Efficient Spectrum Sharing in Cognitive Radio Networks with Implicit Power Control

Miao Ma and Danny H.K. Tsang

Department of Electronic and Computer Engineering
The Hong Kong University of Science and Technology
Clear Water Bay, Kowloon, Hong Kong
eemma@ust.hk, eetsang@ust.hk

Abstract. Cognitive radio technology solves the spectrum under-utilization problem by enabling the secondary users access the spectrum holes opportunistically. How to efficiently share the spectrum holes among the secondary users, therefore, is of interest. Previous studies on spectrum sharing either do not consider interference constraints or assume the links being unidirectional. For simplicity the power control is usually not jointly considered when modeling the spectrum sharing in most of the previous studies. In this paper, we present a cross-layer design by modeling the spectrum sharing and power control with the interference constraints. A binary integer linear programming (BILP) problem is formulated to determine which link will be established, which channel will be assigned to each established link and which power level each established link will use for transmission. Different from the previous work, we assume links being bidirectional because we believe the link level acknowledgements in an ad-hoc network are a must. Moreover, we propose an implicit power control approach, where the power level for each link is predefined and implicitly embedded in the formulation, which makes the problem formulation very simple. Numerical results show that the power control helps to reduce the interference and therefore significantly (up to 56.3% in the simulated scenario) improves the total spectrum utilization.

Keywords: Cognitive radio, cross-layer design, spectrum sharing, power control, interference constraints.

1 Introduction

Cognitive radio technology [1] [2] [3] provides a novel way to solve the spectrum under-utilization problem. In cognitive radio (CR) networks there are two types of users: primary users and secondary users. A primary user is the rightful owner of a channel[1], while a secondary user periodically scans the channels, identifies the currently unused channels and accesses the channels opportunistically. The secondary users organize among themselves an ad-hoc network, and communicate with each other using these identified available channels. Different from the

[1] We use the term channel and spectrum interchangeably in this paper.

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existing multi-channel multi-radio (MCMR) networks where the set of available channels at each node is identical, in CR networks the set of available channels is different from node to node.

In this paper, we are interested in the cross-layer design which takes into account the interference constraints, spectrum sharing and power control. The main issues we are going to address include:

(1) How does a node determine which neighbor it will communicate with?

(2) How does a node decide which channel on which this communication will take place?

(3) How does a node decide which power level it will use for each link?

There has been some research work on spectrum sharing in CR networks. Wang et al. [4] formulated the channel allocation problem among secondary users as a list-coloring problem with the objective of maximizing the total spectrum utilization. Zheng et al. [5] developed a graph-theoretical model to characterize the spectrum access problem under a number of different optimization functions. Thoppian et al. [6] presented a formulation of MAC-layer scheduling in CR network. All the formulations in [4], [5] and [6] do not consider interference constraints. Hou et al. [7] modeled the spectrum sharing and sub-band division, scheduling and interference constraints, and flow routing; but they assume unidirectional links, which is not a very realistic assumption. The impact of power control is not considered in all the above work [4], [5], [6], [7].

Shi et al. [8] developed a formal mathematical model for scheduling feasibility under the influence of power control; the formulation is a cross-layer design optimization problem encompassing power control, scheduling and flow routing. However, the authors also consider unidirectional links; and particularly, the problem formulation becomes substantially larger and complicated since the authors regard the power level at each node as a decision variable.

In this paper, we propose a cross-layer optimization framework to jointly design the spectrum sharing and power control with the interference constraints. We do not include the flow routing in our formulation since we assume that the traffic demands are unknown. Different from the work in [7] and [8], we consider bidirectional links and adopt an 802.11-style protocol interference model, because we believe the link level acknowledgements in an ad-hoc network are a must [10]. Most importantly, we propose an implicit power control approach, where the links are classified into different classes and where a class determines the power level being chosen by this link, rather than explicitly regarding the power level at each node as a decision variable. As a result, the power level for each link is implicitly embedded in the formulation, which makes the formulation much simpler than that in [8]. To evaluate the performance, we study two types of scenarios: one is homogeneous node location and the other is heterogeneous node location. Numerical results show that for both scenarios the power control can significantly improve the total spectrum utilization.

The rest of this paper is organized as follows. In Section 2, we describe the assumptions and system model. The modeling of interference constraints and spectrum sharing is presented and a binary integer linear programming (BILP) formulation is proposed in Section 3. Section 4 studies the impact of power control. Section 5 presents numerical results. Finally, Section 6 concludes the paper.

2 Assumptions and System Model

We consider a cognitive radio (CR) network with n secondary users. There are M orthogonal channels in the network, denoted by the set C and the cardinality $|C| = M$. Each secondary user individually detects the available channels, and the set of available channels that can be used for communication is different from node to node. Let C_i and m_i denote the set and the number of available channels observed by node i, respectively. We have $C_i \subseteq C$ and the cardinality $|C_i| = m_i \leq M$.

Each secondary user i (where $1 \leq i \leq n$) has a programmable number of radio interfaces, denoted by γ_i. We assume that the radio interface is able to tune in a wide range of channels, but at a specific time each radio interface can only operate on one channel [9].

2.1 Bidirectional Links

We represent the CR network with an undirected graph $G = (N, E)$, where N is the set of secondary users denoted by the vertices of the graph, and E is the set of edges between two vertices (i.e., secondary users). As long as a pair of secondary users are within the maximum transmission range, we draw an edge between them. As a result, E includes all the possible links. The secondary users form among themselves an ad-hoc network. We consider bidirectional links, rather than unidirectional links, due to two reasons [10]:

(1) Wireless channels is lossy. We can not assume a packet can be successfully received by a neighbor unless the neighbor acknowledges it. In an ad-hoc network, the link level acknowledgements are necessary.

(2) Medium access controls such as IEEE 802.11 implicitly rely on bi-directionality assumptions. For example, a RTS-CTS exchange is usually used to perform virtual carrier sensing.

Thus if node i can transmit data to node j and vice versa, then we represent this by a link, denoted by $e : i \leftrightarrow j$, between node i and node j. Moreover, we let C_e and δ_e denote the set and the number of available channels for the link e, respectively. We have $C_e = C_i \cap C_j$, and the cardinality $|C_e| = \delta_e$.

2.2 Power Control and Bi-directionality

We assume each secondary user is equipped with an omnidirectional antenna and each node's transmitter has power control capability. By adjusting the transmission power level, the sender can reach destination nodes located at different distances. Therefore, for any pair of sender i and receiver j there exist a transmission range r_{ij} and an interference range R_{ij}. We have $R_{ij} = (1 + \Delta)r_{ij}$, where Δ is the guard zone to prevent a neighboring node from being assigned a same channel [11].

To ensure the bi-directionality, we assume that for each bidirectional link, say $e : i \leftrightarrow j$, both nodes i and j transmit at the same power. This is because all physical paths taken by radio waves from node i to node j can be reversed, it follows that if two nodes i and j transmit at the same power, then if j can hear i, i can also hear j [10]. Therefore, for each link $e : i \leftrightarrow j$ we have $r_{ij} = r_{ji}$ and the power control is on each link individually.

2.3 Static Node Location with a Centralized Server

In this paper, we focus on a model with static node location. We also assume the set of available channel at each secondary user is static. This corresponds to the applications with a slow varying spectrum environment (e.g., TV broadcast bands).

We assume that there exists a centralized server in the CR network. Each secondary user reports its location and the set of available channels to the spectrum server. The spectrum management and power control, therefore, is simple and coordinated.

Table 1 lists the notations used in this paper.

Table 1. Notations

Symbol	Meaning
N	set of secondary users
E	set of possible links
G	network graph
C	set of available channels
C_i	set of available channels at node i
C_e	set of available channels at link e
n	number of secondary users $\|N\|$
M	number of available channels $\|C\|$
m_i	number of available channels at node i, i.e., $\|C_i\|$
γ_i	number of radio interfaces at node i
δ_e	number of available channels at link e, i.e., $\|C_e\|$
β_e	max number of channels can be assigned to link e
t_i	min number of active links at node i
E_i	the set of links incident on node i
I_e	the set of links that interfere with link e
P_{ij}	transmission power at node i to node j
r_{ij}	transmission range at node i to node j
R_{ij}	interference range at node i to node j
K	number of discrete levels of transmission range
Δ	guard zone
d_{ij}	distance between i and j
α	path loss exponent
η	detection power threshold at the receiver
F	the set of clusters
A_i	the set of nodes belonging to the i-th cluster
B_{jk}	the set of inter-cluster links between j-th and k-th clusters

3 Modeling of Spectrum Sharing

In this section, we model the interference constraints and spectrum sharing and present a binary integer linear programming (BILP) formulation. Spectrum sharing can be done either in time domain or frequency domain. In this paper, we consider frequency domain channel assignment. Spectrum sharing is to determine which link is going to be active and which channel will be assigned to each active link. Our target is to *activate as many links as possible* to increase *channel reuse*.

3.1 Link Assignment

For direct communication, two secondary users need to be within transmission range of each other, and each will tune one of its radio interface to a common channel. We say link e is active only if some channel m has been assigned to link e. We define a 0-1 binary variable x_e^m as follows:

$$x_e^m = \begin{cases} 1 & \text{if link } e \text{ is active on channel } m, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

3.2 Interference Model

Protocol Interference Model. Protocol interference model [11] considers the links being unidirectional, and only receiver is required to be free of interference for successful transmission.

Suppose that node i transmits data to node j. Let d_{ij} denote the distance between node i and node j. This transmission from node i to node j is successful if and only if

(i) The distance between node i and node j is no more than the transmission range, i.e., $d_{ij} \leq r_{ij}$.

(ii) For any node k to node h being assigned a same channel, the receiving node j must be out of the interference range, i.e., $d_{kj} > R_{kh}$.

802.11-Style Protocol Interference Model. Different from protocol interference model, an 802.11-style protocol interference model considers the links being bidirectional, and both the sender and the receiver are required to be free of interference for successful transmission.

Let e denote a link between nodes i and j, and e' denote another link between nodes k and h. The transmission on link e is successful if and only if

(i) The distance between nodes i and j is no more than the transmission range, i.e., $d_{ab} \leq r_{ab}$ for $ab = ij, ji$.

(ii) For any link $e' : k \leftrightarrow h$ being assigned a same channel, the receiving nodes i and j must be out of the interference range, i.e., $d_{ab} > R_{kh}$ for $ab = ki, kj$ and $d_{ab} > R_{hk}$ for $ab = hi, hj$.

Note that the second requirement implicitly includes the cases where link e and link e' have a node in common (i.e., $i = k$ or $i = h$ or $j = k$ or $j = h$).

For ease of presentation, we define two *link sets* as follows: we let E_i denote the set of links incident on node i, and I_e denote the set of links which interfere with link e. We have

$$\begin{aligned} E_i &= \{i \leftrightarrow j : d_{ij} \leq r_{ij}\} \cap \{i \leftrightarrow j : d_{ij} \leq r_{ji}\}, \\ I_e &= \{e' : d_{ki} \leq R_{kh}\} \cup \{e' : d_{kj} \leq R_{kh}\} \cup \\ &\quad \{e' : d_{hi} \leq R_{hk}\} \cup \{e' : d_{hj} \leq R_{hk}\}. \end{aligned}$$

Note also that in our model we have $r_{ij} = r_{ji}$ and $R_{kh} = R_{hk}$ (i.e., for any pair of sender and receiver, the sender and receiver transmit at the same power) to ensure bi-directionality.

Comparison. Compared with the protocol interference model, the 802.11-style protocol interference model is a more realistic model, which well reflects the fact that 802.11 may usually use a RTS-CTS exchange to perform virtual carrier sensing.

In this paper, we consider bidirectional links and adopt 802.11-style protocol interference model.

3.3 Constraints

Interference Constraint. *Interference* only occurs among the links sharing *the same channel*. According to the 802.11-style protocol interference model, if link e is active on channel m, then channel m cannot be assigned to any link e' as long as $e' \in I_e$. Hence, we have

$$x_e^m + x_{e'}^m \leq 1 \; (m \in C_e \cap C_{e'}, e' \in I_e, e \in E). \tag{2}$$

Link-Channel Constraint. It is possible to have multiple links between the same pair of nodes (provided that the number of radio interface can support this), because a pair of nodes may share two or more channels. But we can restrict each link to be assigned no more than β_e channels (where $\beta_e \leq \delta_e$). This leads to the following constraint:

$$\sum_{m \in C_e} x_e^m \leq \beta_e \; (e \in E). \tag{3}$$

Node-Radio Constraint. A node can establish multiple links with its neighboring nodes if it can tune each of its radio interface to a different channel. But the number of established links at each node is constrained by the number of its radio interfaces. This leads to the following constraint:

$$\sum_{e \in E_i} \sum_{m \in C_e} x_e^m \leq \gamma_i \; (i \in N). \tag{4}$$

Additional Constraint. In addition, we can add a *node-connectivity* constraint to make sure that each node has established at least t_i (where $t_i \geq 1$) links. This leads to the following constraint:

$$\sum_{e \in E_i} \sum_{m \in C_e} x_e^m \geq t_i \ (i \in N). \tag{5}$$

In fact, any linear constraint can be added to the formulation whenever necessary. In Section V we will show that for the heterogeneous node location scenario we can add some *inter-cluster connectivity constraints* to guarantee the connectivity between the clusters.

3.4 Problem Formulation

The objective of spectrum sharing is to maximize the total spectrum utilization. In this paper we are interested in studying the impact of the power control on the *channel reuse* in different scenarios, and therefore we choose the objective function as the total number of active links[2]. This problem can be formulated as:

$$\max \sum_{e \in E} \sum_{m \in C_e} x_e^m \tag{6}$$

Subject to:

$$x_e^m = 0, 1 \qquad (m \in C_e, e \in E), \tag{7}$$

$$x_e^m + x_{e'}^m \leq 1 \ (m \in C_e \cap C_{e'}, e' \in I_e, e \in E), \tag{8}$$

$$\sum_{m \in C_e} x_e^m \leq \beta_e \qquad (e \in E), \tag{9}$$

$$\sum_{e \in E_i} \sum_{m \in C_e} x_e^m \leq \gamma_i \qquad (i \in N), \tag{10}$$

$$\sum_{e \in E_i} \sum_{m \in C_e} x_e^m \geq t_i \qquad (i \in N), \tag{11}$$

where β_e, γ_i and t_i are constants. x_e^m (binary integer) are optimization variables. The objective function is a linear function and all the constraints are linear. The optimization problem is in the form of *binary integer linear programming* (BILP) and can be solved by LINGO [12].

[2] The objective function, however, can certainly be chosen as $\left(\max \sum_{e \in E} \sum_{m \in C_e} x_e^m \cdot B_e^m\right)$, where B_e^m denotes the bandwidth for the channel m at link e. Note that B_e^m can be homogeneous or heterogeneous (where heterogeneous means the bandwidth is link-dependent and/or channel-dependent), and either case can be easily extended without much technical difficulty.

4 Impact of Power Control

In this section, we investigate the impact of power control on spectrum sharing. Power control is to decide which power level a node is going to use for each link individually.

4.1 Transmission Range and Interference Range

Recall that we assume all transmitters have power control capabilities, and the sender can reach destination nodes located at different distances by adjusting the transmission power level. We also assume all receivers have the same signal detection power threshold, denoted by η. A data transmission is successful only if the receiving power exceeds the detection power threshold.

For data transmission between node i to node j, a widely used model for power propagation gain G_{ij} is

$$G_{ij} = \frac{1}{d_{ij}^{\alpha}}, \tag{12}$$

where α denotes the path loss exponent. The typical value of α is between 2 and 4, depending on the characteristics of the communication medium.

Suppose that node i transmits data with the power P_{ij} to node j, then based on $P_{ij} \cdot G_{ij} \geq \eta$, we obtain the transmission range $r_{ij} = \big(\frac{P_{ij}}{\eta}\big)^{1/\alpha}$ and interference range $R_{ij} = (1 + \Delta) \cdot \big(\frac{P_{ij}}{\eta}\big)^{1/\alpha}$. In case that the expected transmission range r_{ij} is known, we obtain the transmission power P_{ij} as follows,

$$P_{ij} = \eta \cdot r_{ij}^{\alpha}. \tag{13}$$

4.2 Discrete Power Level

In reality, the transmission power can not be continuously adjusted. Under a quantization approach, the transmission power adjustment is only allowed for a series of discrete levels, which gives rise to that the transmission range also has a series of discrete levels.

Without loss of generality, we assume the transmission range is evenly divided and consists of at most K discrete levels r_y $(1 \leq y \leq K)$ which corresponds to the transmission power P_y, where r_K and P_K are the maximum transmission range and transmission power, respectively. We have,

$$r_y = y \cdot \frac{r_K}{K}, \quad y = 1, 2, ..., K, \tag{14}$$

$$P_y = \eta \cdot y^{\alpha} \cdot \big(\frac{r_K}{K}\big)^{\alpha}, \quad y = 1, 2, ..., K. \tag{15}$$

4.3 Implicit Power Control

As we stated earlier, E includes all the possible links between the secondary users. That is, as long as a pair of secondary users are within the maximum transmission range r_K, there exists an edge between them. Then according to a link's length we classify the links in E into K classes. More specifically, we classify a link $e : i \leftrightarrow j$ as a y-class link if $r_{y-1} < d_{ij} \leq r_y$. Furthermore, the two nodes i and j separated by the distance d_{ij} use the same transmission power P_{ij} ($P_{ij} = P_{ji}$) and the same range r_{ij} ($r_{ij} = r_{ji}$) to communicate with each other, and we have $P_{ij} = P_y$ and $r_{ij} = r_y$.

In the BILP formulation, the power level for each link (i.e., y) is not a decision variable. Instead, it is predefined and implicitly embedded in the formulation (i.e., r_y and R_y for a y-class link). Since we classify the links in E into different classes and the class determines the power level being chosen by each link, we can easily obtain E_i for each node i and I_e for each link e. Then by using LINGO to solve the BILP problem, the optimal solution will show which link is active and which channel is assigned to each active link, and in particular, the (predefined) power level for each active link is automatically known according to the link's class. The implicit power control approach, therefore, significantly reduces the complexity of the problem formulation.

Note that without power control, all nodes use the maximum transmission power P_K for their transmissions, and the transmission range and the interference range for all links are r_K and R_K, respectively.

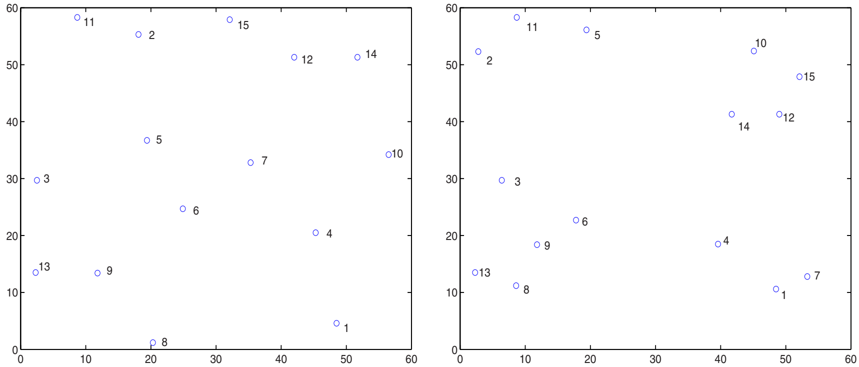
5 Numerical Results

In this section, we present numerical results for the BILP formulation and evaluate the impact of power control on spectrum sharing. We consider a 15-node ad hoc network in a 60×60 area, and study two types of scenarios: one is homogeneous node location and the other is heterogeneous node location, shown in Fig. 1(a) and Fig. 1(b), respectively. We make no claims that these two topologies are representative of typical cognitive radio networks. We have chosen these two simple topologies is to facilitate detailed discussion of the results and for the illustration purpose.

We assume there are 12 channels in the entire network. We also assume the maximum transmission range of each node is the same and $r_K = 30$. Each node has 6 discrete levels of transmission range, corresponding to 5, 10, 15, 20, 25 and 30, respectively. Table 2 summarizes the notations of the symbols and the parameter settings for both homogeneous and heterogeneous scenarios.

5.1 Scenario I: Homogeneous Node Location

First, we consider the case where the nodes are homogeneously distributed across the entire area and the topology is shown in Fig. 1(a). The set of available channels at each node is randomly generated, see Table 3. Note that the set of available channels is different from node to node.



(a) **Homogeneous** scenario (b) **Heterogeneous** scenario

Fig. 1. A 15-node ad hoc network

Table 2. Notations and parameter settings

Symbol	Meaning	Values
A^2	deployment area	$(60m)^2$
M	no. of channels in the network	12
n	no. of secondary users	15
β_e	max no. of channels assigned to link e	1
γ_i	no. of radio interfaces at node i	4
t_i	min no. of active links at node i	2
K	no. of discrete levels of transmission range	6
r_K	maximum transmission range	30
R_K	maximum interference range	45
Δ	guard zone	0.5

We use LINGO to solve the BILP problem. Fig. 3(a) shows the optimal spectrum sharing and Table 4 lists the power level that each active link will use. Fig. 3(a) shows that with power control a total number of 18 active links can be established, and the number in the figure shows which channel is assigned to each active link. It is noticed that, the channels 4, 7, 8, 10, 11 and 12 are reused.

Note that without power control, there are only 14 active links can be established and the power level of all the links is 6. But with power control, as shown in Table 4, 3, 4, 7, 2 and 2 links use power level of 2, 3, 4, 5 and 6, respectively. We comment that with power control each link properly uses its power level, which helps to reduce the interference, and therefore increases the spectrum reuse efficiency. In this example, the power control improves the total spectrum utilization by 28.6%.

Table 3. Set of available channels at each node (i.e., C_i) for **homogeneous** scenario

Node index	Location	Available channels
1	(48.5, 4.6)	1, 3, 4, 6, 8
2	(18.1, 55.3)	4, 7, 11
3	(2.5, 29.7)	1, 2, 5, 6, 12
4	(45.3, 20.5)	1, 4, 5, 8, 10
5	(19.4, 36.7)	3, 5, 8, 9, 10, 11
6	(24.9, 24.7)	2, 3, 5, 7, 9, 10
7	(35.3, 32.8)	1, 2, 6, 7, 9, 12
8	(20.3, 1.2)	2, 4, 6, 11, 12
9	(11.8, 13.4)	2, 4, 5, 6, 7, 11
10	(56.5, 34.2)	3, 6, 9, 11
11	(8.7, 58.3)	1, 5, 7 ,8
12	(42.0, 51.3)	2, 4, 8, 10, 12
13	(2.3, 13.5)	2, 3, 6, 8, 12
14	(51.7, 51.3)	1, 7, 9, 10, 11
15	(32.1, 57.9)	4, 5, 8

5.2 Scenario II: Heterogeneous Node Location

Next, we consider the case where the nodes are heterogeneously dispersed across the entire area and the topology is shown in Fig. 1(b). The set of available channels at each node is also randomly generated, see Table 5.

Different from the homogeneous scenario, the heterogeneous scenario is suitable for *clustering*. That is, we group the nodes into clusters and the hierarchy of clustering could be as deep as the number of power levels. In this example, we can simply group the nodes into four $20m$ clusters. Fig. 2(a) and Fig. 2(b) show the intra-cluster links (i.e., the link's length is less than or equal to 20) and inter-cluster links (i.e., the link's length is more than 20 but less than 30), respectively. If we let F denote the set of clusters, A_i denote the set of nodes belonging to the i-th cluster and B_{jk} denote the inter-cluster links between the j-th and k-th clusters ($B_{jk} = B_{kj}$), we have $F = \{1, 2, 3, 4\}$, and $A_1 = \{2, 5, 11\}$, $A_2 = \{3, 6, 8, 9, 13\}$, $A_3 = \{1, 4, 7\}$, $A_4 = \{10, 12, 14, 15\}$. In addition, $B_{12} = \{3 \leftrightarrow 2, 3 \leftrightarrow 11, 3 \leftrightarrow 5\}$, $B_{14} = \{5 \leftrightarrow 10, 5 \leftrightarrow 14\}$, $B_{23} = \{4 \leftrightarrow 6, 4 \leftrightarrow 9\}$ and $B_{34} = \{4 \leftrightarrow 12, 4 \leftrightarrow 14, 7 \leftrightarrow 12\}$.

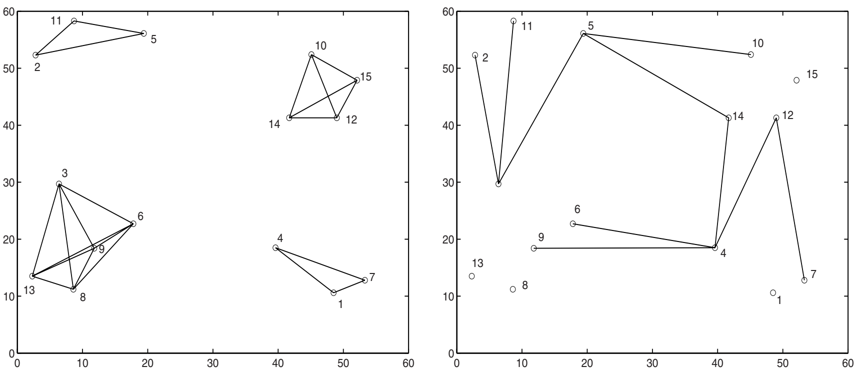
Note that in the BILP formulation, the objective function is to establish as many links as possible to increase channel reuse. As long as a channel is assigned to a link, the objective function is increased by one. With power control, a short link uses a small power level and therefore incurs a shorter interference range, as compared with a long link. As a result, a short link will have more chances to be assigned a channel. But in the heterogeneous scenario, establishing an inter-cluster link is a must to guarantee the connectivity between the clusters. This reminds us to add the following *inter-cluster connectivity constraints* into BILP formulation:

Table 4. Power level of each active link

Homogeneous scenario		**Heterogeneous** scenario	
Active link	Power level	Active link	Power level
$2 \leftrightarrow 11$	2	$1 \leftrightarrow 7$	2
$9 \leftrightarrow 13$	2	$2 \leftrightarrow 11$	2
$12 \leftrightarrow 14$	2	$6 \leftrightarrow 9$	2
$5 \leftrightarrow 6$	3	$8 \leftrightarrow 9$	2
$6 \leftrightarrow 7$	3	$8 \leftrightarrow 13$	2
$8 \leftrightarrow 9$	3	$10 \leftrightarrow 15$	2
$12 \leftrightarrow 15$	3	$12 \leftrightarrow 14$	2
$1 \leftrightarrow 4$	4	$12 \leftrightarrow 15$	2
$2 \leftrightarrow 5$	4	$1 \leftrightarrow 4$	3
$3 \leftrightarrow 9$	4	$3 \leftrightarrow 6$	3
$3 \leftrightarrow 13$	4	$3 \leftrightarrow 9$	3
$4 \leftrightarrow 7$	4	$4 \leftrightarrow 7$	3
$7 \leftrightarrow 12$	4	$5 \leftrightarrow 11$	3
$10 \leftrightarrow 14$	4	$6 \leftrightarrow 8$	3
$7 \leftrightarrow 10$	5	$9 \leftrightarrow 13$	3
$11 \leftrightarrow 15$	5	$10 \leftrightarrow 12$	3
$1 \leftrightarrow 8$	6	$10 \leftrightarrow 14$	3
$6 \leftrightarrow 13$	6	$14 \leftrightarrow 15$	3
		$2 \leftrightarrow 5$	4
		$3 \leftrightarrow 13$	4
		$2 \leftrightarrow 3$	5
		$4 \leftrightarrow 6$	5
		$5 \leftrightarrow 10$	6
		$5 \leftrightarrow 14$	6
		$7 \leftrightarrow 12$	6

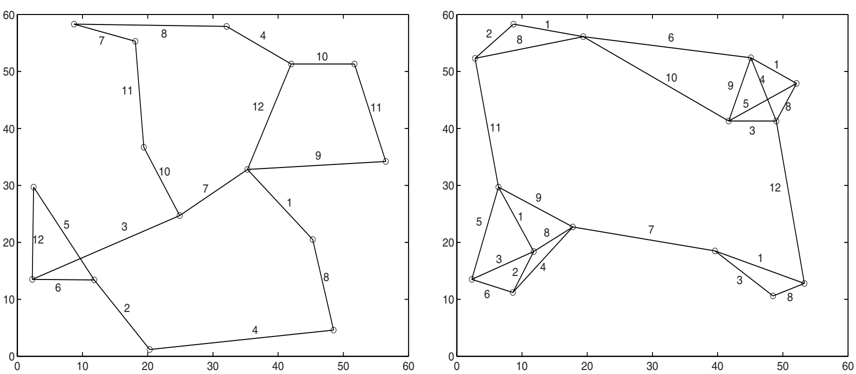
Table 5. Set of available channels at each node (i.e., C_i) for **heterogeneous** scenario

Node index	Location	Available channels
1	(48.5, 10.6)	1, 3, 4, 6, 8, 10, 11
2	(2.8, 52.3)	2, 3, 4, 7, 8, 9, 11
3	(6.4, 29.7)	1, 2, 5, 6, 9, 11, 12
4	(39.6, 18.5)	1, 3, 4, 5, 7, 8, 10, 12
5	(19.4, 56.1)	1, 3, 5, 6, 7, 8, 10, 11
6	(17.8, 22.7)	2, 3, 4, 5, 7, 8, 9, 10
7	(53.3, 12.8)	1, 2, 6, 7, 8, 9, 12
8	(8.6, 11.2)	1, 2, 4, 6, 9, 11, 12
9	(11.8, 18.4)	1, 2, 3, 4, 6, 8, 9, 11
10	(45.1, 52.4)	1, 3, 4, 6, 9, 10, 11
11	(8.7, 58.3)	1, 2, 5, 7, 8, 9, 12
12	(49.0, 41.3)	2, 3, 4, 6, 8, 10, 12
13	(2.3, 13.5)	2, 3, 5, 6, 8, 9, 12
14	(41.7, 41.3)	1, 3, 4, 5, 7, 9, 10, 11
15	(52.1, 47.9)	1, 3, 4, 5, 8, 9, 12



(a) Intra-cluster links (b) Inter-cluster links

Fig. 2. Possible links for **heterogeneous** scenario



(a) **Homogeneous** scenario (b) **Heterogeneous** scenario

Fig. 3. Optimal spectrum sharing with power control

$$\sum_{e \in B_{jk}} \sum_{m \in C_e} x_e^m \geq 1 \; (B_{jk} \neq \emptyset, k > j, k \in F, j \in F). \tag{16}$$

The purpose of the joint power control and clustering is to assign the channels properly, so that most of the intra-cluster communication is at a lower power level, and a higher power level is used for an inter-cluster link.

Fig. 3(b) shows the optimal spectrum sharing and again the channel is given next to the link in the figure. Table 4 describes the power level that each active link will use. It is observed that, there are 25 active links can be established.

Moreover, among these 25 active links there are 8, 10, 2, 2 and 3 links whose power levels are 2, 3, 4, 5 and 6, respectively.

Note that without power control, there are only 16 active links established and the power level of all links is 6. But with power control, power level 2 is chosen whenever a link's length is within $(5, 10]$, and power level 3 is selected whenever a link's length is within $(10, 15]$, etc. Therefore, as Table 4 illustrates, 8 links use power level 2 and 10 links use power level 3. Since there are many short links in heterogenous scenario due to the clustering structure, the power control significantly improves the spectrum sharing efficiency. In this example, the power control improves the spectrum utilization by 56.3%.

6 Conclusion

In this paper, we present a cross-layer design by modeling the spectrum sharing and power control with the interference constraints in cognitive radio networks. We formulate an optimization problem in the form of binary integer linear programming (BILP). The model is general and any linear constraint can be added to the formulation whenever necessary. Different from the previous work, we consider bidirectional links and adopt an 802.11-style protocol interference model. Particularly, we present an implicit power control approach, where the power level for each link is predefined and implicitly embedded in the formulation, which makes the problem formulation very simple. Numerical results show that the power control significantly improves the total spectrum utilization. For the heterogeneous node location the joint design of power control and clustering helps to assign the channels properly. For our future work, we are continuing to study the spectrum sharing problem in CR networks and our next step is to consider how to jointly design the spectrum sharing, power control and flow routing.

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