Throughput Analysis of IEEE 802.11 DCF in the Presence of Transmission Errors

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Abstract. This paper introduces an accurate analysis using three-dimensional Markov chain modeling to compute the IEEE 802.11 DCF throughput under heavy traffic conditions and absence of hidden terminals for both access modes, basic and *rts/cts*. The proposed model considers the impact of retry counts of control and data frames jointly on the saturated throughput. Moreover, It considers the impact of transmission errors by taking into account the strength of the received signal and using the BER model to convert the SNR to a bit error probability.

Keywords: IEEE 802.11 DCF, Markov model, BEB, BER.

1 Introduction

The IEEE 802.11 wireless LAN is currently the most popular product on the market and it is widely deployed in hot-spots and offices. The Medium Access Control (MAC) plays a key role in determining the channel efficiency and quality of service for upper layer applications. It defines two medium access methods, the mandatory Distributed Coordination Function (DCF) and the optional Point Coordination Function (PCF). DCF defines two modes to access the wireless channel, basic access mode and ready-to-send (rts)/clear-to-send (cts) access mode. PCF is out of scope of this paper.

In literature, there is a lot of research work done in modeling the IEEE 802.11 DCF and studying its throughput. The first Markov chain model introduced by Bianchi [2] has become the most common method for calculating the saturated throughput for single hop wireless networks. Bianchi proposed a two dimensional Markov chain model to calculate the saturated throughput of the IEEE 802.11 DCF under the assumptions of error-free channels, no hidden terminals, no capture effects, and unlimited packet retransmissions. Tay [3] developed a simple mathematical model to derive the probability of collision and the saturated throughput under idealistic assumptions: packet retransmissions are unlimited and no channel errors. The number of transmission per packet is considered as geometric distribution. Wu [4] extended Bianchi model by considering the impact of packet's retransmission limits on throughput analysis. Neither [2] nor [4] studied the impact of transmission errors on the performance of DCF. For basic access mode, Chatzimisios [9] considered the impact of the retry limit setting

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and the transmission errors on the DCF. In [9], transmission errors considered as a constant frame error probabilities for data frames but ignored for control frames. Tickoo [7] introduced an analytical model for evaluating the DCF packet queuing delays under finite load conditions by modeling each node as a discrete time G/G/1 queue. Vukovic [8] simplified Wu model [4] into a one dimensional Markov chain and computed the throughput of three backoff mechanisms.

To the best of our knowledge, the proposed analytical models for IEEE 802.11 DCF did not consider the actual specification as it is described in the standard [1]. The proposed models either evaluated the basic access mode by considering the data retry limit and the transmission errors or evaluated the *rts/cts* access mode by considering the *rts* retry limit and the transmission errors but ignoring the data retry limit. Moreover, most of the existing work assumed a Gaussian wireless error channel with a constant Bit Error Rate (BER), whereas all transmitted frames have the same Frame Error Rate (FER). In fact, The radio channel introduces significant complexity to the design and performance analysis of wireless LANs due to multipath fading.

The main contribution of this work is to develop and analyze an analytical performance model for IEEE 802.11 DCF protocol, which considers the long and short retry counts as well as the quality of the received data. We introduce a new 3-dimensional Markov chain that directly integrates backoff process as well as short and long retry counts into one model. The x-dimension used to model the backoff process, the y-dimension used to model the short retry count (backoff stage) and the z-dimension allows us to accurately model the long retry count (backoff layer). Frame error probabilities are calculated using the shadowing propagation model and added to the state transition probabilities. The steady state probabilities of the Markov chain are solved and used to derive the transmission probabilities, which we then use to derive the throughput.

This paper is organized as follows: Section II presents the DCF backoff procedure. Sections III presents the impact of transmission errors on the BER. Section IV introduces our model and develops an analytical analysis to derive the DCF throughput. Section V provides analysis results. Finally, section VI concludes the paper.

2 IEEE 802.11 Distributed Coordination Function (DCF)

DCF is a contention-based access scheme uses Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). Carrier sensing can be performed at the physical layer and at the MAC sublayer. Priority levels for access to the channel are provided through the use of Interframe Spaces such as Short InterFrame Space (SIFS), DCF InterFrame Space (DIFS) and Extended InterFrame Space (EIFS). The backoff procedure is used for collision avoidance, where each station (STA) waits for a backoff time (a random time interval in units of time slots) before any frame transmission.

In 802.11 DCF, a STA with new packet to transmit monitors the channel, if the channel is sensed to be idle for an interval larger than DIFS period, the STA transmits the packet. Otherwise, if the channel is sensed busy (either immediately or during the DIFS), the STA defers its transmission and keeps monitoring the channel until it becomes idle for a DIFS period. Then, the STA generates a random backoff period before transmitting the packet. To avoid channel capture problem, random backoff period is selected between successive transmissions.

DCF uses a Binary Exponential Backoff (BEB) algorithm to resolve channel contention. The backoff time is randomly and uniformly chosen from the range $(0, CW-1)$ slots. The Contention Window (CW) value depends on the number of retransmissions. It starts with a minimum value (CW_{min}) and doubles after each unsuccessful transmission up to a maximum value $CW_{max} = 2^m CW_{min}$, (m is an integer number that limits the value of the contention window). Backoff time is a slotted time, the duration of each slot (σ) is carefully set equals to the time needed by any STA to detect the transmission of other STAs within a certain range. Therefore, σ selection takes in consideration the signal propagation time, the necessary time for the antenna transceiver to switch from a transmit state to a receive state and other considerations. The backoff time is decremented once every time slot for which the channel is detected idle, frozen when a transmission is detected on the channel, and resumed when the channel is sensed idle again for a DIFS period. The STA transmits when the backoff time reaches zero. Time duration between successive empty time slots is variable and depends on the status of the medium. Two successive empty time slots should be proceeded by an idle DIFS period.

802.11 DCF sets a threshold for the number of retransmissions, as the number of retransmission exceeds this threshold, the frame is dropped from the MAC queue. As CW reaches it is maximum value, it keeps on this value for the subsequent retransmission attempts.

In the basic access mode, as the backoff time equals zero, a source node transmits a data frame and waits for a timeout period in order to receive an acknowledgment packet (ACK) from a destination node. The destination node waits for a SIFS period immediately following the successful reception of the data frame and replies with a positive ACK to indicate that the data packet has been received correctly. If the source node does not receive the ACK, the data frame is assumed to be lost and the source node doubles its CW and reschedules the frame retransmission according to the backoff rules. When the data frame is being transmitted, other nodes hearing the data frame transmission adjust their Network-Allocation Vector (NAV), which is used for virtual carrier sense at the MAC layer, correctly based on the duration field value in the received data frame. This includes the SIFS and the ACK frame transmission time, which are following the data frame.

In the *rts/cts* access mode, two small control packets, *rts* and *cts*, are handshaked between a source and a destination nodes prior to a transmission of an actual data frame in order to capture the channel, to prevent other nodess from transmission and to shorten the collision time interval. A node that needs to transmit a packet follows the rules of backoff mechanism. As the backoff counter reaches zero, the source node sends an *rts* frame to the destination node. As the destination node receives the *rts* frame, it responds with a *cts* frame after a SIFS period. The source node is allowed to transmit its data frame if and only if it received the *cts* frame correctly. Successful data transmission is acknowledged by the destination node. If the source node does not receive the ACK, the data frame is assumed to be lost and the source node doubles its CW and reschedules the frame retransmission. When the *rts* and *cts* frames are transmitted, other STAs update their NAVs based on the *rts* from the source node and the *cts* from the destination node, which helps to overcome the hidden terminal problem. In fact, the node that is able to receive the *cts* frame correctly can avoid collisions even when it is unable to sense the data transmissions from the source node. If a collision occurs with two or more *rts* frames, less bandwidth is wasted as compared to the situation when larger data frames are collided.

In both access modes, if the ACK frame is received correctly, the transmitting node resets its CW to CW_{min} and reenters the backoff process if it has further frames in its MAC queue. The transmitted data packet is dropped from the MAC queue after specific number of retransmission attempts. The standard proposed two thresholds, which are maintained by each STA and take an initial value of zero for every new packet: STA short retry count ($ssrc$) and STA long retry count ($slrc$). $slrc$ represents the maximum number of retransmission attempts of the data frame, which is incremented with each unsuccessful data frame transmission. $ssrc$ represents the maximum number of retransmission attempts for the *rts* control frame, which is incremented with each unsuccessful data or *rts* frame transmission. As either of these two limits is reached first, the frame is discarded from the MAC queue, the CW is reset to CW_{min}, and both retry limits are set to zero.

2.1 Transmission Errors

The radio channel introduces significant complexity to the design and performance analysis of wireless LANs due to multipath fading. A signal arriving at the receiver is assumed to be valid if its received power (P_r) is above a certain threshold. Otherwise, it will be regarded as noise and could not be received and processed by the MAC later. The free-space and the two-ray propagation models assume the received power as a deterministic function of distance. They model the communication range as an ideal circle around the transmitter node, if the receiver is inside the circle, it receives all packets. Otherwise, it loses all packets. Given a transmission power (P_t), the average received power $Pr\left(d\right)$ at a distance d from the transmitting antenna is given by:

$$P_r\left(d\right)=\begin{cases}\frac{P_tG_tG_r\lambda^2}{(4\pi d)^{\alpha}L} & :\ Free-space\ model,\ (\alpha=2)\\ \frac{P_tG_tG_rh_t^2h_r^2}{d^{\alpha}L} & :\ Two-ray\ model\ \ (\alpha=4)\end{cases} \quad (1)$$

(G_t, h_t) and (G_r, h_r) are the gains and the heights of the transmitting and the receiving antennas respectively, $L(L \geq 1)$ is the system loss, λ is the wavelength and α is the path loss exponent. α reflects how fast the signal decays.

In fact, the received power at a certain distance is a random variable due to multipath propagation effects (fading effects) [6]. The above two propagation

Table 1. Path loss exponent and shadowing deviation σ_{dB} for different environments ([6])

Path loss exponent		Shadowing deviation	
Environment	α	Environment	σ_{dB}
Free space	2	Outdoor	4--12
In building line of sight	1.6--1.8	Office	7--9.6
Obstructed in building	4--6	Factory, line-of-sight	3--6
Obstructed in factories	2--3	Factory, obstructed	6.8

models predict the mean received power at distance d. A more general and widely-used model is called the shadowing model. The shadowing model consists of two parts. The first part is known as path loss model, which predicts the mean received power at a distance d, denoted by $\overline{P_r(d)}$. The second part is a log-normal random variable (X_{dB}), also called a Gaussian distribution with zero mean and standard deviation (shadowing deviation) σ_{dB}, which reflects the variations of the received power at a certain distance. The overall shadowing model is:

$$[P_r(d)]_{dB} = \left[\overline{P_r(d)}\right]_{dB} + X_{dB} \tag{2}$$

α and (X_{dB}) are usually experimentally determined by field measurements. Tables 1 shows some typical values of the α and σ_{dB} for different environments. $\overline{P_r(d)}$ is represented by:

$$\left[\overline{P_r(d)}\right]_{dB} = [P_r(d_0)]_{dB} + 10log_{10}\left(\frac{d_0}{d}\right)^{\alpha} \tag{3}$$

where $P_r(d_0)$ is the power received at a close-in reference point (close-in distance d_0). At d_0. The space-loss model is used to predict $P_r(d_0)$. Equation (2) shows that the fading environment as well as the transmission distance have a great effect on the received power and consequently different STAs can have different BERs and different FERs.

In wireless communication, if the received power is less than a Carrier Sense Threshold (CSThresh), the receiver cannot detect this signal [1]. In our proposed model, we consider the receiver sensitivity equals to CSThresh. Receiver sensitivity is the received signal power with which BER is approximately 10^{-5}. To achieve this BER, the signal to noise ratio (SNR) should be approximately $10dB$ for Binary Phase Shift Keying (BPSK) modulation scheme. So, we can calculate the receiver's noise (N) from the receiver sensitivity ($Rx_{sensitivity}$) as:

$$[N]_{dB} = [Rx_{sensitivity}]_{dB} - 10 \tag{4}$$

Since IEEE 802.11b uses Direct Sequence Spread Spectrum (DSSS) physical layer and BPSK modulation, the BER is given by:

$$P_b = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{5}$$

where E_b is the transmitted signal energy per bit, N_0 is the noise spectral density, and $erfc(.)$ is the standard complementary error function. E_b/N_0 is the normalized version of SNR. The SNR and the E_b/N_0 of the received packet are:

$$[SNR]_{dB} = [P_r(d)]_{dB} - N \tag{6}$$

$$\frac{E_b}{N_0} = SNR \times \frac{W}{R_b} \tag{7}$$

where R_b is the transmission bit rate and W is the channel bandwidth. Then, P_b is calculated using equation (5).

FER depends on the probability of BER (P_b) and number of bits per frame. Assume frame lengths of *rts, cts, data* and *ack* frames are constant and equal to l_{rts}, l_{cts}, l_{data} and l_{ack} bits respectively. Given that the bit errors are uniformly distributed over the whole frame and $P_{(b,l)}$ is the BER at STA l, $P_{(e,l)}^{rts}$, $P_{(e,l)}^{cts}$, $P_{(e,l)}^{data}$ and $P_{(e,l)}^{ack}$ are:

$$\begin{cases} P_{(e,l)}^{rts} &= 1 - \left(1 - P_{(b,l)}\right)^{l_{rts}} \\ P_{(e,l)}^{cts} &= 1 - \left(1 - P_{(b,l)}\right)^{l_{cts}} \\ P_{(e,l)}^{data} &= 1 - \left(1 - P_{(b,l)}\right)^{l_{data}} \\ P_{(e,l)}^{ack} &= 1 - \left(1 - P_{(b,l)}\right)^{l_{ack}} \end{cases} \tag{8}$$

3 System Performance Analysis

3.1 Network Model Assumptions

We assume that the network consists of a finite number of n STAs, where all STAs run IEEE 802.11 DCF mechanism and use the same channel access mode, basic or *rts/cts*. Moreover, all STAs are in direct communications (no hidden STAs) and they operate under heavy traffic conditions (each STA has always frames available to transmit). The received frames may have errors due to high BERs or collisions. FERs of data frames and control frames are independent.

Collision occurs due to simultaneous transmission by two or more STAs (a STA's frame transmission encounters collision if at least one of the remaining $n-1$ STAs transmit simultaneously). Assuming a STA l transmits with probability τ_l and it collides with an independent probability $P_{(c,l)}$, then the probability of collision can be expressed as:

$$P_{(c,l)} = 1 - \prod_{v=1}^{n-1} (1 - \tau_v) \tag{9}$$

3.2 Transmission Probability

Let $b(t)$ be a stochastic process represents the backoff time counter, $s(t)$ be a stochastic process represents the backoff stage, and $l(t)$ be a stochastic process represents the backoff layer (number of data frame retransmission attempts) for a given STA at time slot t. The 3-dimensional process $\{b(t), s(t), l(t)\}$ can

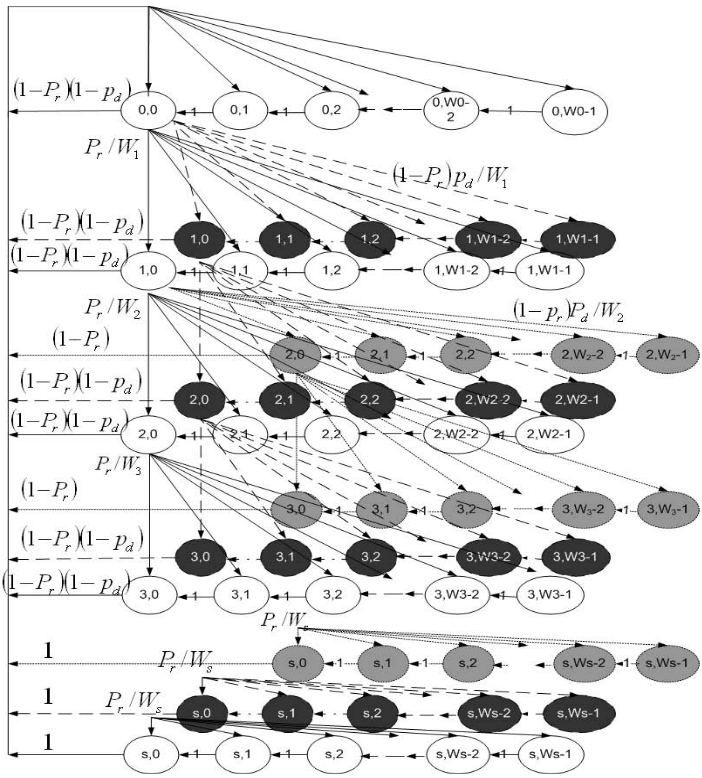

Fig. 1. Markov chain model for the backoff window size

be modeled with a discrete-time Markov chain shown in fig. 1. Let $b_{i,k}^{(j)} = lim_{t\to\infty} P\left\{s\left(t\right)=i, b\left(t\right)=k, l\left(t\right)=j\right\}, j\in\left(0,d-1\right), i\in\left(j,s\right), k\in\left(0,W_i-1\right)$ be the stationary distribution of the Markov chain. In the proposed model, there are d layers, transition from layer j to layer *j+1* is triggered by unsuccessful data frame transmission. Each layer j has *s-j+1* stages, transition from the backoff stage i to the next backoff stage *i+1* in the same layer j is triggered by unsuccessful *rts* frame transmission. Each stage i has $W_i - 1$ backoff states, transmission from state *k+1* to state k represents the backoff counter decrement process.

At each STA l, transmission probabilities $p_{(r,l)}$ and $p_{(d,l)}$ between stages and layers, respectively, are constant and independent regardless of the number of packet retransmission. Let $p_{(r,l)}$ represents the probability that a *rts* frame collides with another *rts* frame or the transmitted *rts* or *cts* frame is an erroneous frame. Let $p_{(d,l)}$ represents the probability that the transmitted *data* or *ack* frame is an erroneous frame. Assuming these events are independent, $p_{(r,l)}$ and $p_{(d,l)}$ are expressed as:

$$\begin{cases} p_{(r,l)} = 1-\left(1-P_{(c,l)}\right)\left(1-P_{(b,l)}\right)^{l_{rts}+l_{cts}} \\ p_{(d,l)} = 1-\left(1-P_{(b,l)}\right)^{l_{data}+l_{ack}} \end{cases} \tag{10}$$

From fig. 1, the only non null one-step transition probabilities are:

$$\begin{cases} P\left\{i^{(j)},k|i-1^{(j)},0\right\} = \frac{p_{(r,l)}}{W_i} & j \in (0,d-1), i \in (j+1,s) \\ P\left\{i^{(j)},k|i-1^{(j-1)},0\right\} = \frac{\left(1-p_{(r,l)}\right)p_{(d,l)}}{W_i} & j \in (1,d-1), i \in (j,s) \\ P\left\{i^{(j)},k|i^{(j)},k+1\right\} = 1 & j \in (0,d-1), i \in (j,s) \\ P\left\{0^{(0)},k|s^{(j)},0\right\} = \frac{1}{W_0} & j \in (0,d-1) \\ P\left\{0^{(0)},k|i^{(j)},0\right\} = \frac{\left(1-p_{(r,l)}\right)\left(1-p_{(d,l)}\right)}{W_0} & j \in (0,d-2), i \in (j,s-1) \\ P\left\{0^{(0)},k|i^{(j)},0\right\} = \frac{\left(1-p_{(r,l)}\right)}{W_0} & j = d-1, i \in (j,s-1) \end{cases} \quad (11)$$

Assume the current backoff process is at backoff layer $d-1$, backoff stage $i-1$ and backoff state $k+1$. First and second equations show that moving from current backoff stage to the next backoff stage within the current or next backoff layer is triggered by unsuccessful channel reservation or an erroneous data frame transmission respectively. Third equation accounts for the fact that, at the beginning of each time slot, the backoff time is decremented. Fourth equation indicates that once the backoff stage reaches the stage s, the backoff process is reset regardless of data transmission status. Fifth equation accounts for the fact that, at backoff layers $L_0, \ldots, L_{d-2}$, successful data transmission followed by reseting the backoff process. Finally, the last equation holds for layer L_{d-1}, the *slrc* count reaches its maximum limit and as a result the backoff process is reset regardless of the data transmission status. Reseting the backoff process implies $CW = CW_{min}$, $slrc = 0$ and $ssrc = 0$. Those transition probabilities can be expressed in a highly reduced mathematical form in terms of $b^{(0)}_{(0,0)}$ and the two conditional probabilities $p_{(r,l)}$ and $p(d,l)$:

$$b^{(j)}_{i,k} = \frac{(W_i-k)}{W_i} b^{(0)}_{0,0} \times \begin{cases} \sum_{w=0}^{d-2} \sum_{r=w}^{s-1} p^{r-w}_{(r,l)} . p^{w}_{(d,l)} . \left(1-p_{(r,l)}\right)^{w+1} . \left(1-p_{(d,l)}\right) . a_{r,w} + \\ \sum_{w=0}^{d-1} p^{s-w}_{(r,l)} . p^{w}_{(d,l)} . \left(1-p_{(r,l)}\right)^{w} . a_{s,w} + \\ \sum_{r=d-1}^{s-1} p^{r-d+1}_{(r,l)} . p^{d-1}_{(d,l)} . \left(1-p_{(r,l)}\right)^{d} . a_{r,d-1} & i=0, j=0 \\ p^{i-j}_{r} . p^{j}_{(d,l)} . \left(1-p_{(r,l)}\right)^{j} . a_{i,j} & Otherwise \end{cases} \quad (12)$$

where $u(x)$ is a unit step function[1] and $a_{i,j}$ is a positive integer coefficient equals to:

$$a_{i,j} = \begin{cases} 0 & ; i < 0,\ j < 0 \\ 1 & ; j = 0,\ i = 0 \\ a_{j,i-1} + a_{j-1,i-1} & ; Otherwise \end{cases} \quad (13)$$

Based on the fact that, transmission is only allowed when the backoff timer value is zero, the transmission probability τ_l (the probability that a STA l transmits in a randomly chosen time slot) is:

$$\begin{aligned} \tau_l &= \sum_{j=0}^{d-1} \sum_{i=j}^{s} b^{(j)}_{i,0} \\ &= b^{(0)}_{0,0} \left[\sum_{j=0}^{d-1} \sum_{i=j}^{s} p^{i-j}_{(r,l)} . p^{j}_{(d,l)} . \left(1-p_{(r,l)}\right)^{j} . a_{i,j} \right] \end{aligned} \quad (14)$$

[1] if $x \leq 0$, $u(x) = 0$, otherwise $u(x) = 1$.

where $b_{0,0}^{(0)}$ is determined by imposing the normalization condition as follows:

$$\sum_{j=0}^{d-1} \sum_{i=j}^{s} \sum_{k=0}^{W_i-1} b_{i,k}^{(j)} = 1 \tag{15}$$

Assume the size of data and control frames are constant, then $P_{(f,l)}^{rts}$, $P_{(f,l)}^{cts}$, $P_{(f,l)}^{data}$ and $P_{(f,l)}^{ack}$ are constant. Let $K_{(1,l)} = \left(1 - P_{(b,l)}\right)^{l_{rts}+l_{cts}}$ and $K_{(2,l)} = \left(1 - P_{(b,l)}\right)^{l_{data}+l_{ack}}$, then $p_{(r,l)}$ and $p_{(d,l)}$ are expressed as:

$$\begin{cases} p_{(r,l)} = 1 - \prod_{v=1}^{n-1} (1 - \tau_v) \times K_{(1,l)} \\ p_{(d,l)} = 1 - K_{(2,l)} \end{cases} \tag{16}$$

Equations (14) and (16) represent a nonlinear system, which is hard to solve especially as number of nodes is increased. Hence, an iteration method (algorithm 1) similar to the one proposed in [10] is required to solve such system of non-linear equations. Our proposed iteration method starts initially with $\tau_l = 1$ for each STA l. Each iteration consists of two loops, the first loop computes $p_{(r,l)}$ for each STA l and the second loop recomputes a new transmission probability τ_l from the computed $p_{(r,l)}$ in the previous step. The algorithm repeats all steps until the solution is reached.

Algorithm 1. Iteration Method

$k = 0$
for all node l **do**
  compute $P_{(b,l)}$, $K_{(1,l)}$ and $K_{(2,l)}$
  let $\tau_l^0 = 1$
end for
repeat
  $k = k + 1$
  for all node l **do**
    compute $p_{(r,l)}^k$ from equation (16)
  end for
  for all node l **do**
    using the computed $p_{(r,l)}^k$, calculate τ_l^k from equation (14)
  end for
until $(\tau_l^k - \tau_l^{k-1} \leq \epsilon)$ for all node l

In the basic access mode, no control frames are used to reserve the channel. The STA's frame transmission encounters collision if at least one of the remaining $n-1$ STAs transmit simultaneously. All previous derivations are valid for the basic access mode by assuming $p_{(r,l)} = 0$, $p_{(d,l)} = 1 - \prod_{v=1}^{n-1} (1 - \tau_v) \times K_{(2,l)}$ and $s = d - 1$.

Our model is a general one, where the impact of transmission errors on the data and control frames is considered for both access modes. Other proposed models [2,4,9] can be derived from our model as shown in table 2.

Table 2. Evaluation Parameters

$\tau_l = \tau$, $K_{(1,l)} = K_1$, $K_{(2,l)} = K_2$ $p_{(r,l)} = p_r, p_{(d,l)} = p_d$				
rts/cts access mode $p_r = 1 - (1-\tau)^{n-1} K_1$, $p_d = 1 - K_2$				
s	d	K_1	K_2	Model
∞	1	1	1	[2]
$ssrc$	1	1	1	[4]
basic access mode $p_r = 0$, $p_d = 1 - (1-\tau)^{n-1} K_2$				
s	d	K_1	K_2	Model
∞	∞	x	1	[2]
$slrc - 1$	slrc	x	1	[4]
$slrc - 1$	$slrc$	x	$(1 - P_{BER})^{l_{data}+l_{ack}}$	[9]

3.3 The Throughput

Let S be the normalized throughput defined as the fraction of time the channel is used to successfully transmit useful payload bits $E(P)$. $E(P)$ is the average packet payload size. In this paper, we assume a constant packet payload size $(E(P) = P)$. To compute S we need to identify the expected events that may occur in a randomly chosen time slot and their corresponding probabilities and durations. In each slot, one of the following three events may occur: an empty time slot (σ) with a probability P_{idle}, a successful data transmission with duration T_s and probability P_s, or an unsuccessful data transmission due to one of the following reasons: collision, an error data frame, an error *ack* frame, an error *rts* frame, or an error *cts* frame. Each reason has its own duration (T_k) and probability (P_k). Then, S can be expressed as:

$$S = \frac{P_s.E(P)}{P_{Idle}.\sigma + P_s.T_s + \sum_k (P_k.T_k)} \tag{17}$$

In the *rts/cts* access mode, the sequence of a data frame transmission is "rts-cts-data-ack", therefore, P_s represents the probability that only one STA transmits on the channel and that transmission is free from *rts*, *cts*, *data* and *ack* errors. Unsuccessful transmission could occur due to collision, an error *rts* frame, an error *cts* frame, an error data frame, or an error *ack* frame with probabilities and durations (P_1, T_1), (P_2, T_2), (P_3, T_3), (P_4, T_4) and (P_5, T_5) respectively. Let $H = h_{rtr} + h_{phy} + h_{mac}$ be the packet header $(l_{data} = H + P)$, and δ be the propagation delay. Equations (18) and (19) show time durations and their corresponding probabilities respectively:

$$\begin{cases} T_{idle} = \sigma \\ T_s \quad = l_{rts}/R + \delta + SIFS + l_{cts}/R + \delta + SIFS + l_{data}/R + \delta + SIFS \\ \qquad + l_{ack}/R + \delta + DIFS \\ T_1 \quad = l_(rts)/R + \delta + EIFS \\ T_2 \quad = l_(rts)/R + \delta + EIFS \\ T_3 \quad = l_(rts)/R + \delta + SIFS + l_(cts)/R + EIFS \\ T_4 \quad = l_(rts)/R + \delta + SIFS + l_(cts)/R + \delta + SIFS + l_(data)/R\delta + EIFS \\ T_5 \quad = l_(rts)/R + \delta + SIFS + l_(cts)/R + \delta + SIFS + l_(data)/R + \delta + SIFS \\ \qquad + l_(ack)/R + \delta + EIFS \end{cases} \tag{18}$$

$$\begin{cases} P_{idle} = \prod_{i=1}^{n} (1-\tau_i)^n \\ P_s \quad = \sum_{i=1}^{n} \tau_i \left(1 - P_{(e,i)}^{data}\right)\left(1 - P_{(e,i)}^{ack}\right)\left(1 - P_{(e,i)}^{rts}\right)\left(1 - P_{(e,i)}^{cts}\right) \\ \qquad \times \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \\ P_1 \quad = 1 - \prod_{i=1}^{n} (1-\tau_i)^n - \sum_{i=1}^{n} \tau_i \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \\ P_2 \quad = \sum_{i=1}^{n} \tau_i P_{(e,i)}^{rts} \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \\ P_3 \quad = \sum_{i=1}^{n} \tau_i \left(1 - P_{(e,i)}^{rts}\right) P_{(e,i)}^{cts} \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \\ P_4 \quad = \sum_{i=1}^{n} \tau_i \left(1 - P_{(e,i)}^{rts}\right)\left(1 - P_{(e,i)}^{cts}\right) P_{(e,i)}^{data} \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \\ P_5 \quad = \sum_{i=1}^{n} \tau_i \left(1 - P_{(e,i)}^{rts}\right)\left(1 - P_{(e,i)}^{cts}\right)\left(1 - P_{(e,i)}^{data}\right) P_{(e,i)}^{ack} \\ \qquad \times \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \end{cases} \tag{19}$$

In the basic access mode, the sequence of a data frame transmission is "*data-ack*", therefore, P_s represents the probability that only one STA transmits on the channel and that transmission is free from *data* and *ack* errors. Unsuccessful transmission occurs due to collision, an error data frame, or an error *ack* frame with probabilities and durations (P_1, T_1), (P_2, T_2), and (P_3, T_3) respectively. Equations (20) and (21) show time durations and their corresponding probabilities respectively:

$$\begin{cases} T_{idle} = \sigma \\ T_s \quad = l_{data}/R + \delta + SIFS + l_{ack}/R + \delta + DIFS \\ T_1 \quad = l_{data}/R + \delta + EIFS \\ T_2 \quad = l_{data}/R + \delta + EIFS \\ T_3 \quad = l_{data}/R + \delta + SIFS + l_{ack}/R + \delta + EIFS \end{cases} \tag{20}$$

$$\begin{cases} P_{idle} = \prod_{i=1}^{n} (1-\tau_i)^n \\ P_s \quad = \sum_{i=1}^{n} \left[\tau_i \left(1 - P_{(e,i)}^{data}\right)\left(1 - P_{(e,i)}^{ack}\right) \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j)\right] \\ P_1 \quad = 1 - \prod_{i=1}^{n} (1-\tau_i)^n - \sum_{i=1}^{n} \tau_i \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \\ P_2 \quad = \sum_{i=1}^{n} \tau_i P_{(e,i)}^{data} \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \\ P_3 \quad = \sum_{i=1}^{n} \tau_i \left(1 - P_{(e,i)}^{data}\right) . P_{(e,i)}^{ack} \prod_{j\in\{1,\dots,n\}-i} (1-\tau_j) \end{cases} \tag{21}$$

Table 3. System Parameters

Physical Layer		MAC Layer	
Parameter	Value	Parameter	Value
Path Loss Model	Shadowing	Packet payload P	8184 bits
Shadowing Mean	0	$ssrc$	7
X_{dX}	6.8dB	$slrc$	4
Propagation delay (δ)	$1\mu sec.$	W_{min}	32 time slots
Noise Factor	10dB	m	5
close-in distance	1 m	Routing header (h_{rtr})	160 bits
P_t	0.03162 W	MAC header (h_{mac})	272 bits
Receiver sensitivity	-102dBW	Physical header (h_{phy})	192 bits
Frequency	2.472GHz	l_{ack}	112 + h_{phy} bits
α	3.6	l_{rts}	160 + h_{phy} bits
G_t	1dBi	l_{cts}	112 + h_{phy} bits
G_r	1dBi	Slot (σ)	$50\mu sec.$
Modulation Scheme	BPSK	SIFS	$28\mu sec.$
Channel bit rate (R)	1Mbps	DIFS	2 * Slot + SIFS
Antenna Loss (L)	1dB	EIFS	SIFS + DIFS + l_{ack}/R

4 Analysis Results

In our analytical analysis, all STAs are static, run the same 802.11 DCF access mode and have the same bit rate. We assume that nodes are placed randomly within a certain area ($67x67\ m^2$) and communication node pairs are selected randomly. At the same time, placed STAs act as senders and may acts as receivers, whereas each STA transmits data packets to a single receiver. Unless otherwise specified, Table 3 shows the the system parameters used in analytical analysis, the reported values are for the Direct Spread Sequence Spectrum (DSSS) physical layer used in 802.11b standard.

Throughput analysis is divided into two parts. In the first part, the impact of retry counts (*ssrc* and *slrc*) on DCF throughput is analyzed. In the second part, throughput is analyzed under three different models, error-free channel, constant BER, and shadow fading.

Fig. 2 shows normalized throughput versus number of STAs for basic access mode. It shows that throughput depends on the number of STAs as well as *slrc* values. Throughput for WLAN decreases as the number of STAs connected within the network increases because the probability of collision increases. On the other hand, at low *slrc* values, the transmitted packets are dropped frequently from the MAC queue due to collision/transmission errors. As a result, all STAs have low CW values. Usually, At low CW values and large number of STAs, the collision probability is high. For large *slrc* values, STAs will working with higher CW sizes and hence the probability of collision is reduced. Fig 2 clearly shows the throughput enhancement as *slrc* increases. For *rts/cts* access mode, fig. 3 shows that the throughput does not strongly depend on the number of STAs. Additionally, the performance of the network at $ssrc = 6$ and at $ssrc = \infty$

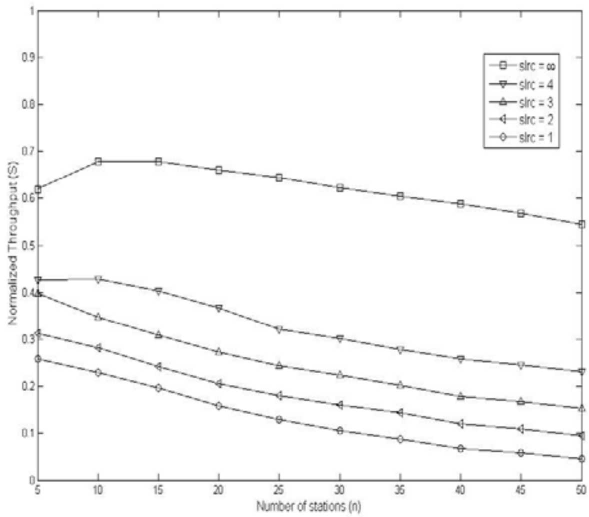

Fig. 2. Normalized saturation throughput for basic access mode

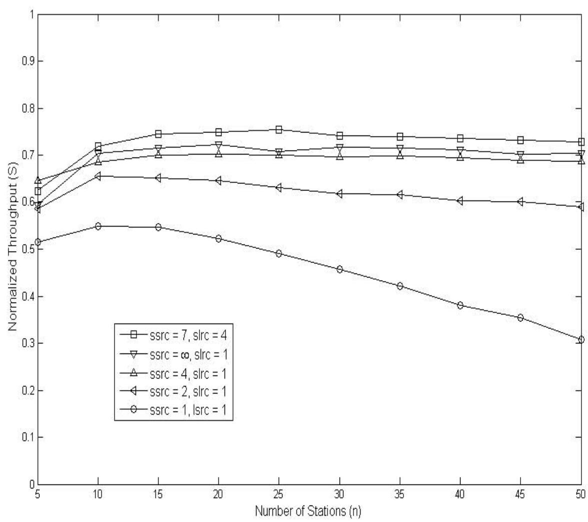

Fig. 3. Normalized saturation throughput for *rts/cts* access mode

are comparable. Although the probability of collision increases with increasing number of STAs but less bandwidth is wasted as compared to the situation when larger data frames are collided as in the basic access mode.

By assuming identical transmission power for all nodes, the received power and consequently the end-to-end BER are proportional to transmission distance. As the received power falls behind the receiver sensitivity, the receiver looses the

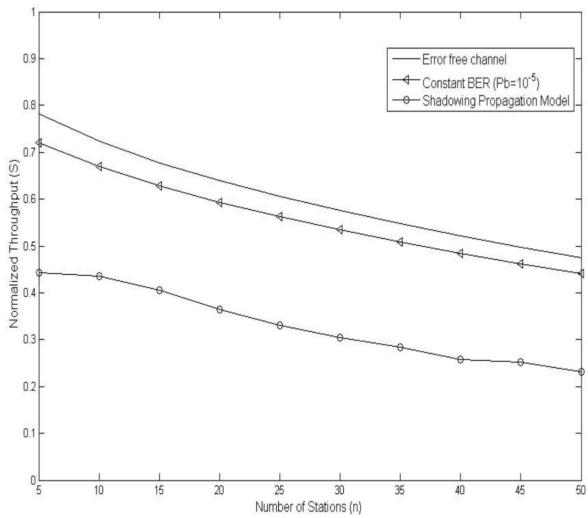

Fig. 4. Normalized throughput versus number of STAs for basic access mode

ability to detect and decode the received signal correctly. Due to the random distribution of STAs over the considered area, there is about 25% of connections have a $P_b > 0.01$. Those connections cannot transmit their data frames sussessfuly because P_e^{data} and P_e^{rts} equal *1* and *0.945* respectively.

In the basic access mode, all STAs have the ability to reserve the medium for transmission but the success of transmission requires no collision as well as low BER . Fig. 4 shows the degradation in throughput under shadow fading compared to constant BER and error-free channel. Under shadow fading, STAs have different BERs, therefore, their abilities to correctly decode the received frames are unequal. Moreover, increasing number of STAs increases the collision probability. Transmission errors and collision affects the throughput severely because the necessary time to recover from the collision is the same as the time needed to recover from the unsuccessful transmission ($T3 = T4$). By comparing the throughput at $n = 5$ and $n = 50$, the reduction in the throughput is 40% for error-free channel and 48% for shadow fading. This unequal reduction in the throughput figures the impact of BER.

From fig. 5, Throughput for WLAN decreases as the number of STAs connected within the network increases because the probability of collision increases. As expected, reduction in throughput is small compared to the basic mode because the time needed to recover from collision is less than 10% of the time needed in the basic mode. However, the fading channel has higher throughput than the error-free channel at high number of STAs. This can be attributed to the fact that under fading about *25%* of STAs cannot reserve the channel and consequently they have large CWs compared to the other STAs. As a result, STAs with low CW have the ability to reserve the channel quickly. In conclusion, the average idle time for the fading channel is small

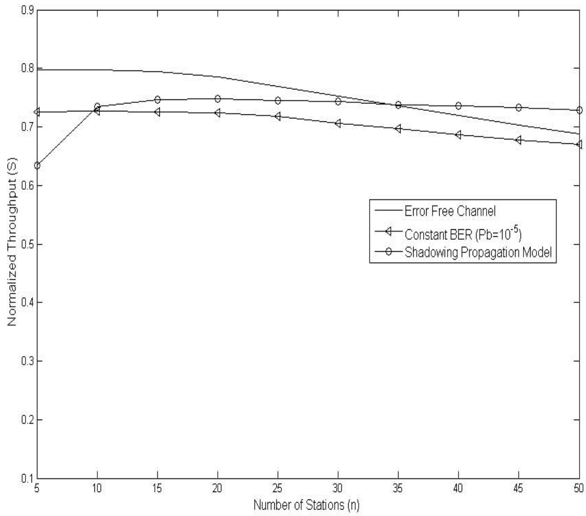

Fig. 5. Normalized throughput versus number of STAs for *rts/cts* access mode

compared to the error-free channel. At low number of STAs, the error-free channel has better performance than the fading channel because the probability of collision is low and consequently the idle time is small. In fading channel, transmission errors reduces the idle time.

5 Conclusion

IEEE 802.11 DCF with its two access modes is modeled and analyzed. The impacts of transmission errors and retry counts of control and date frames on the DCF throughput are considered. We have shown that the throughput of the basic access mode strongly depends on the system parameters (the network size and the long retry count) and it is sensitive to the transmission errors. On the other hand, the *rts/cts* access mode shows stable throughput and it is insensitive to the transmission errors. It shows better throughput compared to the error-free and the constant-BER channels. We conclude that *rts/cts* access mode is more suitable for error-prone channels than the basic access mode.

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