Achievable Region in Slotted ALOHA Throughput for One-Relay Two-Hop Wireless Network Coding

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Abstract. This paper presents achievable regions in slotted ALOHA throughput both without and with network coding for one-relay two-hop wireless networks between two end node groups. In this paper, there are no restrictions on the total traffic and the number of end nodes per group. It follows that the relay node will be generally involved with asymmetric bidirectional traffic. This paper derives closed-form expressions of the throughput and packet delay per group both without and with network coding from a theoretical perspective regardless of whether the buffer on the relay node is saturated or not. Furthermore, we show that the maximum throughput per group with network coding can be achieved at the boundary of the relay buffer saturation and unsaturation which is expressed as the solution of a polynomial equation in two group node traffics. As a result, we clarify the enhancement of the achievable region in slotted ALOHA throughput by applying network coding.

Keywords: one-relay two-hop wireless network, slotted ALOHA, network coding, throughput, packet delay, queueing system.

1 Introduction

Wireless relay access systems are expected to solve the digital divide problems [1] and implement communication services for smart grid applications [2]. There are many challenging issues to enhance the system capacity with respect to wireless relay techniques, for example cooperative diversity, multiuser multi-input and multi-output (MIMO), network coding, and so on.

Coding at a node on multihop wired or wireless networks is called *network coding*. Ahlswede *et al.* clarified that the maximum achievable flow for any single-source multicast network can be achieved by applying network coding [3]. Li *et al.* proved that the maximum flow can be achieved even if coding is limited to any linear combination of the information from the input links over a sufficiently large finite-field alphabet [4]. Ho *et al.* proposed a random linear network coding

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in general multisource multicast networks and showed that such random linear coding can approach to the multicast capacity as the length of code increases [5]. Many applications of network coding in scheduling, switching, and random access systems have been studied.

Katti *et al.* demonstrated that XOR-based network coding, which encodes multiple MAC packets by a bitwise XOR operation, and *opportunistic listening*, in which a node hears even undirected packets from its adjacent nodes, achieve high throughput on wireless multihop networks when implemented on wireless LAN terminals [6]. Matsuda *et al.* resolved the broadcast storm problem in dense wireless ad hoc networks, in which a carrier sense multiple access (CSMA) protocol is employed, by applying randomized network coding based on additions and multiplications in finite field [7]. Hasegawa *et al.* proposed a scheme jointly applying packet aggregation and network coding for voice over IP (VoIP) applications in multihop wireless networks and illustrated the increase of supportable VoIP sessions through the implementation to IEEE 802.11 networks [8]. Many nodes on wireless multihop networks will employ random access protocols on the MAC layer. Therefore, we should analyze the performance of network coding on multihop wireless networks employing random access protocols and clarify the design parameters that are crucial for maximizing the achievable throughput or minimizing the packet delay.

Before the development of network coding, several work has been presented for the analysis of random access protocols in multihop wireless networks. Gitman analyzed the capacity of two-hop wireless slotted ALOHA networks for bidirectional traffic [11]. Tobagi clarified the uplink throughput and packet delay for two-hop wireless access systems in the cases of slotted ALOHA [12] and CSMA [13]. These results provided some important design parameters for maximizing the throughput and minimizing the packet delay. Sagduyu *et al.* showed the achievable throughput region and throughput optimization in random access mode employing network coding over wireless multihop networks for several source packet transmission schemes in the case of saturated buffers [9]. Le *et al.* proposed a fundamental coding structure and clarified the encoding number of packets in that structure taking into consideration the physical environments [10].

We analyzed the throughput and packet delay for network coding employing slotted ALOHA with two-hop bidirectional traffic at the relay node and clarified that the throughput can be maximized by adapting the transmission probability of the relay node to a rational function of end node traffic [14,15]. However, the above results are limited to two kinds of specific wireless relay access networks. In this paper, we generalize the above results to any one-relay two-hop wireless access networks between two end node groups. There are no restrictions on total traffic and number of end nodes per group. As a result, we provide achievable regions in slotted ALOHA throughput both with and without network coding for any one-relay two-hop wireless access networks.

First, this paper derives expressions of the throughput and packet delay per group both with and without network coding by solving queueing systems for

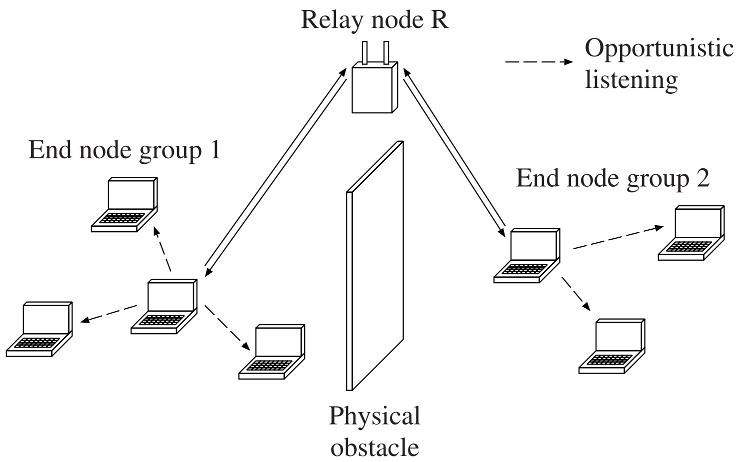

Fig. 1. One-relay two-hop wireless access networks between two end node groups

the relay node buffer behavior regardless of whether the buffer on the relay node is saturated or not. Furthermore, we show that the maximum throughput per group with network coding can be achieved at the boundary of the buffer saturation and unsaturation which is expressed as the solution of a polynomial equation in two group node traffics. Consequently, we clarify the enhancement of the achievable region in slotted ALOHA throughput by applying network coding to any one-relay two-hop wireless access networks.

2 Previous Work

2.1 System Description

We consider two-hop wireless relay access networks that consist of one relay node R and two end node groups 1 and 2, as shown in Fig. 1. For convenience, let us define $\bar{v} \in \{1, 2\}$ as the complemental element of v. Let us consider only bidirectional traffic via R and there is no closed traffic inside groups 1 and 2; here, traffic is defined as the average number of new transmissions and retransmissions per slot. Let us define the number of end nodes in group v as $n_v \geq 1$ for any $v \in \{1, 2\}$. Let us assume that all the end nodes in group v have the same traffic which is described as $0 < g_v \leq 1$ for any $v \in \{1, 2\}$. The total traffic in group v is described as $G_v = n_v g_v$ for any $v \in \{1, 2\}$.

All the nodes have one transceiver with an antenna. Hence, a node can be either transmitting or receiving, but not both simultaneously. We assume that any end node and R are in line-of-sight and within transmission range but any end node in a group is not in line-of-sight due to some physical obstacles and is out of transmission range with all the end nodes in the other group. Further if packets are transmitted from an end node in group v to R and from R to an end node in group $\bar{v}$ simultaneously and the node in group $\bar{v}$ is in receiver mode, then the node in group $\bar{v}$ can receive the packet from R successfully because we assume that the interference power from group v to group $\bar{v}$ is sufficiently small as compared with the received power of packet from R to group $\bar{v}$. R is a decode-and-forward repeater with an infinite storage capacity and the buffer at R is a FIFO queue. If there are always any packets in the buffer, the buffer is called *saturated* and otherwise it is called *unsaturated*.

All the nodes are synchronized with a slot time and all data packets begin their transmissions only at the beginning of slots and have a constant length. A collision occurs if two or more than packets are simultaneously received at a node or any packets are arrived on a transmitting node. A packet is lost only by the collision but not the bit errors because we assume that the received signal power is sufficiently large as compared with the thermal noise power at the receiver. If a packet is successfully received by its receiver, then an acknowledgement (ACK) packet is transmitted to the transmitter immediately. It is assumed that any ACK packets are not lost by the bit errors as well as data packets. A data packet time, an ACK packet time, and their propagation delays are disjointly included in a slot, and the propagation delay and ACK packet time are negligibly small as compared to the slot time. As a result, when a packet is transmitted at the beginning of a slot, the transmitter can observe whether the packet is successfully received or not in the end of that slot. If a collision occurs, the packet is retransmitted until the packet is received successfully. All the nodes have two modes: *transmission* mode and *backlogged* mode. Nodes are switched from transmission to backlogged mode if they make a collision of their new arrival packet transmission or they receive any packets even if the packets are undirected because they think that the channel will be busy. On the contrary, they are switched from backlogged to transmission mode if they can transmit a new arrival packet successfully because they think that the channel will be idle. End nodes transmit packets in the transmission mode with a constant probability g_{t} in each slot if their buffer is nonempty whereas they transmit packets in the backlogged mode with g_{r} in each slot. Note that all retransmission packets from end nodes are transmitted with g_{r} in each slot. In this case, the relay node R is always in backlogged mode because it generates no new arrival packets. R transmits and retransmits with a probability q_{r} in each slot if the buffer at R is nonempty.

A system without network coding is called a *non-NC system* whereas that with network coding is called an *NC system*. In the non-NC systems, R transmits the packet with probability q_{r} at the head of the buffer in each slot. On the other hand, in the NC systems, R transmits a *coding packet* or *native packet* with probability q_{r} in each slot if there are any packets in the buffer. R has two *virtual buffers* 1 and 2 for NC systems. Let us define the packet whose source is in group v as a packet v for any $v \in \{1, 2\}$. If a packet v is transmitted in a slot from group v and R receives the packet successfully, R stores the received packet in virtual buffer v for any $v \in \{1, 2\}$. If both the virtual buffers are nonempty, the transmitted packet is encoded from two packets corresponding to the heads of the virtual buffers by the bitwise XOR operation and is called a coding packet. If one of virtual buffers is nonempty and the other is empty, the transmitted packet is the packet at the head of its buffer and is called a native packet. End nodes in groups 1 and 2 store the packets received from the same group in a *packet pool*, which is a buffer to store undirected packets heard by using opportunistic listening [6]. If the destination receives a coding packet from R successfully, then the destination decodes the desired packet from its source by performing the bitwise XOR operation for the received coding packet and the packet in the packet pool.

The transmission probability q_{r} at R and the retransmission probability g_{r} at the end nodes are design parameters for one-relay two-hop non-NC and NC systems. This paper clarifies the relationship between such design parameters and performance parameters such as *throughput* S_v and *packet delay* D_v for each end node group v. S_v is defined as the average number of successful transmissions via R per slot time from group v to $\bar{v}$. D_v from group v to $\bar{v}$ via R is defined as the sum of packet transmission time, retransmission delay from group v to R, queueing delay of a packet v in R, and access delay from R to group $\bar{v}$. The total throughput S is defined as $S_1 + S_2$.

2.2 Achievable Region of Direct Communication Systems

We model each end node's packet transmission as a sequence of independent Bernoulli trials. The achievable throughput region in slotted ALOHA for wireless systems in which all the end nodes are within the transmission range has been presented [16, 17]. The probability in a slot that only one end node transmits in group v will be expressed as $\gamma_v = G_v(1 - g_v)^{n_v - 1}$ for any $v \in \{1, 2\}$. The probability in a slot that any end nodes do not transmit in group v will be expressed as $\eta_v = (1 - g_v)^{n_v}$ for any $v \in \{1, 2\}$. The throughput S_v is obtained as

$$S_v = \gamma_v \eta_{\bar{v}} = G_v(1 - g_v)^{n_v - 1}(1 - g_{\bar{v}})^{n_{\bar{v}}} \tag{1}$$

for any $v \in \{1, 2\}$. The packet delay D_v is obtained as

$$D_v = 1 + \frac{1}{g_{\mathrm{r}}} \left(\frac{1}{(1 - g_v)^{n_v - 1}(1 - g_{\bar{v}})^{n_{\bar{v}}}} - 1 \right) \tag{2}$$

for any $v \in \{1, 2\}$.

The necessary and sufficient condition has been obtained for maximizing the total throughput.

Theorem 1. *The maximum achievable throughput is obtained if and only if*

$$G_1 + G_2 = 1 \tag{3}$$

holds.

If (3) holds, the group throughputs will be expressed as

$$S_1 = G_1 \left(1 - \frac{G_1}{n_1}\right)^{n_1 - 1} \left(1 - \frac{1 - G_1}{n_2}\right)^{n_2}, \tag{4}$$

$$S_2 = (1 - G_1) \left(1 - \frac{G_1}{n_1}\right)^{n_1} \left(1 - \frac{1 - G_1}{n_2}\right)^{n_2 - 1}, \tag{5}$$

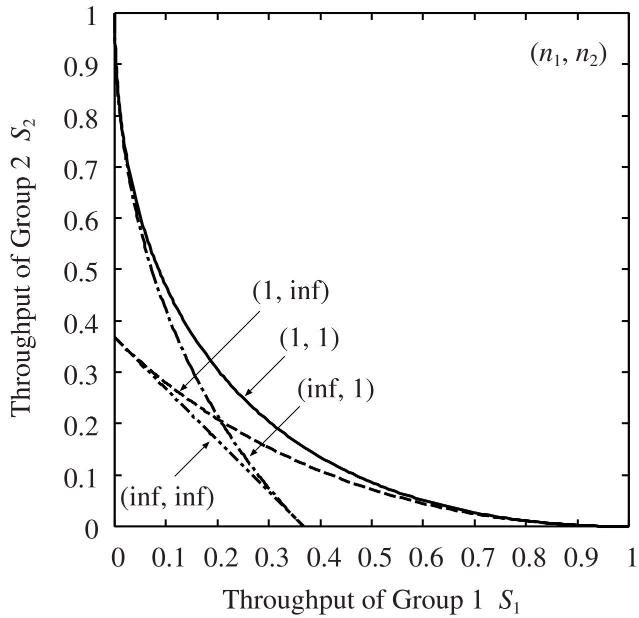

Fig. 2. The boundaries of achievable regions in slotted ALOHA throughput for direct communication systems

by using the end node traffic g_1. In particular, if both n_1 and n_2 approach to infinity,

$$S_1 = \frac{G_1}{\mathrm{e}}, \quad S_2 = \frac{1 - G_1}{\mathrm{e}} \tag{6}$$

are obtained.

Figure 2 illustrates the four kinds of achievable region boundaries in throughput for the direct communication systems. From Fig. 2, the throughput makes severe degradation with the increase of end nodes number because the increase of end nodes number reduces to the increase of packet collisions.

3 Achievable Region of Non-NC Systems

We model each end node's packet transmission as a sequence of independent Bernoulli trials. This section analyzes the throughput and packet delay per group of non-NC systems for any one-relay two-hop wireless access networks. Let us describe the buffer state at R as the sequence of packet group sources $\boldsymbol{v}_1^n = v_1 v_2 \cdots v_n$, $v_i \in \{1, 2\}$ in the buffer and define the empty buffer state $\boldsymbol{v}_1^0$ at R as 0. The state at the kth slot is described as V_k for any $k \geq 0$ and let us assume that $V_0 = 0$. Figure 3 illustrates the Markov chain of V_k. The state transition probabilities in Fig. 3 are expressed as $\lambda_{0,v} = \gamma_v \eta_{\bar{v}}$, $\lambda_v = (1 - q_{\mathrm{r}})\gamma_v \eta_{\bar{v}}$, and

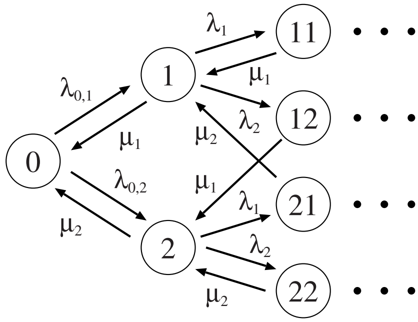

Fig. 3. Markov chain of buffer states for non-NC systems

$\mu_v = q_\mathrm{r}\eta_{\bar{v}}$ for any $v \in \{1, 2\}$. The *utilization factor* of a packet v is expressed as $\rho_v = \lambda_v/\mu_v = (1 - q_\mathrm{r})\gamma_v/q_\mathrm{r}$. The steady-state probability of a state $\boldsymbol{v}_1^n$ is defined as $Q(\boldsymbol{v}_1^n)$ and that of number n of packets v in the buffer at R is defined as $P_v(n)$. The number of packets v in state $\boldsymbol{v}_1^n$ is defined as $n_v(\boldsymbol{v}_1^n)$ for any $v \in \{1, 2\}$. The traffic on R is expressed as $q = q_\mathrm{r}(1 - Q(0))$. The throughput of group v is expressed as $S_v = \mu_v Q(v*)$ for any $v \in \{1, 2\}$, where $v*$ stands for any sequence whose head is the packet v.

The following lemma and theorem are obtained with respect to the throughput.

Lemma 1. *The buffer at R is unsaturated if and only if*

$$q_\mathrm{r} > \frac{\gamma_1 + \gamma_2}{1 + \gamma_1 + \gamma_2} \tag{7}$$

holds. The steady-state probabilities of the buffer states

$$Q(0) = 1 - \frac{\gamma_1 + \gamma_2}{q_\mathrm{r}(1 + \gamma_1 + \gamma_2)}, \tag{8}$$

$$Q(\boldsymbol{v}_1^n) = \frac{\rho_1^{n_1(\boldsymbol{v}_1^n)}\rho_2^{n_2(\boldsymbol{v}_1^n)}Q(0)}{1 - q_\mathrm{r}}, \ (n \geq 1), \tag{9}$$

are derived.

Theorem 2. *If the buffer at R is unsaturated,*

$$S_v = \frac{\gamma_v \eta_{\bar{v}}}{1 + \gamma_1 + \gamma_2} \tag{10}$$

is derived for any $v \in \{1, 2\}$. Otherwise,

$$S_v = \frac{q_\mathrm{r}\gamma_v \eta_{\bar{v}}}{\gamma_1 + \gamma_2} \tag{11}$$

is derived for any $v \in \{1, 2\}$.

The proofs of Lemma 1 and Theorem 2 will be obtained as an extension of those of [15, Lemma 3 and Theorem 3]. The following lemma and theorem are obtained with respect to the packet delay.

Lemma 2. *If the buffer at R is unsaturated, the steady-state probabilities of number of packets v in the buffer at R are derived as*

$$P_v(0) = \frac{1 - q_\mathrm{r}(1 - \rho_{\bar{v}})}{(1 - q_\mathrm{r})(1 - \rho_{\bar{v}})} Q(0), \tag{12}$$

$$P_v(n) = \frac{\rho_v^n Q(0)}{(1 - q_\mathrm{r})n!} \frac{\mathrm{d}^n}{\mathrm{d}\rho_{\bar{v}}^n} \frac{\rho_{\bar{v}}^n}{1 - \rho_{\bar{v}}} \tag{13}$$

for any $n \geq 1$.

Theorem 3. *If the buffer at R is unsaturated,*

$$D_v = 1 + \frac{1}{g_\mathrm{r}} \left(\frac{G_v(1 + \gamma_1 + \gamma_2)}{\gamma_v \eta_{\bar{v}}} - 1 \right) + \frac{1}{\eta_{\bar{v}}(q_\mathrm{r} - (1 - q_\mathrm{r})(\gamma_1 + \gamma_2))} \tag{14}$$

is derived for any $v \in \{1, 2\}$. *Otherwise, the packet delay is infinite.*

The proofs of Lemma 2 and Theorem 3 will be obtained as an extension of those of [15, Lemma 4 and Theorem 4].

The unsaturated group throughput (10) is independent of q_r and the saturated group throughput (11) increases monotonically with q_r for given g_1, g_2, n_1, and n_2. As a result, the unsaturated (10) is superior to the saturated (11). For each unsaturated node throughput $s_v = S_v/n_v$, the partial differentials of s_v with respect to g_v and $g_{\bar{v}}$ are expressed as

$$\frac{\partial s_v}{\partial g_v} = \frac{\bar{g}_v^{n_v - 2} \bar{g}_{\bar{v}}^{n_{\bar{v}}} (1 - G_v)(1 + G_{\bar{v}} \bar{g}_{\bar{v}}^{n_{\bar{v}} - 1})}{(1 + \gamma_1 + \gamma_2)^2}, \tag{15}$$

and

$$\frac{\partial s_v}{\partial g_{\bar{v}}} = -\frac{n_v g_{\bar{v}} \bar{g}_v^{n_v - 1} \bar{g}_{\bar{v}}^{n_{\bar{v}} - 1} (1 + G_v \bar{g}_v^{n_v - 1} + \bar{g}_{\bar{v}}^{n_{\bar{v}} - 1})}{(1 + \gamma_1 + \gamma_2)^2}, \tag{16}$$

respectively, where $\bar{g}_v = 1 - g_v$ is the complementary probability of g_v for any $v \in \{1, 2\}$. As a result, the Jacobian is derived as

$$J \propto 1 - G_1 - G_2 - G_1\gamma_2 - G_2\gamma_1. \tag{17}$$

Consequently, the following theorem will be obtained for maximizing the total throughput.

Theorem 4. *The maximum achievable throughput is obtained if and only if*

$$G_1(1 + \gamma_2) + G_2(1 + \gamma_1) = 1 \tag{18}$$

holds.

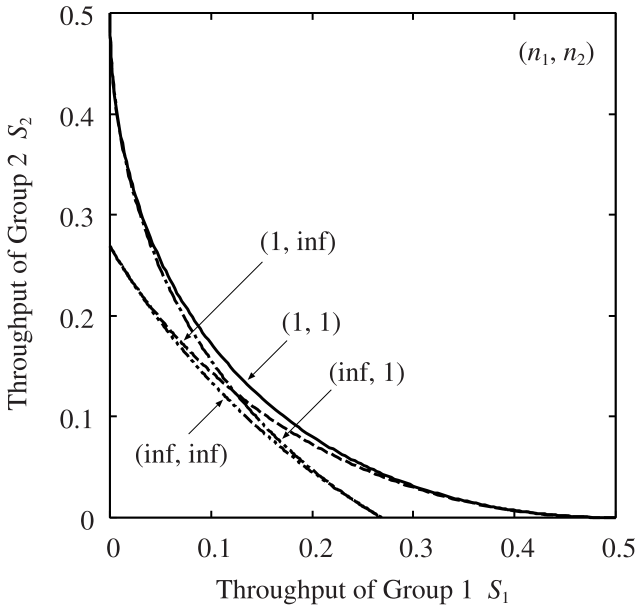

Fig. 4. The boundaries of achievable regions in slotted ALOHA throughput for non-NC systems

The boundary of achievable throughput region in slotted ALOHA for non-NC systems can be obtained by assigning the solution (g_1, g_2) of (18) to the unsaturated throughput (10). The solution (g_1, g_2) of (18) will be obtained by exploiting some zero-finding algorithms.

If both n_1 and n_2 are equal to 1, i.e. there is a single end node per group, (18) will reduce to

$$G_2 = \frac{1 - G_1}{1 + 2G_1} \tag{19}$$

and the group throughputs will be expressed as

$$S_1 = \frac{3G_1^2}{2(1 + G_1 + G_1^2)}, \quad S_2 = \frac{(1 - G_1)^2}{2(1 + G_1 + G_1^2)}, \tag{20}$$

by using the end node traffic g_1. If group 1 has an infinite number of nodes and group 2 has a single node, (18) will reduce to

$$G_2 = \frac{1 - G_1}{1 + G_1 + G_1 \mathrm{e}^{-G_1}} \tag{21}$$

and the group throughputs will be expressed as

$$S_1 = \frac{G_1^2 \mathrm{e}^{-G_1}(2+\mathrm{e}^{-G_1})}{2+G_1\mathrm{e}^{-G_1}(2+G_1+G_1\mathrm{e}^{-G_1})}, \tag{22}$$

$$S_2 = \frac{(1-G_1)\mathrm{e}^{-G_1}}{2+G_1\mathrm{e}^{-G_1}(2+G_1+G_1\mathrm{e}^{-G_1})}, \tag{23}$$

by using the end node traffic g_1.

Figure 4 illustrates the four kinds of achievable region boundaries in slotted ALOHA throughput for non-NC systems. The maximum achievable throughput, i.e. capacity for one-relay two-hop non-NC scheduling systems is exactly half that for direct communication scheduling systems. On the other hand, the capacity for non-NC slotted ALOHA systems is more than half that for direct communication slotted ALOHA systems, particularly when the nodes number increases. This will be because a node will interfere all other nodes in the direct communication systems whereas the interference of a node is limited to all other end nodes in the same group for non-NC systems.

4 Achievable Region of NC Systems

We model each end node's packet transmission as a sequence of independent Bernoulli trials. This section analyzes the throughput and packet delay per group of NC systems for any one-relay two-hop wireless access networks. In contrast to non-NC systems, q_r at R is a design parameter that is crucial for maximizing the achievable throughput of NC systems as in [14, 15].

Let us describe the buffer state at R as a pair (n, m) of packet numbers in the virtual buffers 1 and 2. The state at the kth slot is described as $W_k = (X_k, Y_k)$ for any $k \geq 0$ and let us assume that $W_0 = (0, 0)$. Figure 5 illustrates the two-dimensional Markov chain of W_k. The state transition probabilities $\lambda_{0,v}$, λ_v, and μ_v for any $v \in \{1, 2\}$ are defined as in the non-NC systems. The new state transition probabilities are expressed as $\mu = q_\mathrm{r}\eta_1\eta_2$ and $\mu_{0,v} = q_\mathrm{r}(1-\eta_v)\eta_{\bar{v}}$ for any $v \in \{1, 2\}$. The utilization factor ρ_v for any $v \in \{1, 2\}$ is defined as in the non-NC systems. The steady-state probability of state (n, m) is denoted as $P(n, m)$ and that of state n in the virtual buffer v for any $v \in \{1, 2\}$ is denoted as $P_v(n)$. The traffic on R is expressed as $q = q_\mathrm{r}(1 - P(0, 0))$. The throughput of group v is expressed as $S_v = \mu_v(1 - P_v(0))$ for any $v \in \{1, 2\}$. Let us define an indicator δ such as δ is v if $\gamma_v \geq \gamma_{\bar{v}}$ for any $v \in \{1, 2\}$.

The following lemma and theorem are obtained with respect to the throughput.

Lemma 3. *Both the virtual buffers at R are unsaturated if and only if*

$$q_\mathrm{r} > \frac{\gamma_\delta}{1+\gamma_\delta} \tag{24}$$

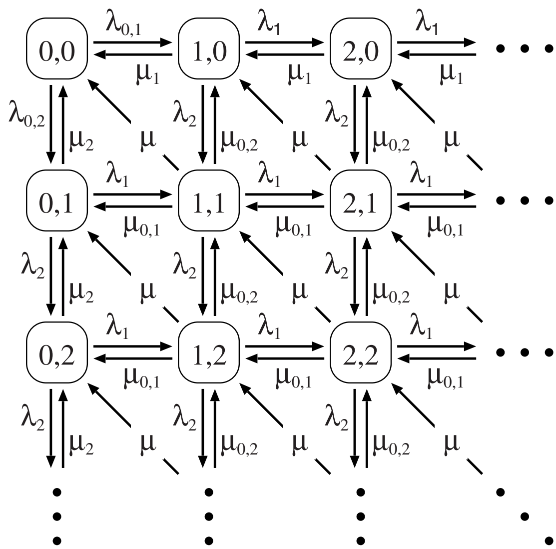

Fig. 5. Markov chain of buffer states for NC systems

holds. Then, for any $v \in \{1, 2\}$, the steady-state probabilities of number of packets in virtual buffer v are derived as

$$P_v(0) = 1 - \rho_v - \gamma_v P(0,0), \tag{25}$$

$$P_v(n) = \rho_v^{n-1}(1-\rho_v)(\rho_v + \gamma_v P(0,0)), \ (n \geq 1). \tag{26}$$

Virtual buffer δ is saturated and $\bar{\delta}$ is unsaturated if and only if

$$\frac{\gamma_{\bar{\delta}}}{1+\gamma_{\bar{\delta}}} < q_{\mathrm{r}} \leq \frac{\gamma_\delta}{1+\gamma_\delta} \tag{27}$$

holds. Then the steady-state probabilities of number of packets in virtual buffer $\bar{\delta}$ are derived as

$$P_{\bar{\delta}}(n) = \frac{\rho_{\bar{\delta}}^n}{q_{\mathrm{r}}}(q_{\mathrm{r}} - (1-q_{\mathrm{r}})\gamma_{\bar{\delta}}) \tag{28}$$

for any $n \geq 0$. Both the virtual buffers are saturated if and only if

$$q_{\mathrm{r}} \leq \frac{\gamma_{\bar{\delta}}}{1+\gamma_{\bar{\delta}}} \tag{29}$$

holds.

Theorem 5. *If both the virtual buffers are unsaturated,*

$$S_v = \gamma_v \eta_{\bar{v}}(1-q) \tag{30}$$

is derived for any $v \in \{1,2\}$. *If virtual buffer* δ *is saturated and* $\bar{\delta}$ *is unsaturated,*

$$S_\delta = q_{\mathrm{r}}\eta_{\bar{\delta}}, \quad S_{\bar{\delta}} = (1-q_{\mathrm{r}})\gamma_{\bar{\delta}}\eta_\delta \tag{31}$$

are derived. If both the virtual buffers are saturated,

$$S_v = q_{\mathrm{r}}\eta_{\bar{v}} \tag{32}$$

is derived for any $v \in \{1,2\}$.

The proofs of Lemma 3 and Theorem 5 will be obtained as an extension of those of [15, Lemma 7 and Theorem 8]. The following theorem is obtained with respect to the packet delay.

Theorem 6. *If the both the virtual buffers are unsaturated,*

$$D_v = 1 + \frac{1}{g_{\mathrm{r}}}\left(\frac{G_v}{\gamma_v\eta_{\bar{v}}(1-q)} - 1\right) + \frac{1}{\eta_{\bar{v}}(q_{\mathrm{r}} - (1-q_{\mathrm{r}})\gamma_v)} \tag{33}$$

is derived for any $v \in \{1,2\}$. *If virtual buffer* δ *is saturated and* $\bar{\delta}$ *is unsaturated,*

$$D_{\bar{\delta}} = 1 + \frac{1}{g_{\mathrm{r}}}\left(\frac{G_{\bar{\delta}}}{\gamma_{\bar{\delta}}\eta_\delta(1-q_{\mathrm{r}})} - 1\right) + \frac{1}{\eta_\delta(q_{\mathrm{r}} - (1-q_{\mathrm{r}})\gamma_{\bar{\delta}})} \tag{34}$$

is derived and the packet delay from group δ *to* $\bar{\delta}$ *is infinite. Otherwise, the packet delay from group* v *to* $\bar{v}$ *is infinite for any* $v \in \{1,2\}$.

The proof of Theorem 6 will be obtained as an extension of that of [15, Theorem 12].

The unsaturated group throughput (30) decreases monotonically with q_{r} unlike the non-NC systems. The saturated group throughput (32) increases monotonically with q_{r}. Consequently, the throughput of group δ will be maximized with $q_{\mathrm{r}} = \gamma_\delta/(1+\gamma_\delta)$ whereas the throughput of group $\bar{\delta}$ will be maximized with $q_{\mathrm{r}} = \gamma_{\bar{\delta}}/(1+\gamma_{\bar{\delta}})$. This paper analyzes the throughput with $q_{\mathrm{r}} = \gamma_\delta/(1+\gamma_\delta)$.

If q_{r} is on the boundary of the buffer saturation and unsaturation for virtual buffer δ, i.e. $q_{\mathrm{r}} = \gamma_\delta/(1+\gamma_\delta)$, the throughput of group v will be obtained as

$$S_v = \frac{\gamma_v\eta_{\bar{v}}}{1+\gamma_\delta} \tag{35}$$

for any $v \in \{1,2\}$ by assigning $q_{\mathrm{r}} = \gamma_\delta/(1+\gamma_\delta)$ to (31). The throughput (35) is superior to the throughput (10) for the non-NC systems because $\gamma_{\bar{\delta}}$ is removed from the denominator polynomial in (10). The Jacobian will be derived as

$$J \propto 1 - G_1 - G_2 - \gamma_\delta G_{\bar{\delta}} \tag{36}$$

and the following theorem will be obtained for maximizing the total throughput.

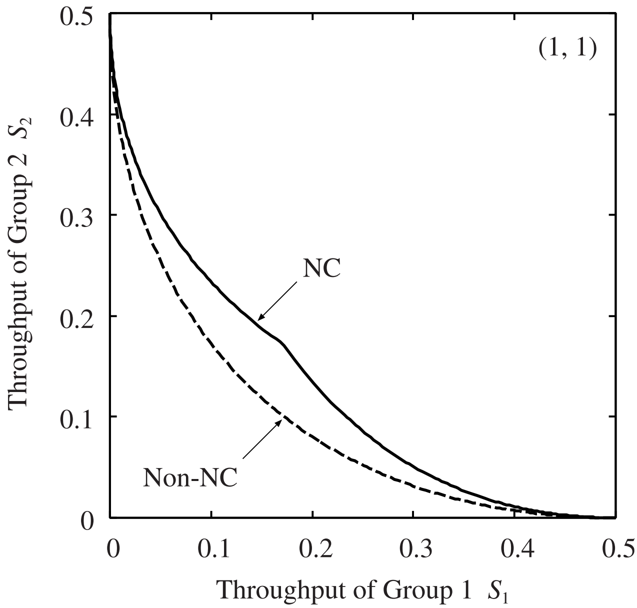

Fig. 6. The boundaries of achievable regions in slotted ALOHA throughput for non-NC and NC systems ($n_1 = n_2 = 1$)

Theorem 7. *The achievable throughput is obtained if and only if*

$$G_1 + G_2 + \gamma_\delta G_{\bar{\delta}} = 1 \tag{37}$$

holds.

The boundary of achievable throughput region in slotted ALOHA for NC systems can be derived by performing the following procedure.

1. Let be $G_1 \leftarrow 1$. Let d be a sufficiently small real number.
2. Calculate
$$G_2 \leftarrow \frac{1 - G_1}{1 + \gamma_1}. \tag{38}$$
3. If $\gamma_1 \geq \gamma_2$, go to 4. Otherwise, go to 7.
4. Calculate
$$S_1 \leftarrow \frac{\gamma_1 \eta_2}{1 + \gamma_1}, \quad S_2 \leftarrow \frac{\gamma_2 \eta_1}{1 + \gamma_1}. \tag{39}$$
5. $G_1 \leftarrow G_1 - d$.
6. If $G_1 < 0$, the procedure is complete. Otherwise, return to 2.
7. Calculate
$$G_1 \leftarrow \frac{1 - G_2}{1 + \gamma_2}. \tag{40}$$

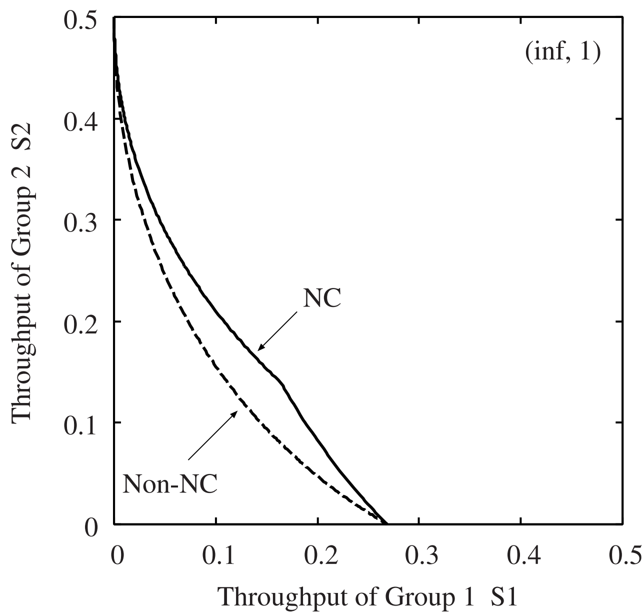

Fig. 7. The boundaries of achievable regions in slotted ALOHA throughput for non-NC and NC systems ($n_1 = \infty$, $n_2 = 1$)

8. If $\gamma_1 < \gamma_2$, go to 9. Otherwise, return to 2.
9. Calculate

$$S_1 \leftarrow \frac{\gamma_1 \eta_2}{1 + \gamma_2}, \quad S_2 \leftarrow \frac{\gamma_2 \eta_1}{1 + \gamma_2}. \tag{41}$$

10. Let be $G_2 \leftarrow G_2 + d$.
11. If $G_2 > 1$, the procedure is complete. Otherwise, return to 9.

The three kinds of achievable throughput regions for the NC systems will be compared with those for the non-NC systems.

Figure 6 illustrates the boundary of achievable throughput region for the NC systems if each group has a single node. If the group traffics are symmetric with each other, the maximum coding gain will be obtained. A cusp appears if the group traffics are symmetric unlike the direct communication systems and non-NC systems. Because the opportunity to perform network coding will be enhanced by balancing the group traffics. Figure 7 illustrates the boundary of achievable throughput region for NC systems if group 1 has an infinite number of end nodes and group 2 has a single end node. We can see that the sufficiently large coding gain will be obtained and a cusp appears as in the case of a single end node for each group. Figure 8 illustrates the boundary of achievable throughput region for NC systems if both the groups have an infinite number of end nodes. From Fig. 8, the sufficiently large coding gain will be obtained even if a number of end nodes are included in a group for one-relay two-hop wireless access networks.

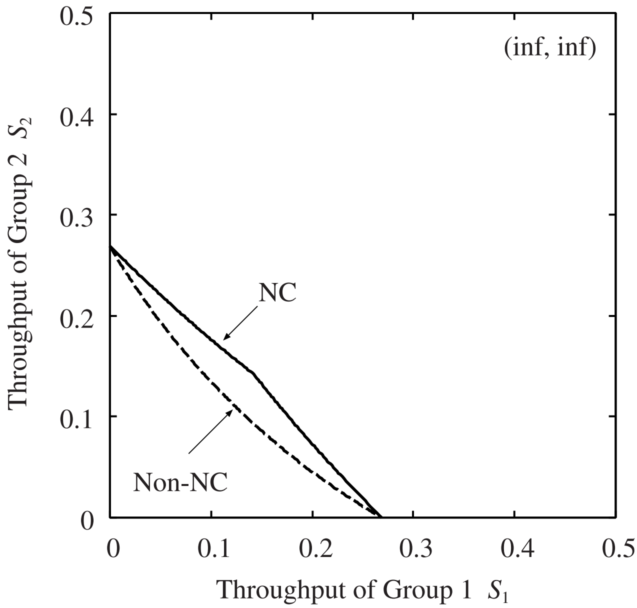

Fig. 8. The boundaries of achievable regions in slotted ALOHA throughput for non-NC and NC systems ($n_1 = n_2 = \infty$)

5 Conclusion

We have shown the achievable regions in slotted ALOHA throughput both without and with network coding for one-relay two-hop wireless networks between two end node groups. In this paper, there are no restrictions on total traffic and number of end nodes per group. We have derived expressions of the throughput and packet delay per group both with and without network coding from a theoretical perspective regardless of whether the buffer on the relay node is saturated or not. Furthermore, we have shown that the maximum throughput per group with network coding can be achieved at the boundary of the buffer saturation and unsaturation which is expressed as the solution of a polynomial equation in two group node traffics. As a result, we have clarified the enhancement of the achievable region in slotted ALOHA throughput by applying network coding even if a number of end nodes are included in a group.

The results in this paper will give some useful performance references for multihop wireless networks employing random access protocols.

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References

1. Ishmael, J., Bury, S., Pezaros, D., Race, N.: Deploying Rural Community Wireless Mesh Networks. IEEE Internet Computing 12(4), 22–29 (2008)
2. Ipakchi, A., Albuyeh, F.: Grid of the Future. IEEE Power and Energy Magazine 7(2), 52–62 (2009)
3. Ahlswede, R., Li, S., Yeung, R.: Network Information Flow. IEEE Transactions on Information Theory 46(4), 1204–1216 (2000)
4. Li, S.R., Yeung, R.W., Cai, N.: Linear Network Coding. IEEE Transactions on Information Theory 49(2), 371–381 (2003)
5. Ho, T., Médard, M., Koetter, R., Karger, D.R., Effros, M., Shi, J., Leong, B.: A Random Linear Network Coding Approach to Multicast. IEEE Transactions on Information Theory 52(10), 4413–4430 (2006)
6. Katti, S., Rahul, H., Hu, W., Katabi, D., Médard, M., Crowcroft, J.: XOR's in the Air: Practical Wireless Network Coding. IEEE/ACM Transactions on Networking 16(3), 497–510 (2008)
7. Matsuda, T., Noguchi, T., Takine, T.: Broadcasting with Randomized Network Coding in Dense Wireless Ad Hoc Networks. IEICE Transactions on Communications E91-B(10), 3216–3225 (2008)
8. Hasegawa, J., Yomo, H., Kondo, Y., Davis, P., Sakakibara, K., Miura, R., Obana, S.: Bidirectional Packet Aggregation and Coding for Efficient VoIP Transmission in Wireless Multi-Hop Networks. IEICE Transactions on Communications E92-B(10), 3060–3070 (2009)
9. Sagduyu, Y.E., Ephremides, A.: Cross-Layer Optimization of MAC and Network Coding in Wireless Queueing Tandem Networks. IEEE Transactions on Information Theory 54(2), 554–571 (2008)
10. Le, J., Lui, J., Chiu, D.M.: How Many Packets can We Encode? — An Analysis of Practical Wireless Network Coding. In: IEEE INFOCOM (2008)
11. Gitman, I.: On the Capacity of Slotted ALOHA Networks and Some Design Problems. IEEE Transactions on Communications COM-23(3), 305–317 (1975)
12. Tobagi, F.A.: Analysis of a Two-Hop Centralized Packet Radio Network—Part I: Slotted ALOHA. IEEE Transactions on Communications COM-28(2), 196–207 (1980)
13. Tobagi, F.A.: Analysis of a Two-Hop Centralized Packet Radio Network—Part II: Carrier Sense Multiple Access. IEEE Transactions on Communications COM-28(2), 208–216 (1980)
14. Umehara, D., Hirano, T., Denno, S., Morikura, M.: Throughput Analysis of Wireless Relay Slotted ALOHA Systems with Network Coding. In: IEEE Global Communications Conference (2008)
15. Umehara, D., Hirano, T., Denno, S., Morikura, M., Sugiyama, T.: Wireless Network Coding in Slotted ALOHA with Two-Hop Unbalanced Traffic. IEEE Journal on Selected Areas in Communications 27(5), 647–661 (2009)
16. Abramson, N.: The Throughput of Packet Broadcasting Channels. IEEE Transactions on Communications COM-25(1), 117–128 (1977)
17. Kleinrock, L.: Queueing Systems, Volume II: Computer Applications. John Wiley and Sons, Chichester (1976)