A Linear Precoding Design for Multi-Antenna Multicast Broadcast Services with Limited Feedback

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Abstract. The provision for spectrally efficient multicast broadcast services (MBS) is one of the key functional requirements for next generation wireless communication systems. The challenge inherent to MBS is to ensure that all MBS users can be served, and one effective solution to this problem is to employ MIMO multicast transmit precoding. In previous works on MIMO multicast transmit precoding design, the authors either assumed 1) perfect transmitter-side channel state information (CSIT) or 2) special channel conditions that facilitate precoder design with imperfect CSIT. In this paper, we focus on transmit precoding design for MBS where the CSIT is obtained via limited feedback. In addition, we analyze the average minimum receive signal-noise-ratio (RxSNR) among the MBS users and study the order of growth with respect to the number of MBS users and the number of feedback bits. Finally, we propose a threshold based feedback reduction scheme and study the tradeoff between feedback cost and performance loss.

Keywords: MIMO, transmit precoding, beamforming, multicast, MBS, limited feedback.

1 Introduction

Ubiquitous multimedia multicast broadcast services (MBS) is an integral part of the vision for next generation wireless communication systems [1, Section 7.5] [2]. In multicast transmission, the transmitter sends common information to multiple users, and the transmission rate needs to be adapted to support all the users. However, in a wireless network the MBS users have diverse channel conditions, so the multicast rate can be severely limited by the user with the worst channel. To enhance the spectral efficiency of wireless multicasting, some multi-antenna transmission techniques are proposed in recent literature [3, 4, 5, 6, 7, 8] and references therein. In particular, one effective technique is max-min fair (MMF) transmit precoding (or beamforming) where the multi-antenna

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base station (BS) scales the transmitted signal with a precoder to enhance the minimum received signal-to-noise ratio (RxSNR) among all the users.

There are two fundamental technical challenges associated with MMF transmit precoding design: first, transmitter-side channel state information (CSIT) of all the MBS users is required; and second, as shown in [3, Appendix I], [9, Section 2], the precoder optimization problem is NP-hard. To address these issues, some prior works proposed precoder designs assuming perfect CSIT or special channel conditions that facilitate precoder design with imperfect CSIT. In [3], given that the BS has perfect CSIT, the authors propose a precoder design that works around the NP-hard nature of the problem via a semi-definite relaxation (SDR) approach and a randomization process. It is shown that this SDR-randomization based algorithm can be solved in polynomial time to produce effective precoders. When the BS has imperfect CSIT, the precoder design needs to suitably account for the CSIT error. In [4], the authors consider transmit precoding design for the case of far-field line-of-sight propagation conditions. By exploiting the special properties of line-of-sight channels, the authors develop a precoder design with approximate knowledge of the user directions. On the other hand, in [6,7] the authors consider transmit precoding design given that the BS is furnished with a channel vector estimate with small Euclidean distance distortion. By adopting robust beamforming concepts, the authors develop a precoder design based on the worst case CSIT error.

In practical communication systems, the channel conditions are substantially different from those considered in the aforementioned prior works. In an urban environment typically there is no line-of-sight channel between the BS and the mobile users [10]. Moreover, when channel reciprocity does not hold as is the case for frequency division duplex (FDD) systems, the BS is commonly provided with CSIT via rate-constrained *limited feedback* from the user [2,11]. There are a number of prior works on limited feedback precoder design for unicast transmission. For example, in [12] the authors model joint power and precoder adaptation of a MIMO link with limited feedback as a vector quantization (VQ) problem and propose a modified Lloyd's algorithm to obtain the optimal solution. On the other hand, in [13] the authors consider the Grassmannian packing approach in MIMO precoder design with limited feedback for single-user systems, whereas in [14,15] the authors consider MIMO precoder design for multi-user systems. However, the limited feedback precoder design for unicast applications in the prior works cannot enhance the worst case user performance and thus are not suitable for MBS scenario.

In this paper, we consider a MIMO MBS system with one BS having N transmit antennas and K MBS users having a single receive antenna. We focus on transmit precoding design to maximize the minimum RxSNR among the MBS users where the CSIT is obtained via limited feedback. Specifically, we have the following emphases:

1. We devise a MIMO transmit precoding design for MBS by means of precoder optimization with limited feedback.

2. We derive a closed-form lower bound for the average minimum RxSNR among the MBS users and study the order of growth with respect to (w.r.t.) the number of MBS users and the number of bits for limited feedback.
3. We devise a threshold-based feedback reduction scheme and study the trade-off between feedback cost and performance loss.

The rest of this paper is organized as follows. In Section 2, we define our system model and problem statement. In Section 3, we introduce our proposed transmit precoding design and, in Section 4, we analyze the system performance. In Section 5, we present our proposed threshold-based feedback reduction scheme. Finally, in Section 6 and Section 7, we provide our numerical simulation results and concluding remarks, respectively.

We adopt the following notations. Θ^N denotes the set of unit vectors in $\mathbb{C}^{N\times 1}$; $\mathbb{R}_+$ denotes the set of nonnegative real numbers; upper and lower case letters denote matrices and vectors, respectively; $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\dagger$ denote conjugate, transpose, and Hermitian transpose, respectively; $\mathrm{Tr}(\cdot)$ and $\mathrm{rank}(\cdot)$ denote matrix trace and rank, respectively; $\mathbf{X} \succeq 0$ denotes a positive semi-definite matrix; $\mathbf{I}_N$ denotes an $N \times N$ identity matrix; $\mathbb{E}[\cdot]$ denotes expectation; $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes complex Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$; $\mathcal{O}(\cdot)$ denotes the Big O notation, and $\Omega(\cdot)$ denotes the Big Omega notation where $f(n) = \Omega(g(n))$ if there are constants c and n_0 such that $f(n) \geq cg(n)\ \forall n > n_0$; and $\Pr(\cdot)$ denotes the probability of the given event.

2 System Model and Problem Statement

2.1 System Model

We consider a communication system where a BS delivers MBS to K users as shown in Fig. 1. The BS has N transmit antennas, whereas the users have one receive antenna and experience quasi-static Rayleigh flat fading. For simplicity of exposition, in the following we focus on the k^{th} user, whose downlink channel vector is denoted by $\mathbf{h}_{(k)} \in \mathbb{C}^{N\times 1}$. We assume independent and identically distributed (i.i.d.) channels with unit variance such that $\mathbf{h}_{(k)} \sim \mathcal{CN}(0, \mathbf{I}_N)$ and remains unchanged within a fading block. Prior to multicast transmission, the

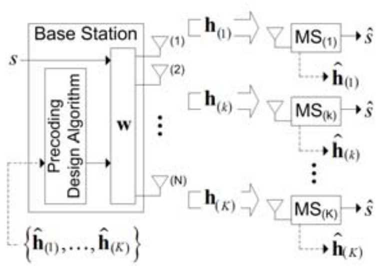

Fig. 1. MIMO MBS system with limited feedback

BS scales the MBS data symbol s with a precoder $\mathbf{w} \in \Theta^N$, and the received symbol is given by $y_{(k)} = \mathbf{h}_{(k)}^T \mathbf{w}^* s + z_{(k)}$, where $z_{(k)}$ is the additive noise with variance N_0. Let $\mathbb{E}\left[|s|^2\right] = P$. The instantaneous RxSNR is given by

$$\gamma_{(k)} = \frac{\mathbb{E}\left[|s|^2\right]}{\mathbb{E}\left[\left|z_{(k)}\right|^2\right]} \left|\mathbf{h}_{(k)}^T \mathbf{w}^*\right|^2 = \frac{P}{N_0} \left|\mathbf{h}_{(k)}^T \mathbf{w}^*\right|^2 \tag{1}$$

and we define $\frac{P}{N_0}$ as the transmit SNR (TxSNR). Since the MBS users' channels experience independent fading, the users' RxSNR can be substantially different. The multicast transmission rate is limited by the user with the worst RxSNR. Thus, to enhance the performance of MBS, we are interested for the BS to design the precoder $\mathbf{w}$ to enhance the minimum RxSNR among the users. In order to facilitate the BS with designing $\mathbf{w}$, in each fading block the users provide the BS with CSIT via limited feedback.

2.2 Limited Feedback Scheme

In order to focus on the effects of limited feedback, we make the following assumption.

Assumption 1. *The k^{th} MBS user has perfect knowledge of channel vector $\boldsymbol{h}_{(k)}$.*

The channel vector of the k^{th} user can be represented as [15, 14]

$$\mathbf{h}_{(k)} = \sqrt{\zeta_{(k)}}\mathbf{g}_{(k)}, \tag{2}$$

where we define $\mathbf{g}_{(k)} = \mathbf{h}_{(k)}/|\mathbf{h}_{(k)}|$ as the *channel direction vector* and $\zeta_{(k)} = |\mathbf{h}_{(k)}|^2$ as the *channel gain*, which are independent. We propose a limited feedback scheme where, for a given channel realization $\mathbf{h}_{(k)}$, the user feeds back to the BS the channel vector estimate

$$\widehat{\mathbf{h}}_{(k)} = \sqrt{q\left[\mathbf{h}_{(k)}\right]}\widehat{\mathbf{g}}_{(k)}, \tag{3}$$

which is constituted of a *quantized channel direction vector* $\widehat{\mathbf{g}}_{(k)} \in \Theta^N$ and a *channel quality metric* $q[\mathbf{h}_{(k)}] \in \mathbb{R}_+$ defined as follows.

Quantized channel direction vector. The quantized channel direction vector $\widehat{\mathbf{g}}_{(k)}$ is obtained from $\mathbf{g}_{(k)}$ using a codebook based method. Specifically, let the codebook $\mathcal{A} = \{\mathbf{a}_1, \ldots, \mathbf{a}_{2^b} : \mathbf{a}_i \in \Theta^N\}$ be available to the BS and the MBS users. The user quantizes $\mathbf{g}_{(k)}$ to a codevector in $\mathcal{A}$ according to the maximal squared absolute inner product criterion

$$\widehat{\mathbf{g}}_{(k)} = \arg\max_{\mathbf{a}\in\mathcal{A}} \left|\mathbf{g}_{(k)}^T \mathbf{a}^*\right|^2, \tag{4}$$

and sends the b-bit index corresponding to $\widehat{\mathbf{g}}_{(k)}$ to the BS.

Channel quality metric. The channel quality metric $q[\mathbf{h}_{(k)}]$ encapsulates both the channel gain $\zeta_{(k)}$ and the effects of quantization on $\widehat{\mathbf{g}}_{(k)}$. Since $q[\mathbf{h}_{(k)}]$ is a positive real scalar, it could be accurately represented with a small number of quantization bits. As such, we make the following assumption.

Assumption 2. *We assume the channel quality metric is accurately conveyed from the MBS user to the BS.*

Note that the same assumption that the channel quality is perfectly known to the BS is often made in the literature of SDMA (e.g. [14, 15] and references therein) since the number of bits required for quantizing the channel quality is much less than the number of bits required for quantizing the channel direction vector.

2.3 Problem Formulation

At the BS, given CSIT of the K users $\mathcal{H}_{(K)} = \left\{\widehat{\mathbf{h}}_{(1)}, \ldots, \widehat{\mathbf{h}}_{(K)}\right\}$, the BS selects a precoder to maximize the minimum expected RxSNR among the users. Let $\mathbf{w}[\mathcal{H}_{(K)}]$ denote the precoder action for the CSIT realization $\mathcal{H}_{(K)}$ and let $\mathcal{W} = \{\mathbf{w}[\mathcal{H}_{(K)}] \in \Theta^N : \forall \mathcal{H}_{(K)} \in \mathbb{C}^{N\times K}\}$ denote the precoding policy, which is the set of all possible precoder actions for all possible CSIT realizations. The precoder design optimization problem is formally written as:

Problem 1 (Precoder optimization for MBS).

$$\mathcal{W}^\star = \arg\max_{\mathcal{W}} \mathbb{E}\left[\min_{k=1,\ldots,K} \mathbb{E}\left[\gamma_{(k)} \middle| \mathcal{H}_{(K)}\right]\right].$$

In Problem 1, note that optimization w.r.t. the precoding policy is equivalent to optimization w.r.t. the actions for a given CSIT realization. As a result, Problem 1 is equivalent to

$$\mathbf{w}^\star = \arg\max_{\mathbf{w}\in\Theta^N} \min_{k=1,\ldots,K} \mathbb{E}\left[\gamma_{(k)} \middle| \mathcal{H}_{(K)}\right] \tag{5}$$

and we shall present a transmit precoding design for obtaining the solution to Problem 1 in the following section.

3 Transmit Precoding Design

To facilitate transmit precoding design with CSIT obtain via limited feedback, we first deduce the expected RxSNR of MBS users given CSIT. After that, we present the solution for precoder optimization for MBS.

3.1 Expected RxSNR of MBS Users Given CSIT

From (1) and (2) the instantaneous RxSNR of the k^{th} user can be expressed as

$$\gamma_{(k)} = \frac{P}{N_0}\zeta_{(k)}\left|\mathbf{g}_{(k)}^T\mathbf{w}^*\right|^2. \tag{6}$$

In Theorem 1, we deduce a lower bound for the expected RxSNR given CSIT and derive the channel quality metric for limited feedback.

Theorem 1 (Lower bound of the average RxSNR). *The expected RxSNR of the k^{th} user given CSIT $\widehat{\boldsymbol{h}}_{(k)}$ is given by $\mathbb{E}\left[\gamma_{(k)}\middle|\widehat{\boldsymbol{h}}_{(k)}\right] \geq \frac{P}{N_0}\left|\widehat{\boldsymbol{h}}_{(k)}^T\boldsymbol{w}^*\right|^2 = \frac{P}{N_0}q\left[\boldsymbol{h}_{(k)}\right]\left|\widehat{\boldsymbol{g}}_{(k)}^T\boldsymbol{w}^*\right|^2$. The channel quality metric is given by $q[\boldsymbol{h}_{(k)}] = \zeta_{(k)}\eta_{(k)}$, where*

$$\begin{aligned}\eta_{(k)} &= \frac{1}{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}-\theta_{(k)}}\frac{\cos^2\left(\phi_{(k)}+\theta_{(k)}\right)}{\cos^2\left(\phi_{(k)}\right)}d\phi_{(k)} \\ &= \left(1-\tfrac{2}{\pi}\theta_{(k)}\right)\cos\left(2\theta_{(k)}\right)+\tfrac{2}{\pi}\sin\left(2\theta_{(k)}\right)\left(\ln\left(\sin\left(\theta_{(k)}\right)\right)+\tfrac{1}{2}\right),\end{aligned} \tag{7}$$

and $\theta_{(k)} = \arccos|\boldsymbol{g}_{(k)}^T\widehat{\boldsymbol{g}}_{(k)}^*|$.

Proof. As per (6), the expected RxSNR of the k^{th} user given $\zeta_{(k)}$ and $\widehat{\mathbf{g}}_{(k)}$ can be expressed as

$$\mathbb{E}\left[\gamma_{(k)}\middle|\,\zeta_{(k)},\widehat{\mathbf{g}}_{(k)}\right] = \frac{P}{N_0}\zeta_{(k)}\mathbb{E}\left[\left|\mathbf{g}_{(k)}^T\mathbf{w}^*\right|^2\middle|\widehat{\mathbf{g}}_{(k)}\right]. \tag{8}$$

We obtain a lower bound for $\mathbb{E}[|\mathbf{g}_{(k)}^T\mathbf{w}^*|^2|\widehat{\mathbf{g}}_{(k)}]$ as follows. By definition, $\widehat{\mathbf{g}}_{(k)}$ and $\mathbf{g}_{(k)}$ are related by an angle $0 \leq \theta_{(k)} \leq \frac{\pi}{2}$ according to

$$\left|\mathbf{g}_{(k)}^T\widehat{\mathbf{g}}_{(k)}^*\right|^2 = \upsilon_{(k)} = \cos^2\left(\theta_{(k)}\right) \tag{9}$$

where $0 \leq \upsilon_{(k)} \leq 1$. Similarly, the precoder $\mathbf{w}$ and $\widehat{\mathbf{g}}_{(k)}$ are related by an angle $0 \leq \phi_{(k)} \leq \frac{\pi}{2}$ according to

$$\left|\widehat{\mathbf{g}}_{(k)}^T\mathbf{w}^*\right|^2 = u_{(k)} = \cos^2\left(\phi_{(k)}\right) \tag{10}$$

where $0 \leq u_{(k)} \leq 1$. As illustrated in Fig. 2, $\mathbf{w}_{(k)}$ and $\mathbf{g}_{(k)}$ are related by an angle $\varphi_{(k)}$ according to $\phi_{(k)} - \theta_{(k)} \leq \varphi_{(k)} \leq \phi_{(k)} + \theta_{(k)}$ for $0 \leq \phi_{(k)} - \theta_{(k)}$ and $\phi_{(k)} + \theta_{(k)} \leq \frac{\pi}{2}$ [15]. Equivalently, since $|\mathbf{g}_{(k)}^T\mathbf{w}^*|^2 = \cos^2(\varphi_{(k)})$,

$$\cos^2\left(\phi_{(k)}+\theta_{(k)}\right) \leq \left|\mathbf{g}_{(k)}^T\mathbf{w}^*\right|^2 \leq \cos^2\left(\phi_{(k)}-\theta_{(k)}\right). \tag{11}$$

From (11)

$$\left|\mathbf{g}_{(k)}^T\mathbf{w}^*\right|^2 \geq \begin{cases}\cos^2\left(\phi_{(k)}+\theta_{(k)}\right) & \text{for } 0 \leq \phi_{(k)} < \frac{\pi}{2}-\theta_{(k)} \\ 0 & \text{for } \frac{\pi}{2}-\theta_{(k)} \leq \phi_{(k)} \leq \frac{\pi}{2}\end{cases} \tag{12}$$

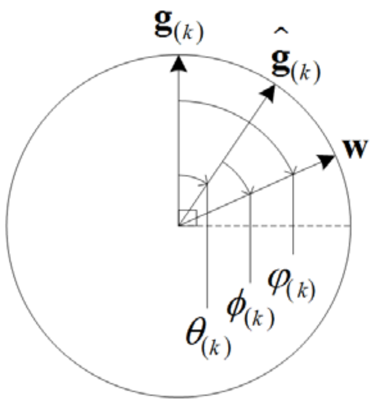

Fig. 2. Quantization model. The angle between $\widehat{\mathbf{g}}_{(k)}$ and $\mathbf{g}_{(k)}$ is $\theta_{(k)}$. The angle between $\mathbf{w}$ and $\widehat{\mathbf{g}}_{(k)}$ is $\phi_{(k)}$. The angle between $\mathbf{w}$ and $\mathbf{g}_{(k)}$ is $\varphi_{(k)}$.

and it follows from (10) that

$$\left|\mathbf{g}_{(k)}^T \mathbf{w}^*\right|^2 \geq \frac{\cos^2\left(\phi_{(k)} + \theta_{(k)}\right)}{\cos^2\left(\phi_{(k)}\right)} \left|\widehat{\mathbf{g}}_{(k)}^T \mathbf{w}^*\right|^2 \tag{13}$$

for $0 \leq \phi_{(k)} < \frac{\pi}{2} - \theta_{(k)}$. The average of (13) over $\phi_{(k)}$ gives

$$\mathbb{E}\left[\left|\mathbf{g}_{(k)}^T \mathbf{w}^*\right|^2 \middle| \widehat{\mathbf{g}}_{(k)}\right] \geq \underbrace{\frac{1}{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}-\theta_{(k)}} \frac{\cos^2\left(\phi_{(k)} + \theta_{(k)}\right)}{\cos^2\left(\phi_{(k)}\right)} d\phi_{(k)}}_{=\eta_{(k)}} \left|\widehat{\mathbf{g}}_{(k)}^T \mathbf{w}^*\right|^2. \tag{14}$$

Therefore, from (8) and (14) $\mathbb{E}\left[\gamma_{(k)} \middle| \zeta_{(k)}, \widehat{\mathbf{g}}_{(k)}\right] \geq \frac{P}{N_0} \underbrace{\zeta_{(k)} \eta_{(k)}}_{=q[\mathbf{h}_{(k)}]} \left|\widehat{\mathbf{g}}_{(k)}^T \mathbf{w}^*\right|^2$, and it follows that $\mathbb{E}\left[\gamma_{(k)} \middle| \widehat{\mathbf{h}}_{(k)}\right] \geq \frac{P}{N_0} \left|\widehat{\mathbf{h}}_{(k)}^T \mathbf{w}^*\right|^2 = \frac{P}{N_0} q\left[\mathbf{h}_{(k)}\right] \left|\widehat{\mathbf{g}}_{(k)}^T \mathbf{w}^*\right|^2$.

From Theorem 1, when the BS uses the CSIT of the K MBS users $\mathcal{H}_{(K)}$ to select the precoder $\mathbf{w}$, the expected RxSNR of the k^{th} user satisfies

$$\mathbb{E}\left[\gamma_{(k)} \middle| \mathcal{H}_{(K)}\right] \geq \frac{P}{N_0} \left|\widehat{\mathbf{h}}_{(k)}^T \mathbf{w}^*\right|^2. \tag{15}$$

3.2 Optimization Solution for MBS Precoder Design

To obtain the optimization solution for MBS precoder design, we apply the lower bound for the expected RxSNR of MBS users given CSIT as per (15) to recast the optimization (5) as

$$\mathbf{w}^{\star} = \frac{P}{N_0} \arg \max_{\mathbf{w}\in\Theta^N} \min_{k=1,\ldots,K} \left|\widehat{\mathbf{h}}_{(k)}^T \mathbf{w}^*\right|^2 . \tag{16}$$

Rewriting (16) in standard form

$$\begin{aligned} \mathbf{w}^{\star} = \min_{\substack{\mathbf{w}\in\Theta^N \\ \gamma\in\mathbb{R}_+}} & -\gamma \\ \text{s.t.} \quad & \gamma - \tfrac{P}{N_0}\left|\widehat{\mathbf{h}}_{(k)}^T \mathbf{w}^*\right|^2 \le 0, k \in [1, K], \end{aligned} \tag{17}$$

we see that this problem is non-convex because the constraints are non-convex. More importantly, this problem is NP-hard as shown in [3, Appendix I], [9, Section 2]. In order to work around the issue of NP-hardness, we employ the SDR-randomization approach [3, 4, 5, 6, 7, 9] to generate an effective precoder solution.

We apply SDR to (17) as follows. Let $\widehat{\mathbf{H}}_{(k)} = \widehat{\mathbf{h}}_{(k)}^* \widehat{\mathbf{h}}_{(k)}^T \in \mathbb{C}^{N\times N}$, and let $\mathbf{W} = \mathbf{w}^*\mathbf{w}^T \in \mathbb{C}^{N\times N}$ which is rank one and positive semidefinite. Thus, (17) can be rewritten as

$$\begin{aligned} \mathbf{W}^{\star} = \min_{\substack{\mathbf{W}\in\mathbb{C}^{N\times N} \\ \gamma\in\mathbb{R}_+}} & -\gamma \\ \text{s.t.} \quad & \gamma - \tfrac{P}{N_0}\operatorname{Tr}\left(\mathbf{W}\widehat{\mathbf{H}}_{(k)}\right) \le 0, k \in [1, K] \\ & \operatorname{Tr}(\mathbf{W}) = 1, \mathbf{W} \succeq 0 \\ & \operatorname{rank}(\mathbf{W}) = 1. \end{aligned} \tag{18}$$

In this equivalent formulation, the rank-one constraint is non-convex but all other constraints are affine. By SDR, we drop the rank-one constraint and obtain the semidefinite problem (SDP)

$$\begin{aligned} \widetilde{\mathbf{W}}^{\star} = \min_{\substack{\mathbf{W}\in\mathbb{C}^{N\times N} \\ t\in\mathbb{R}_+}} & -\gamma \\ \text{s.t.} \quad & \gamma - \tfrac{P}{N_0}\operatorname{Tr}\left(\mathbf{W}\widehat{\mathbf{H}}_{(k)}\right) \le 0, k \in [1, K] \\ & \operatorname{Tr}(\mathbf{W}) = 1 \\ & \mathbf{W} \succeq 0, \end{aligned} \tag{19}$$

which can be solved in polynomial time using interior point methods [16, Section 11.6].

Note that if $\operatorname{rank}(\widetilde{\mathbf{W}}^{\star}) = 1$, then it can be decomposed as $\widetilde{\mathbf{W}}^{\star} = (\mathbf{w}^{\star})^*(\mathbf{w}^{\star})^T$ and we obtain as $\mathbf{w}^{\star}$ the principle eigenvector of $\widetilde{\mathbf{W}}^{\star}$. Conversely, if $\operatorname{rank}(\widetilde{\mathbf{W}}^{\star}) \neq 1$, then we obtain an effective solution $\mathbf{w}^{\star}$ from $\widetilde{\mathbf{W}}^{\star}$ via the randomization process as follows. Let $\mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{\dagger}$ denote the eigen-decomposition of $\widetilde{\mathbf{W}}^{\star}$. By randomization, we generate a set of random candidate precoders in the eigenspaces of $\widetilde{\mathbf{W}}^{\star}$ and select the *best* solution. Specifically, let $\{\widehat{\mathbf{w}}_{(r)}\}_{r=1}^{R}$ denote a set of R random unit vectors given by

$$\widehat{\mathbf{w}}_{(r)} = \frac{\mathbf{Q}\boldsymbol{\Lambda}^{\frac{1}{2}}\mathbf{v}_{(r)}}{\left|\mathbf{Q}\boldsymbol{\Lambda}^{\frac{1}{2}}\mathbf{v}_{(r)}\right|}, \quad r = 1, \ldots, R, \tag{20}$$

where $\mathbf{v}_{(r)} \sim \mathcal{CN}(0, \mathbf{I}_N)$. We select as $\mathbf{w}^\star$ the candidate vector that maximizes the minimum expected RxSNR among the MBS users according to

$$\mathbf{w}^\star = \frac{P}{N_0} \arg \max_{\mathbf{w} \in \left\{\widehat{\mathbf{w}}_{(r)}\right\}_{r=1}^{R}} \min_{k=1,\ldots,K} \left|\widehat{\mathbf{h}}_{(k)}^T \mathbf{w}^*\right|^2 . \tag{21}$$

Remark 1. We remark that solving the SDP and the randomization process are both of polynomial complexity so the overall SDR-randomization approach is of polynomial complexity.

4 Performance Analysis

In this section, we derive a closed-form lower bound on the average minimum RxSNR among the MBS users based on the proposed transmit precoding design. From the performance lower bound, we study the order of growth of the average minimum RxSNR w.r.t. the number of MBS users K, the number of bits for limited feedback b, and the number of transmit antennas N.

Note that the performance of transmit precoding is lower bounded if the MBS users' channel vectors are orthogonal, and as per [17, Section III-B] a *well-designed* precoder can capture at least a fraction of $\frac{1}{K}$ of each user's channel gain. Thus, using the precoder solution $\mathbf{w}^\star$ obtained as per (16), the minimum expected RxSNR given CSIT among the MBS users is

$$\gamma^\star = \frac{P}{N_0} \min_{k=1,\ldots,K} \left|\widehat{\mathbf{h}}_{(k)}^T \left(\mathbf{w}^\star\right)^*\right|^2 \geq \frac{P}{N_0}\frac{1}{K} \min_{k=1,\ldots,K} \left|\widehat{\mathbf{h}}_{(k)}\right|^2 .$$

From Theorem 1, $|\widehat{\mathbf{h}}_{(k)}|^2 = q[\mathbf{h}_{(k)}] = \eta_{(k)}\zeta_{(k)}$, so

$$\gamma^\star \geq \frac{P}{N_0}\frac{1}{K} \min_{k=1,\ldots,K} \eta_{(k)}\zeta_{(k)}, \tag{22}$$

where the channel gain $\zeta_{(k)}$ is χ^2 distributed with $2N$ degrees-of-freedom (DOF), and $\eta_{(k)}$ is given in (7).

To analyze the average minimum RxSNR, we first have the following lemmas.

Lemma 1 (Mean of ordered statistics [18, Section 4.6]). *Given K i.i.d. random variables $\left\{x_{(k)}\right\}_{k=1}^{K}$ with cumulative distribution function (CDF) $F_x(x)$, the mean of* $\min_{k=1,\ldots,K} x_{(k)}$ *grows like*

$$\mu_{x,1:K} = \mathbb{E}\left[\min_{k=1,\ldots,K} x_{(k)}\right] = \mathcal{O}\left(F_x^{-1}\left(\frac{1}{K+1}\right)\right), \tag{23}$$

where $F_x^{-1}(\cdot)$ is the inverse CDF of $\left\{x_{(k)}\right\}_{k=1}^{K}$.

Lemma 2 (Inverse CDF of $\eta_{(k)}$ and $\zeta_{(k)}$). *With N transmit antennas and b bits for limited feedback, the inverse CDF of $\eta_{(k)}$ has the following order of growth*

$$F_{\eta}^{-1}\left(\frac{1}{K+1}\right) = \mathcal{O}\left(\exp\left(-3\left(1-\left(\frac{1}{K+1}\right)^{\frac{1}{2^b}}\right)^{\frac{1}{2N-1}}\right)\right), \tag{24}$$

and the inverse CDF of $\zeta_{(k)}$ has the following order of growth

$$F_{\zeta}^{-1}\left(\frac{1}{K+1}\right) = \mathcal{O}\left(\frac{N!}{K+1}\right)^{\frac{1}{N}}. \tag{25}$$

Proof. Please refer to Appendix I.

Based on Lemma 1 and Lemma 2, we summarize in Theorem 2 the lower bound of the average minimum RxSNR among the MBS users.

Theorem 2 (Lower bound on the average RxSNR). *With K MBS users, N transmit antennas and b bits for limited feedback, the closed-form lower bound of the average minimum RxSNR among the MBS users is given by*

$$\begin{aligned}\mathbb{E}\left[\gamma^{\star}\right] &\geq \tfrac{P}{N_0}\tfrac{1}{K}\mathbb{E}\left[\min_{k=1,\ldots,K}\eta_{(k)}\zeta_{(k)}\right] \geq \tfrac{P}{N_0}\tfrac{1}{K}F_{\eta}^{-1}\left(\tfrac{1}{K+1}\right)F_{\zeta}^{-1}\left(\tfrac{1}{K+1}\right)\\ &= \Omega\left(\tfrac{P}{N_0}\left(\tfrac{1}{K}\right)^{1+\frac{1}{N}}(N!)^{\frac{1}{N}}\exp\left(-3\left(1-\left(\tfrac{1}{K}\right)^{\frac{1}{2^b}}\right)^{\frac{1}{2N-1}}\right)\right).\end{aligned} \tag{26}$$

Proof. The proof is omitted due to the limited pages.

Remark 2 (Order or growth w.r.t. the number of MBS users, K). From (26), it can be shown that, for fixed b and N, the order of growth of $\mathbb{E}\left[\gamma^{\star}\right]$ w.r.t. K is $\Omega((\frac{1}{K})^{1+\frac{1}{N}})$. The order of growth relationship is verified by simulation in Fig. 4(a).

Remark 3 (Order or growth w.r.t. the number of feedback bits, b). From (26), it can be shown that, for fixed K and N, the order of growth of $\mathbb{E}\left[\gamma^{\star}\right]$ w.r.t. b is $\Omega(\exp(-3(1-(\frac{1}{K})^{\frac{1}{2^b}})^{\frac{1}{2N-1}}))$. The order of growth relationship is verified by simulation in Fig. 4(b).

5 Threshold-Based Feedback Reduction Scheme

In the preceding discussion, we formulate the precoder design problem involving feedback of all K MBS users in the system. Intuitively, since the system performance – the minimum RxSNR – is limited by MBS users with the worst channels, some MBS users with strong channels may not need to be considered in the precoder design problem without affecting the system performance. This motivates a threshold-based feedback reduction scheme where the users, based on their channel conditions, selectively feedback to the BS to be considered in the precoder design. As a consequence, we can reduce the system feedback cost. In the threshold-based feedback reduction scheme, the BS broadcasts a common system threshold. Each MBS user locally compares its channel gain with the system threshold and does not attempt to feedback when the channel gain exceeds the system threshold. Based on the CSIT collected from the MBS users that attempt to feedback, the BS solves Problem 1 w.r.t. these users only.

We are interested to quantify the potential performance loss and the feedback cost savings by virtue of the threshold-based feedback reduction scheme. We first have the following lemmas.

Lemma 3 (Distribution of precoder). *Suppose $L \leq K$ MBS users attempt to feedback to the BS and let the CSIT be denoted by $\mathcal{H}_{(L)} = \left\{\widehat{\boldsymbol{h}}_{(1)}, \ldots, \widehat{\boldsymbol{h}}_{(L)}\right\}$. The precoder solution $\boldsymbol{w}^\star$ w.r.t. these L users is uniformly distributed on Θ^N with randomness induced by CSIT $\mathcal{H}_{(L)}$.*

Proof. The proof is omitted due to the limited pages.

Lemma 4 (Conditional distribution of RxSNR). *Suppose the k^{th} MBS user does not attempt to feedback to the BS and the BS designs the precoder considering all MBS users that feedback. For the k^{th} user, the conditional CDF of the RxSNR $\gamma_{(k)}$, conditioned on channel gain $\zeta_{(k)}$, is given by*

$$Pr\left(\gamma_{(k)} \leq \gamma \middle| \zeta_{(k)}\right) = 1 - \left(1 - \frac{\gamma}{\frac{P}{N_0}\zeta_{(k)}}\right)^{N-1} \tag{27}$$

for $\gamma \in [0, \frac{P}{N_0}\zeta_{(k)}]$, where the randomness of $\gamma_{(k)}$ is induced by the CSIT of the users that feedback.

Proof. The proof is omitted due to the limited pages.

Based on Lemma 3 and Lemma 4, we can derive the average performance loss and the average feedback cost savings using the threshold-based feedback reduction scheme. Specifically, with a system threshold ζ_0, the k^{th} MBS user does not attempt to feedback if its channel gain $\zeta_{(k)} > \zeta_0$. The decision of the k^{th} user of not attempting to feedback may or may not cost performance loss. Intuitively, the motivation of the threshold-based scheme is that if a user's instantaneous channel is very strong, it is very likely that this user will not be the "bottleneck user" in the MBS system and hence, there should be no loss of performance even if this user does not feedback. Hence, if the actual instantaneous RxSNR of the k^{th} user $\gamma_{(k)}$ (which depends on the precoder adopted by the BS after considering all the other MBS users that feedback) is larger than is larger than $\frac{P}{N_0}\widehat{\zeta}$ (the best possible RxSNR after precoding), then the decision of not feeding back will not cost any performance loss in the MBS group because the k^{th} user will not be the bottleneck. Otherwise, the decision will cost some performance penalty as the user(s) not feeding back becomes the bottleneck user(s). We define this event as the *miss event*. Theorem 3 summarizes the main results on the probability of miss and the feedback cost.

Theorem 3 (Probability of miss vs. feedback cost). *The system performance loss (quantified by the* probability of miss*) is given by*

$$Pr\{miss\} \equiv Pr\left(\gamma_{(k)} \leq \frac{P}{N_0}\widehat{\zeta}\,\middle|\,\zeta_{(k)} > \zeta_0\right) = 1 - \exp\left(-\zeta_0\right). \tag{28}$$

On the other hand, the probability of feedback *is given by*

$$Pr\left(\zeta_{(k)} > \zeta_0\right) = 1 - F_\zeta\left(\zeta_0\right), \tag{29}$$

where $F_\zeta(\zeta)$ is the CDF of the channel gain (cf. (32)). Hence, the average feedback cost (i.e. the average number of users that feedback) is

$$K Pr\left(\zeta_{(k)} > \zeta_0\right) = K\left(1 - F_\zeta\left(\zeta_0\right)\right). \tag{30}$$

Proof. The proof is omitted due to the limited pages.

Remark 4. By setting the system threshold to be $\zeta_0 = \mathcal{O}(\ln K)$, we have the average feedback cost $\mathcal{O}((\ln K)^{N-1})$ and the average probability of miss given by $\mathcal{O}(1/K \exp(-\ln(K)/K))$. As a result, to maintain asymptotic optimal performance, the average feedback cost per user is asymptotically negligible.

6 Simulation Results

In this section, we evaluate the performance of the proposed transmit precoding design with numerical results. Specifically, we consider 1) the average minimum RxSNR among MBS users applying the proposed transmit precoding design involving feedback from all MBS users, and 2) the performance of the feedback reduction scheme in terms of the feedback overhead versus the probability of miss.

For the purpose of illustration, in our simulations we assume that the TxSNR is 20dB, up to $K = 30$ MBS users, $b = 10$ or $b = 18$ feedback bits, and up to $N = 9$ transmit antennas. Moreover, we assume that a random codebook [19] is adopted for limited feedback. In order to obtain the precoder solution, during the randomization process at least $R = 100$ candidate vectors are tested.

6.1 Performance Involving Feedback from All MBS Users

In Fig. 3, we compare the average minimum RxSNR with the proposed transmit precoding design, with (baseline 1) transmit precoding with perfect CSIT, and with (baseline 2) no precoding. It can be seen that with the proposed transmit precoding design there is substantial RxSNR gain over no precoding. In particular, there is an RxSNR gain in excess of 5dB given $K = 20$ MBS users, $b = 10$

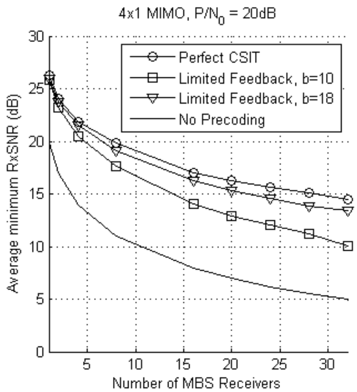

Fig. 3. Average minimum RxSNR using the proposed multicast transmit precoding algorithm. The BS transmits with $N = 4$ antennas and the TxSNR is 20dB.

bits for limited feedback, and $N = 4$ transmit antennas. On the other hand, it can be see that the performance with the proposed transmit precoding design improves with the number of feedback bits and approaches the performance with transmit precoding with perfect CSIT.

In Fig. 4(a) and Fig. 4(b), we illustrate the order of growth of the average minimum RxSNR w.r.t. the number of MBS users and feedback bits, respectively. As per Theorem 2, the average minimum RxSNR scales on the order of $\Omega((\frac{1}{K})^{1+\frac{1}{N}})$ w.r.t. to the number of users as shown in Fig. 4(a) and scales on the order of $\Omega(\exp(-3(1-(\frac{1}{K})^{\frac{1}{2^b}})^{\frac{1}{2N-1}}))$ w.r.t. to the number of feedback bits as shown in Fig. 4(b).

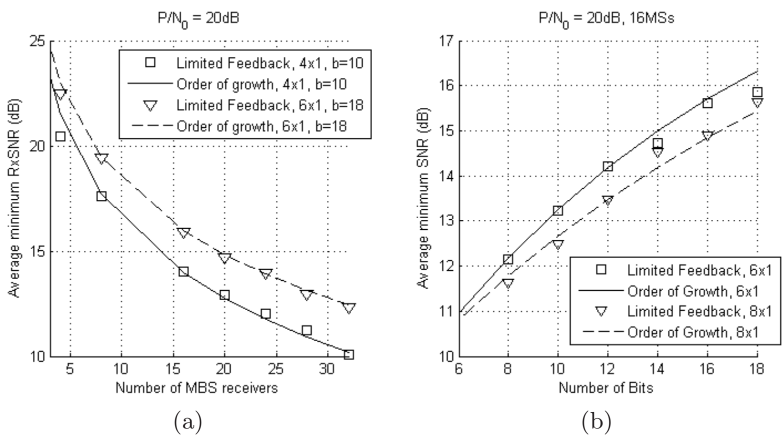

Fig. 4. Order of growth of average minimum RxSNR w.r.t. (a) the number of MBS users K and (b) the number of feedback bits b.

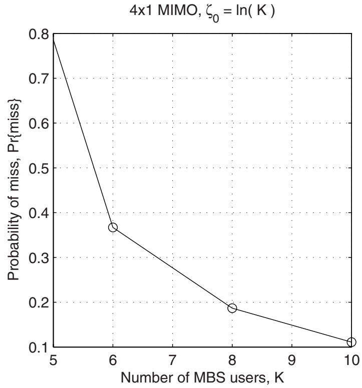

Fig. 5. Probability of miss using the threshold based feedback reduction scheme. The BS transmits with $N = 4$ antennas and the system threshold is $\zeta_0 = \ln(k)$.

6.2 Performance with Feedback Reduction Scheme

In Fig. 5, we show the probability of miss using the threshold based feedback reduction scheme. For the purpose of illustration, we let the BS transmit with $N = 4$ antennas and set the system threshold to be $\zeta_0 = \ln(K)$. As expected, as the number of MBS users K increases, the best possible RxSNR after precoding decreases, and so users with strong instantaneous channels do not have to feedback without affecting system performance.

7 Conclusions

In this paper we consider a MIMO MBS system and propose a transmit precoding design with limited feedback. Specifically, given CSIT obtained via limited feedback, we optimize the precoding policy for enhancing the average minimum RxSNR among MBS users, while suitably accounting for the CSIT error. In order to analyze system performance, we derived a closed-form lower bound and the order of growth expressions for the average minimum RxSNR among the MBS users, and these results agree closely with numerical simulation results. Finally, we further deduce a threshold based feedback reduction scheme, which allows us to effectively reduce system feedback overhead with negligible performance loss.

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Appendix I: Proof of Lemma 2

Inverse CDF of $\eta_{(k)}$. As per (7), it can be shown that $\eta_{(k)}$ decreases with increasing $\theta_{(k)}$, which is the angle between the channel direction vector $\mathbf{g}_{(k)}$ and the quantized channel direction vector $\widehat{\mathbf{g}}_{(k)}$ obtained via limited feedback. We are interested in obtaining the system performance lower bound, and we assume an upper bound model for $\theta_{(k)}$ where $\widehat{\mathbf{g}}_{(k)}$ is obtained using a random codebook [19]. From [19, (11)], the CDF of $\cos^2(\theta_{(k)}) = v_{(k)}$ is given by

$$F_v(v) = \left(1 - (1-v)^{N-1}\right)^{2^b}, \quad v \in [0,1], \tag{31}$$

and the CDF of $\theta_{(k)}$ is given by $F_\theta(\theta) = \Pr\left(v \geq \cos^2(\theta)\right) = 1 - \left(1 - \sin^{2(N-1)}(\theta)\right)^{2^b}$, $\theta \in \left[0, \frac{\pi}{2}\right]$. It can be shown that $\eta_{(k)} \approx e^{-3\theta_{(k)}}$, so the CDF of $\eta_{(k)}$ is given by $F_\eta(\eta) = \Pr\left(\theta \geq -\frac{\ln(\eta)}{3}\right) = \left(1 - \sin^{2(N-1)}\left(-\frac{\ln(\eta)}{3}\right)\right)^{2^b}$, $\eta \in [0,1)$, and the inverse CDF of $\eta_{(k)}$ is given by $F_\eta^{-1}(q) = \exp\left(-3\arcsin\left(1 - q^{\frac{1}{2^b}}\right)^{\frac{1}{2N-1}}\right)$, $q = \frac{1}{K+1}$. For sufficiently large K, $1 - q^{\frac{1}{2^b}}$ approaches 0. Recall that the Maclaurin series of $\arcsin(x) \approx x$ for small x, and so

$$F_\eta^{-1}(q) = \mathcal{O}\left(\exp\left(-3\left(1 - q^{\frac{1}{2^b}}\right)^{\frac{1}{2N-1}}\right)\right), \text{ where } q = \frac{1}{K+1}.$$

Inverse CDF of $\zeta_{(k)}$. The channel gain $\zeta_{(k)}$ is χ^2 distributed with $2N$ DOF, so its CDF is given by

$$F_\zeta(\zeta) = 1 - e^{-\zeta}\sum_{n=0}^{N-1}\frac{\zeta^n}{n!}, \ \zeta \in [0,\infty). \tag{32}$$

Since $e^\zeta = \sum_{n=0}^{\infty}\frac{\zeta^n}{n!}$, (32) can be rewritten as

$$F_\zeta(\zeta) = 1 - e^{-\zeta}\left(e^\zeta - \sum_{n=N}^{\infty}\frac{\zeta^n}{n!}\right) = e^{-\zeta}\sum_{n=N}^{\infty}\frac{\zeta^n}{n!} \tag{33}$$

For small ζ, we can neglect the higher order terms in (33) to get $F_\zeta(\zeta) = \mathcal{O}\left(\frac{\zeta^N}{N!}\right)$ (cf. [20, (50)]). Let $F_\zeta(\zeta) = \frac{1}{K+1}$ and for sufficiently large K, $F_\zeta^{-1}\left(\frac{1}{K+1}\right) = \mathcal{O}\left(\left(\frac{N!}{K+1}\right)^{\frac{1}{N}}\right)$.