{j,i} \geq 1 \qquad \forall\ i \in V. \tag{17}$$

In Section 5, we will see that using these inequalities enable these models to outperform other models. However, not all of these inequalities are valid for the k-connected case.

Yuan *et al.* [16] presented a mixed-integer program for the minimum energy broadcast problem. They used a multi-commodity flow model, which requires a large number of constraints and is notoriously difficult to solve but has proven to be an effective method for difficult problems such as the Travelling Salesman Problem [13]. In a multi-commodity flow model, a source node s is chosen and has a different flow for each destination. For each flow, the source emits one unit of flow and the chosen destination absorbs one unit, while all other nodes do not emit or absorb.

The model presented in this paper was for the asymmetric broadcast problem, which requires that one node transmit to all other nodes, possibly without receiving anything back. In particular, this means that the model does not require two-way communication between nodes. We have modified the model for the symmetric unicast problem, which require two-way communication to establish a link between two nodes, and also requires all nodes to be able to communicate with each other.

In this model, the indicator variable $z_{i,j}$ is 1 if the transmission range of node i is exactly the distance from i to j (denoted by $p_{i,j}$ in this model). As with the MG1 and MG2 models, this enables the model to easily account for power attenuation without making it nonlinear.

$$\textbf{YBH}: \quad \text{Minimise} \qquad \sum_{(i,j)\in A} p_{i,j} z_{i,j}$$

$$\text{subject to} \qquad \sum_{j:(i,j)\in A} z_{i,j} \leq 1 \qquad \forall\ i \in V, \tag{18}$$

$$\sum_{j:(i,j)\in A} x^d_{i,j} - \sum_{j:(j,i)\in A} x^d_{j,i} = \begin{cases} 1 & \text{if } i = s, \\ -1 & \text{if } i = d, \\ 0 & \text{otherwise}, \end{cases} \qquad \forall\ i \in V,\ \forall\ d \in V \backslash \{s\}, \tag{19}$$

$$y_{i,j} \leq \sum_{p_{i,k}\geq p_{i,j}} z_{i,k} \qquad \forall (i,j) \in A, \tag{20}$$

$$y_{j,i} \leq \sum_{p_{i,k} \geq p_{i,j}} z_{i,k} \qquad \forall (i,j) \in A, \tag{21}$$

$$x_{i,j}^d \leq y_{i,j} \qquad \forall (i,j) \in A, d \in V \backslash \{s\}, \tag{22}$$

$$x_{i,j}^d \geq 0, \tag{23}$$

$$y_{i,j} \in \{0,1\}, \tag{24}$$

$$z_{i,j} \in \{0,1\}. \tag{25}$$

Constraint (18) forces each node to choose its range exactly, while constraint (19) represents the multi-commodity flow constraints. Constraints (20)–(22) ensure that flow only goes through edges between nodes which are connected (which happens when $y_{i,j} = 1$).

3.2 A New Model

We introduce a new model, which is again based on network flow. This model is quite similar to MG1, except that we directly include the transmission range of node i as a continuous variable, r_i. We define the distance between nodes i and j to be $d(i,j)$. We use $y_{i,j}$ to indicate if nodes i and j are connected (with a bi-directional link), and $x_{i,j}$ to denote the flow from i to j. We also restrict the flow variables to be integer.

$$\textbf{BSC}: \quad \text{Minimise} \qquad \sum_i r_i$$

subject to

$$y_{i,j} \leq \frac{r_i}{d(i,j)} \qquad \forall\ i \in V, j \neq i, \tag{26}$$

$$y_{i,j} = y_{j,i} \qquad \forall\ i \in V, j < i, \tag{27}$$

$$x_{i,j} \leq (|V|-1) y_{i,j} \qquad \forall\ i,j \in V, \tag{28}$$

$$\sum_{j:(i,j)\in A} x_{i,j} - \sum_{j:(j,i)\in A} x_{j,i} = \begin{cases} |V|-1 & \text{if } i = s \\ -1 & \text{otherwise} \end{cases} \qquad \forall\ i \in V, \tag{29}$$

$$x_{i,j} \in \mathbb{Z}^+, \tag{30}$$

$$y_{i,j} \in \{0,1\}. \tag{31}$$

Constraints (26) and (27) ensure that the connectivity variables are properly set, and constraints (28) and (29) are the flow constraints, directly analogous to constraints (2)–(4).

We implemented the four models in this section to solve the connected minimum energy problem. The results are given in Section 5.

4 Exact *k*-Connected Models

In this section, we extend these models to the k-connected case. To begin, it is obvious that a network is connected if and only if we can find a spanning tree in it. This leads us to the following observation.

Observation 1. *A network is k-connected if and only if we can remove any combination of $k-1$ nodes and find a spanning tree for the remaining nodes. There are $\binom{|V|}{k-1}$ such combinations.*

This translates easily into extensions for the integer programs in Section 3. Firstly, we define the superset L to contain all sets with $k-1$ nodes. We now have to apply connectivity constraints for each $l \in L$. We do this by attaching a sub- or superscript l to each connectivity variable involved in the constraints, and applying those constraints to the node set $V \backslash l$. We denote the corresponding edge set by A^l, and if we have to choose a source node, we denote it by $s^l \notin l$.

For the MG1 model, we replace constraints (2)–(5) by the following constraints for each $l \in L$:

$$x_{ij}^l \leq (|V|-k)y_{ij} \qquad \forall\ (i,j) \in A^l, \tag{32}$$

$$x_{ij}^l \leq (|V|-k)y_{ji} \qquad \forall\ (i,j) \in A^l, \tag{33}$$

$$\sum_{j:(i,j)\in A^l} x_{ij}^l - \sum_{j:(j,i)\in A^l} x_{j,i}^l = \begin{cases} |V|-k \text{ if } i = s^l, \\ -1 \qquad \text{otherwise,} \end{cases} \tag{34}$$

$$x_{ij}^l \in \mathbb{R}. \tag{35}$$

Similarly, for the MG2 model, we replace constraints (8)–(11) by:

$$z_{ij}^l \leq y_{ij} \qquad \forall\ (i,j) \in A^l, \tag{36}$$

$$z_{ij}^l \leq y_{ji} \qquad \forall\ (i,j) \in A^l, \tag{37}$$

$$\sum_{\{i,j\}\in A^l, i\in S, j\in V\backslash(l\cup S)} z_{i,j}^l \geq 1 \qquad \forall\ S \subset V\backslash l, \tag{38}$$

$$z_{ij}^l \in \{0,1\}. \tag{39}$$

For the YBH model, we set $D^l = V\backslash\{l, s^l\}$ and replace constraints (19), (23)–(24) by:

$$\sum_{j:(i,j)\in A^l} x_{i,j}^{d,l} - \sum_{j:(j,i)\in A^l} x_{j,i}^{d,l} = \begin{cases} 1 & \text{if } i = s^l, \\ -1 & \text{if } i = d, \\ 0 & \text{otherwise,} \end{cases} \qquad \forall\ i \in V\backslash l,\ \forall\ d \in D^l, \tag{40}$$

$$x_{i,j}^{d,l} \leq c_{i,j} \qquad \forall (i,j) \in A^l, d \in D^l, \tag{41}$$

$$x_{ij}^{d,l} \geq 0.$$

Finally, for the BSC model, we replace constraints (28)–(30) with:

$$x_{i,j}^l \leq (|V|-k)y_{i,j} \qquad \forall\ (i,j) \in A^l, \tag{42}$$

$$\sum_{j\in(V\backslash l)} x_{i,j}^l - \sum_{l\in(V\backslash l)} x_{l,i}^l = \begin{cases} |V|-k \text{ if } i = s^l \\ -1 \qquad \text{otherwise} \end{cases} \qquad \forall\ i \in (V\backslash l), l \subset V, \tag{43}$$

$$x_{i,j}^l \in \mathbb{Z}^+. \tag{44}$$

The sizes of these models with respect to the number of variables and constraints are in Tables 1 and 2.

Table 1. The asymptotic number of variables for each problem

	MG1	**MG2**	**YBH**	**BSC**
1-connected	$2n^2$	$2n^2$	n^3	$2n^2$
k-connected	$\frac{1}{(k-1)!}n^{k+1}$	$\frac{1}{(k-1)!}n^{k+1}$	$\frac{1}{(k-1)!}n^{k+2}$	$\frac{1}{(k-1)!}n^{k+1}$

Table 2. The asymptotic number of constraints for each problem

	MG1	**MG2**	**YBH**	**BSC**
1-connected	$3n^2$	2^n	n^3	$\frac{5}{2}n^2$
k-connected	$\frac{2}{(k-1)!}n^{k+1}$	$\frac{1}{(k-1)!}2^n n^{k-1}$	$\frac{1}{(k-1)!}n^{k+1} + n^3$	$\frac{1}{(k-1)!}n^{k+1}$

5 Numerical Results

We wish to determine the practical implications of the theory discussed here. For instance, we would like to know the limits of the models in terms of the size of k and $|V|$ that we can solve with these models in a timely manner.

We implemented all the models in Xpress-Optimiser (version 18.10.00) and ran 50 randomised networks for network sizes ranging from 5 to 20. In Figure 2, we provide an illustration of the optimal networks as the connectivity requirement progresses from 1- to 3-connected. We have used the Euclidean distance to calculate the distance between nodes. This means that the outer nodes generally have a much larger transmission radius than the inner nodes, because they have fewer neighbouring nodes. This issue can be addressed by using toroidal boundary conditions.

All four models were run on the same data sets, allowing us to compare the results here. For the 1-connected problems, the best performer of the four models was the MG1 formulation, which solved network sizes of 18 nodes in a short computation time (under 10 minutes each). The BSC and MG2 formulations produced similar results to each other, with MG2 ahead in computational time, but achieving the same maximum network size. The YBH formulation was clearly the worst performer (Figure 3).

From Figure 3, it is clear that the computation time grows asymptotically exponentially with the network size. We fitted regression lines to the observed data and produced the following relations, where t is the time and n the number of nodes:

$$\begin{aligned}
\text{MG1:}\quad & t \sim 0.00104 \times 1.959^n \\
\text{MG2:}\quad & t \sim 0.000423 \times 2.622^n \\
\text{YBH:}\quad & t \sim 0.000429 \times 3.237^n \\
\text{BSC:}\quad & t \sim 0.0000273 \times 3.593^n
\end{aligned}$$

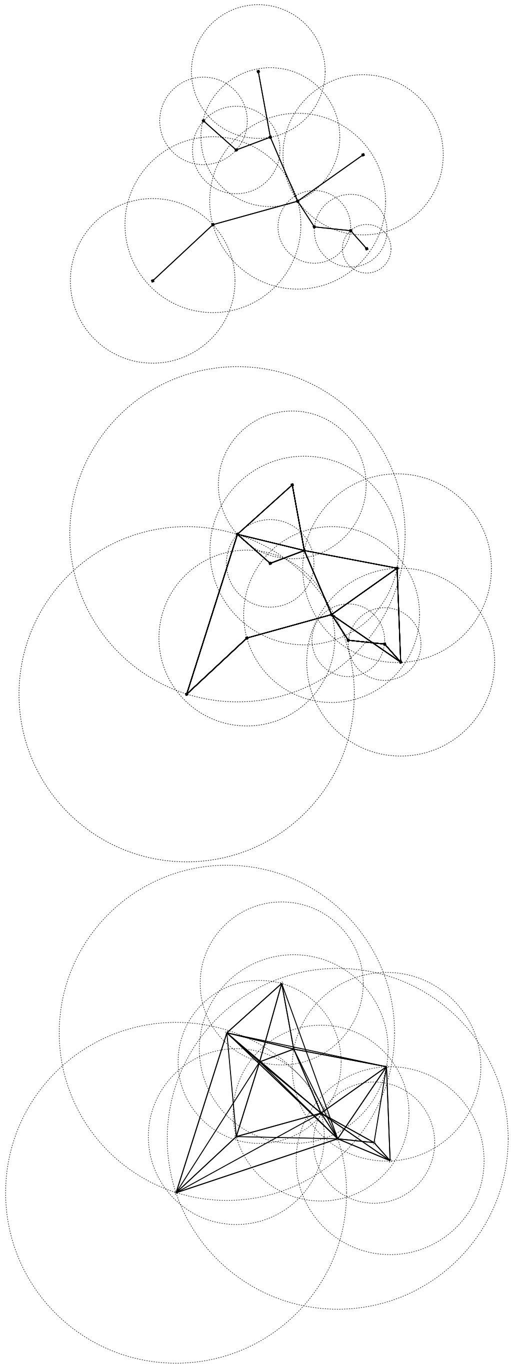

Fig. 2. The 1-, 2- and 3-connected optimal networks for an 11-node problem

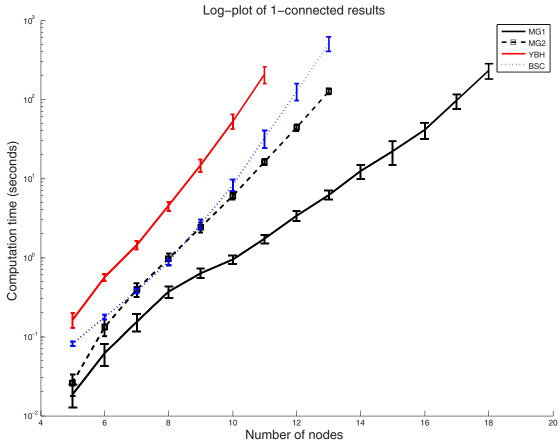

Fig. 3. A log-plot of the 1-connected solution time vs. network size

The MG1 formulation clearly has the smallest growth constant. The next best models, in order, are MG2, YBH, and BSC. It should be noted though that for these models, the available data is limited and they could well have very similar growth constants. However, it is safe to say that the MG1 formulation is the most efficient for the 1-connected problem.

For the 2-connected experiments, we removed the worst performing YBH model and ran the remaining three. Once again, the MG1 model achieved the best results, solving networks of up to 15 nodes in a short time (under 10 minutes) (Figure 5). Again, the computation time appears to have an exponential relationship with the network size. The fitted relationships are:

$$\begin{aligned} \text{MG1:}\quad & t \sim 0.00230 \times 2.178^n \\ \text{MG2:}\quad & t \sim 0.000182 \times 3.701^n \\ \text{BSC:}\quad & t \sim 0.0000269 \times 4.894^n \end{aligned}$$

Again, the models in order from slowest to fastest growth are MG1, MG2, BSC. While the MG2 and BSC models have similar solution times, the BSC model appears to have a substantially higher exponential growth.

For the 3-connected experiments, we ran the same models as before. The results are very similar to that for the 2-connected results: the MG2 and BSC models performed very similarly (Figure 5), while the MG1 model was the fastest and achieved solutions for larger networks within our 10 minute limit. Again, the growth constants show the same pattern: the MG1 model has the smallest growth constant, while the MG2 and BSC models have roughly the same growth.

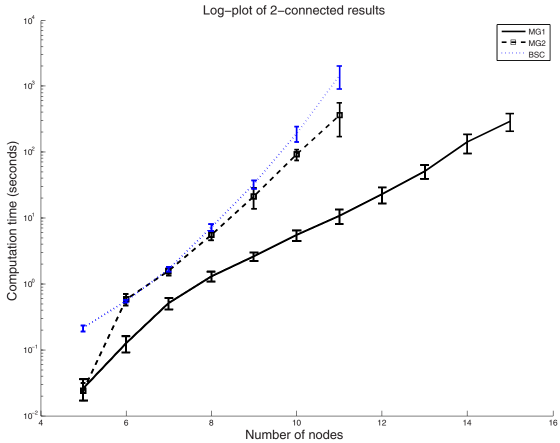

Fig. 4. A log-plot of the 2-connected solution time vs. network size

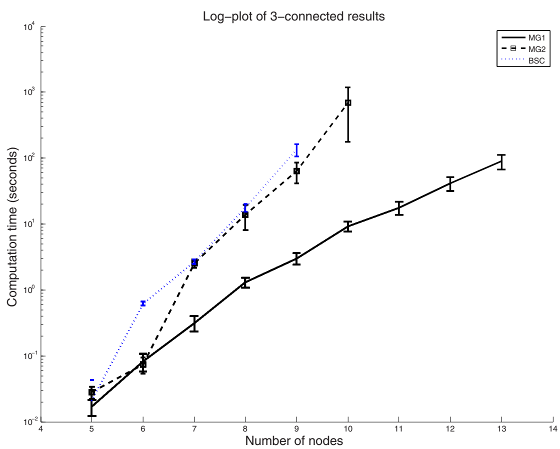

Fig. 5. A log-plot of the 3-connected solution time vs. network size

We compare the most efficient model (MG1) for $k = 1$, 2 and 3 in Figure 5. There is a noticeable intersection between the 2- and 3-connected solutions. This indicates the reduced computation required when the k is close to the size of the network. This can be seen by observing that k-connectivity requires that the network be connected when we remove any set of $k - 1$ nodes. There are $\binom{|V|}{k-1}$ such sets, which decreases when $2k > |V|$. From this figure, we can see that it takes longer to solve the problem as k increases. This is explained in Section 5.1. However, it is not obvious that the growth constant increases with k – this may in fact not be the case.

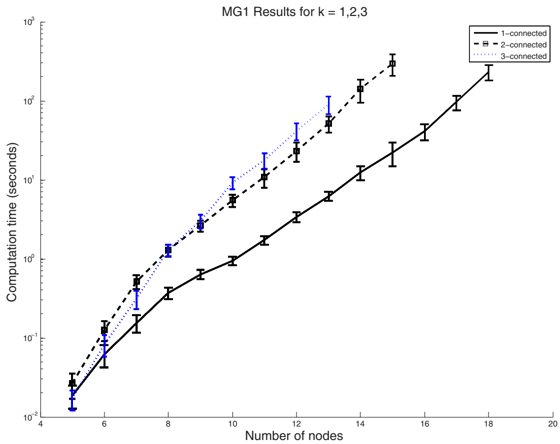

Fig. 6. The 1-, 2-, and 3-connected solution times for MG1

5.1 Computational Complexity

In order to determine the computational complexity of the minimum energy problem, we consider the following decision problem:

> *Does there exist a range assignment such that the network is k-connected, but not $(k+1)$-connected?*

Several polynomial reductions from known $\mathcal{NP}$-complete problems (such as the *vertex-cover* problem) to this decision problem exist ([3], [11], [6], [7]). Further, it is easy to see that we can verify a solution in polynomial time. Therefore the decision problem is $\mathcal{NP}$-complete. The optimisation version of this problem is:

> *What is the minimum transmission range assignment such that the network is k-connected?*

This version of the problem is necessarily $\mathcal{NP}$-hard, as we cannot verify a solution in polynomial time. In other words, this problem does not belong to the class of problems that can be solved on a nondeterministic Turing machine in polynomial time – it is much more difficult. Although Shpungin and Segal's ([15]) approach will, in most cases, have this computational advantage, we should remember that it is an approximate method.

6 Discussion

In this paper, we have collected mixed-integer programs for the minimum energy problem, and then extended them into models for the k-connected minimum energy problem. We implemented these models in XPress-Optimiser, and solved them for varying network sizes.

It is clear that the time taken to solve the minimum energy problem grows exponentially with network size. Of the four models, MG1 appears to be both the fastest and the model with the slowest growth rate for all levels of connectivity. Therefore, we would look to use this model and its extensions in future experiments.

As it stands, the largest network size that we solved is 18 nodes, although this could be extended if we used more computational power. It is clear that a network of this size will have very limited real-world applications, so our models are currently limited to academic and experimental use. It may be possible to increase the solvability of our models with solution techniques such as starting from a heuristic solution, or applying more cutting techniques. We are currently investigating these possibilities.

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