Creating Wheel-Thrown Potteries in Digital Space

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A set of digital potteries generated by our algorithm

Abstract. This paper introduces a novel technique to create digital potteries using certain simple-yet-efficient techniques of digital geometry. Given a *digital generatrix*, the proposed wheel-throwing procedure works with a few primitive integer computations only, wherein lies its strength and novelty. The created digital surface is *digitally connected and irreducible* when the generatrix is an irreducible digital curve segment, which ensures its successful rendition with a realistic finish, whatsoever may be the zoom factor. Thick-walled potteries can also be created successfully and efficiently using non-monotone digital generatrices to have the final product ultimately resembling a real-life pottery.

Keywords: Digital geometry, digital surface of revolution, generatrix, geometry and art, potteries.

1 Introduction

Creating wheel-thrown potteries is a millennium-old artistry, which, with the passing time, has gained an enduring popularity with today's state-of-the-art

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ceramic technology [3,14]. Hence, with the proliferation of digitization techniques in our computerized society, digital creation and artistic visualization of potteries is a call of the day. The existing graphic tools are mostly based on complex trigonometric procedures involving computations in the real space, which have to be tuned properly to suit the discrete nature of the 3D digital space [6,8]. For, the method of *circularly sweeping* a shape/polygon/polyline/generating curve about the axis of revolution is done in discrete steps, which requires a discrete approximation. And choosing k equispaced angles about the y-axis requires the following transformation matrices for a real-geometric realization:

$$M_i = \begin{pmatrix} \cos\theta_i & 0 & \sin\theta_i & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_i & 0 & \cos\theta_i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where, $\theta_i = 2\pi i/k, i = 0, 1, \ldots, k-1$ [8]. Clearly, in order to generate circles of varying radii describing the surface of revolution, the number of steps, k, becomes a concerning issue.

Existing methods for modeling potteries mostly start with an initial cylindrical ‘clay’ piece and use deformation techniques, as described in [9,10]. The deformation is based either on shifting the individual points horizontally or on using devices that accept feedback by means of touch-sensation at multiple points [13]. As a result, the user has a direct control over the geometry of the shape to be constructed, which, however, poses difficulties to represent the surface mathematically or to store it efficiently. Some of these practices may be seen in [6,7,12].

Features of Our Method: The proposed algorithm of *digital wheel-throwing* works purely in the digital domain and banks on a few primitive integer computations only, wherein lies its strength and novelty. Given a digital generatrix as an irreducible digital curve segment, the digital surface produced by it is both connected and irreducible in digital-geometric sense. This, in turn, ensures its successful rendition with a realistic finish that involves conventional processing of quad decomposition, texture mapping, illumination, etc. Another distinguishing feature is that the whole surface can be represented by and stored as a sequence of few control points and the axis of rotation. For, storing these control points in sequence is sufficient to reconstruct the digital generatrix, if it is defined as a sequence of cubic B-splines. The method is robust and efficient, and guarantees easy implementation will all relevant graphic features. The rendered visualization is found to be absolutely free of any bugs or degeneracies, whatsoever may be the zoom factor. Producing a monotone or a non-monotone digital surface of revolution is feasible without destroying its digital connectivity and irreducibility by respective input of a monotone or a non-monotone digital generatrix. To create exquisite digital products resembling on-the-shelf potteries in toto, a double-layered generatrix can be used, which is its ultimate benefit.

2 Proposed Method

A digital curve segment $\mathcal{G} := \{p_i : i = 1, 2, \ldots, n\}$ is a finite sequence of digital points (i.e., points with integer coordinates) [11], in which two points $p(x, y) \in \mathcal{G}$

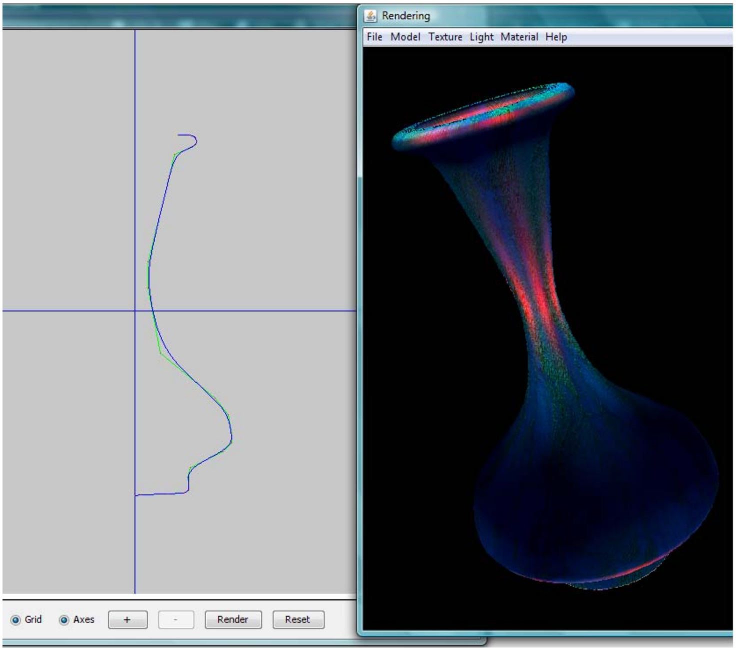


Fig. 1. A snapshot of our algorithm. Left: digital generatrix. Right: a flowerpot.

and $p'(x', y') \in \mathcal{G}$ are *8-neighbors* of each other, if and only if $\max(|x - x'|, |y - y'|) = 1$. To ensure that $\mathcal{G}$ is *simple*, *irreducible*, and *open-ended*, each point $p_i (i = 2, 3, \ldots, n-1)$ should have exactly two neighbors, and each of p_1 and p_n should have one, from $\mathcal{G}$. Thus, the chain code of a point $p_i \in \mathcal{G}$ w.r.t. its previous point $p_{i-1} \in \mathcal{G}$ is given by $c_i \in \{0, 1, 2, \ldots, 7\}$ [5]. In order to generate a digital surface of revolution, $\mathcal{S}$, we consider a digital curve segment $\mathcal{G}$ as the *digital generatrix*, as shown in Fig. 1. The input digital generatrix may be taken either as a sequence of chain codes or as a sequence of control points. Given $m(\geq 4)$ control points as input, the digital generatrix is considered as the digital irreducible curve segment that approximates the sequence of $m - 3$ uniform non-rational cubic B-spline segments interpolating the sequence of these control points [4,8]. The reasons for B-spline interpolation of the generatrix is its unique characteristic of having both parametric and geometric continuities, which ensure the smoothness and the optimal exactness of the fitted curve against the given set of control points. In addition, a local change (insertion/deletion/repositioning) of control point(s) has only a local effect on the shape of the digital generatrix (and a local effect on the generated surface, thereof). This, in fact, simulates the local effect of a "potter's hand" rolling and maneuvering the "clay" on his rotating wheel (Fig. 2).

The sets $N_4(p) := \{(x', y') : |x - x'| + |y - y'| = 1\}$ and $N_8(p) := \{(x', y') : \max(|x - x'|, |y - y'|) = 1\}$ corresponding to a point $p(x, y) \in \mathbb{Z}^2$ define the respective 4-neighborhood (4N) and 8-neighborhood (8N) of p. Similarly, if

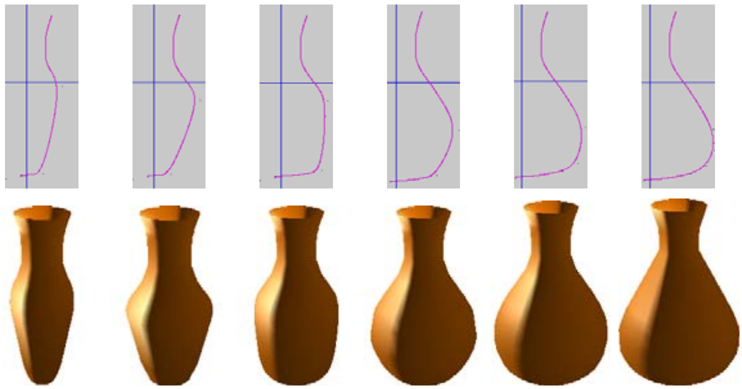


Fig. 2. Simulating the local effect of a "potter's hand" by inserting control points

$p(x, y, z) \in \mathbb{Z}^3$, then $N_6(p) := \{(x', y', z') : |x - x'| + |y - y'| + |z - z'| = 1\}$ and $N_{26}(p) := \{(x', y', z') : \max(|x - x'|, |y - y'|, |z - z'|) = 1\}$ define the respective 6N and 26N of p. Each point in $N_k(p)$ is said to be a *k-neighbor* of p; $k \in \{4, 8\}$ if $p \in \mathbb{Z}^2$ and $k \in \{6, 26\}$ if $p \in \mathbb{Z}^3$. We consider $k = 8$ and $\bar{k} = 4$ in $\mathbb{Z}^2$, and $k = 26$ and $\bar{k} = 6$ in $\mathbb{Z}^3$. Thus, in a (finite) digital set $S \subset \mathbb{Z}^2(\mathbb{Z}^3)$, two points $p \in S$ and $q \in S$ are *k-connected* if and only if there exists a sequence $\langle p := p_0, p_1, \ldots, p_n := q\rangle \subseteq S$ such that $p_i \in N_k(p_{i-1})$ for $1 \leqslant i \leqslant n$. The digital set S is said to be a *connected digital set* if and only if every pair of points in S is k-connected. If a connected digital set S be such that $\bar{S} := \mathbb{Z}^2 \setminus S(\mathbb{Z}^3 \setminus S)$ contains at least two points, $\bar{p} \in \bar{S}$ and $\bar{q} \in \bar{S}$, which are not $\bar{k}$-connected, then S is said possess a *hole*. A hole H is a finite and maximal subset of $\bar{S}$ such that every pair of its points are $\bar{k}$-connected and none of its points is $\bar{k}$-connected with any point from $\bar{S} \setminus H$. If $S \subset \mathbb{Z}^2(\mathbb{Z}^3)$ be a connected digital set containing exactly one hole, namely H, then S forms a *closed digital curve* (*surface*), of possibly arbitrary thickness. A closed digital curve (surface) S is *irreducible* (i.e., of unit thickness) if and only if exclusion of *any* point p from S (and its inclusion to $\bar{S}$) gives rise to a $\bar{k}$-connected path from each point of H to each point of $\bar{S} \setminus H$.

If L be a real line that partitions an irreducible and closed digital curve S into two or more (k-connected) components such that all points in each component are on the same side of L or lying on L, and no two components share a common point, then each such component of S becomes an *open digital curve* (irreducible segment). Partitioning (iteratively) of an open digital curve segment, in turn, gives rise to more and more smaller open segments. Clearly, if the closed digital curve S be such that any horizontal (vertical) line L_y (L_x) always partitions S into at most two components, then S is a y-monotone (x-monotone) digital curve. Similarly, partitioning an irreducible and closed digital surface S by a plane Π produces two or more components such that all points in each component are on the same side of Π or lying on Π, and no two components share a common point. Each component obtained by partitioning S with Π is an *open digital surface* (irreducible). If the closed digital surface S be such that any plane Π_y

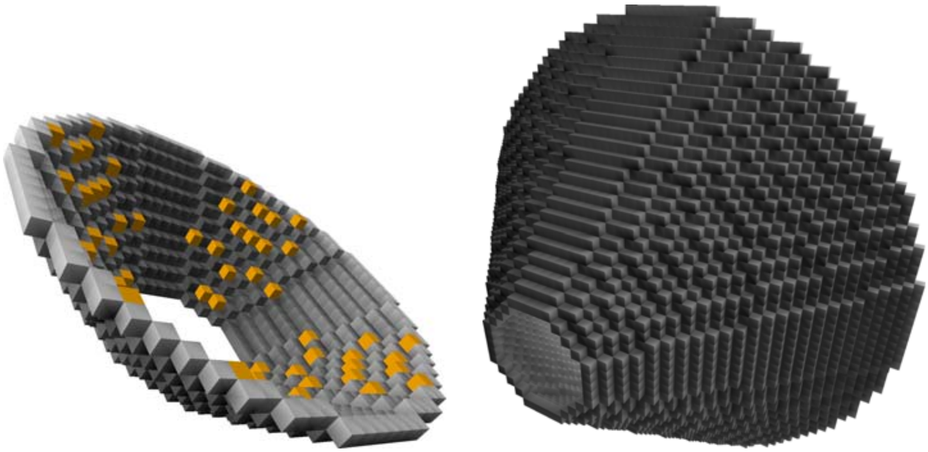


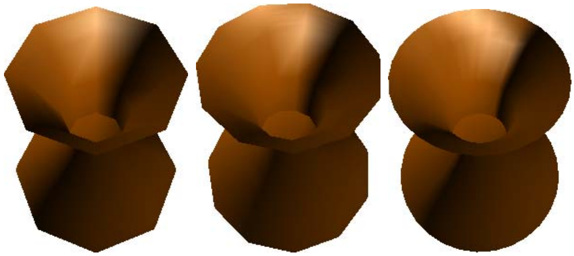
Fig. 3. Shown in yellow are the missing voxels (left), which are detected and included to successfully create the digitally connected and irreducible surface (right)

(Π_x, Π_z) orthogonal to y-axis (x-, z-axis) always partitions S into at most two components, then S is a y-monotone (x-, z-monotone) digital surface. It may be mentioned here that, if the generatrix be an open, irreducible, and y-monotone digital curve segment, C, then the digital surface of revolution, S, produced by our algorithm is also a y-monotone digital surface.

2.1 Algorithm for Digital Wheel-Throwing

We generate a wheel-thrown piece by revolving the digital generatrix $\mathcal{G} := \{p_i : i = 1, 2, \ldots, n\}$ about an axis of revolution given by $\alpha : \langle z = -c, x = a\rangle$, where a and c are two positive integers. In order to achieve this, for each digital point $p_i \in \mathcal{G}$, we construct the digital circle $\mathcal{C}^{\mathbb{Z}}{}_i$ by revolving p_i around the specified axis of revolution, α. Depending on whether the radius and the center of a digital circle are real or integer values, several definitions of digital circles may be seen in the literature [2]. If the radius be $r \in \mathbb{Z}^+$ and the center be $c = o(0, 0)$, then the first octant of the corresponding digital circle is given by $\mathcal{C}_1^{\mathbb{Z}}(o, r) = \left\{(i, j) \in \mathbb{Z}^2 : 0 \leq i \leq j \leq r \wedge |j - \sqrt{r^2 - i^2}| < \frac{1}{2}\right\}$, and so the entire digital circle is $\mathcal{C}^{\mathbb{Z}}(o, r) = \left\{(i, j) : \ \{|i|, |j|\} \in \mathcal{C}_1^{\mathbb{Z}}(o, r)\right\}$. With $p(i_p, j_p) \in \mathbb{Z}^2$ as center, $\mathcal{C}^{\mathbb{Z}}(p, r) = \left\{(i + i_p, j + j_p) : \ (i, j) \in \mathcal{C}^{\mathbb{Z}}(o, r)\right\}$.

As $\mathcal{G}$ is irreducible in nature, two consecutive points p_i and p_{i+1} have their distances, namely r_i and r_{i+1}, measured from α, differing by at most unity. If their distances from α are same, then it is easy to observe that the surface $\mathcal{C}^{\mathbb{Z}}{}_i \cup \mathcal{C}^{\mathbb{Z}}{}_{i+1}$ is digitally connected and irreducible. The problem arises when p_i and p_{i+1} have their respective distances from α differing by unity. Then there may arise some missing voxels trapped between $\mathcal{C}^{\mathbb{Z}}{}_i$ and $\mathcal{C}^{\mathbb{Z}}{}_{i+1}$, which results in digital disconnectedness in the surface $\mathcal{C}^{\mathbb{Z}}{}_i \cup \mathcal{C}^{\mathbb{Z}}{}_{i+1}$, as shown in Fig. 3. Detection



(a) $k = 4$ (b) $k = 7$ (c) $k = 20$

Fig. 4. Approximating generating digital circles by regular $2k$-gons

of these missing voxels is performed to achieve a digitally connected surface, namely $\mathcal{S}_\mathcal{G} := \mathcal{C}^\mathbb{Z}{}_1 \cup \mathcal{C}^\mathbb{Z}{}_2 \cup \ldots \cup \mathcal{C}^\mathbb{Z}{}_n$, as follows.

Case 1 ($r_{i+1} > r_i$): While generating the digital circle $\mathcal{C}^\mathbb{Z}{}_{i+1}$ parallel to the zx-plane corresponding to the point $p_{i+1} \in \mathcal{G}$, there is either an east (E) transition or south-east (SE) transition from the current point $q(x, y, z)$ in Octant 1 ($z \leq x \leq r_{i+1}$) [4]. If we take the respective projections, $\mathcal{C}^{\mathbb{Z}'}_i$ and $\mathcal{C}^{\mathbb{Z}'}_{i+1}$, of $\mathcal{C}^\mathbb{Z}{}_i$ and $\mathcal{C}^\mathbb{Z}{}_{i+1}$ on the zx-plane, then $\mathcal{C}^{\mathbb{Z}'}_i$ and $\mathcal{C}^{\mathbb{Z}'}_{i+1}$ become concentric with their radii differing by unity. As $r_{i+1} > r_i$, the cumulative run-length $\sum_{j=0}^{k} \lambda_{i+1}^{(j)}$ of $\mathcal{C}^{\mathbb{Z}'}_{i+1}$ in Octant 1 is either same as the corresponding cumulative run-length $\sum_{j=0}^{k} \lambda_i^{(j)}$ of $\mathcal{C}^{\mathbb{Z}'}_i$ or greater by unity [2]. Hence, a "missing voxel" between $\mathcal{C}^\mathbb{Z}{}_i$ and $\mathcal{C}^\mathbb{Z}{}_{i+1}$ is formed only if there is a transition towards SE (a change in run, thereof) from a point/pixel in $\mathcal{C}^{\mathbb{Z}'}_{i+1}$ giving rise to a "missing pixel" between $\mathcal{C}^{\mathbb{Z}'}_i$ and $\mathcal{C}^{\mathbb{Z}'}_{i+1}$. We detect such missing pixels by determining whether or not there is a "miss" during each SE transition for $\mathcal{C}^{\mathbb{Z}'}_{i+1}$. More precisely, if the point next to the current point $q'(x, z) \in \mathcal{C}^{\mathbb{Z}'}_{i+1}$ in Octant 1 is $(x-1, z+1)$ and the point $(x-1, z)$ does not belong to $\mathcal{C}^{\mathbb{Z}'}_i$, then we include the point $(x-1, y, z)$ in $\mathcal{C}^\mathbb{Z}{}_{i+1}$ between $(x, y, z) \in \mathcal{C}^\mathbb{Z}{}_{i+1}$ and $(x-1, y, z+1) \in \mathcal{C}^\mathbb{Z}{}_{i+1}$.

Case 2 ($r_{i+1} < r_i$): If the point next to the current point $q'(x, z) \in \mathcal{C}^{\mathbb{Z}'}_{i+1}$ in Octant 1 is $(x-1, z+1)$ and the point $(x, z+1)$ does not belong to $\mathcal{C}^{\mathbb{Z}'}_i$, then we include the point $(x, y, z+1)$ in $\mathcal{C}^\mathbb{Z}{}_{i+1}$ between $(x, y, z) \in \mathcal{C}^\mathbb{Z}{}_{i+1}$ and $(x-1, y, z+1) \in \mathcal{C}^\mathbb{Z}{}_{i+1}$.

2.2 Creating Textured/Painted Potteries

As explained in Sec. 2.1, the surface $\mathcal{S}_\mathcal{G}$ is generated from the digital generatrix $\mathcal{G}$ as an ordered set of voxels. For each digital circle $\mathcal{C}^\mathbb{Z}{}_i$ corresponding to each voxel $p_i \in \mathcal{G}$, the voxels inclusive of the missing voxels are generated in a definite order, starting from Octant 1 and ending at Octant 8. All the circles, namely $\mathcal{C}^\mathbb{Z}{}_1, \mathcal{C}^\mathbb{Z}{}_2, \ldots, \mathcal{C}^\mathbb{Z}{}_n$, in turn, are also generated in order. The above ordering to represent a wheel-thrown piece helps to map textures in a straightforward way. We map adjacent and equal-area rectangular parts from the texture image to

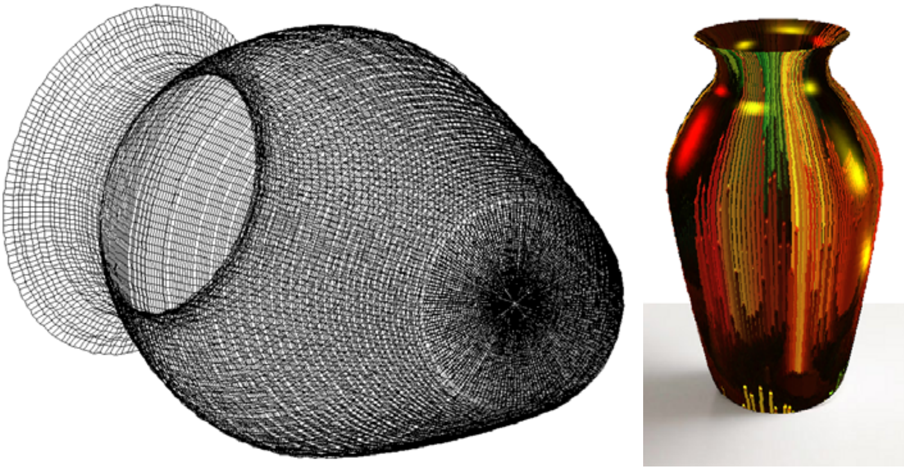


Fig. 5. Quad decomposition of a wheel-thrown digital vase for texturing

each of the quads. To do this, each digital circle is approximated as a regular polygon. Clearly, there lies a trade-off between the number of vertices of the approximate polygon and the rendering speed, since a coarse approximation gives a polyhedral effect (Fig. 4). The nature of approximation error has been studied in a recent work [1], which shows that if we have polygons with $k \geq 20$ or so, then for $r \leq 100$, we have 2% or less error. For higher radii, of course, we should increase k accordingly to have the desired error.

In order to map the adjacent square portions of a texture image to the corresponding adjacent quads representing the digital surface of a pottery, the number of sides, χ, in the polygonal approximation is kept same for all the digital circles corresponding to the digital points in $\mathcal{G}$. To do this, the distances of all the points in $\mathcal{G}$ from the axis of revolution, α, are computed, from which their maximum, $r_{\max}$, is obtained. We use $r_{\max}$ to determine the value of χ. Since the distance between two adjacent digital points in $\mathcal{G}$ is either 1 or $\sqrt{2}$, we determine the value of χ by approximating the circle of radius $r_{\max}$ with a regular polygon having side-length d. Thus, $\chi = \left\lfloor \frac{\pi}{\sin^{-1}\left(\frac{d}{2r_{\max}}\right)} + \frac{1}{2} \right\rfloor$.

Now, for each point on $\mathcal{G}$, we approximate the circle of revolution with a regular polygon with χ sides. For every pair of adjacent points in $\mathcal{G}$, we join the corresponding vertices of the approximate polygon corresponding to the circle of revolution. In this way, we obtain a simple quad-decomposition of the surface of revolution. For a faster rendering, we can approximate the digital generatrix/B-spline by selecting alternate points (or 1 in every $d > 1$). (In our experiments, we have taken $d = 4$.) In this way, the digital continuity of the original image is easily maintained on the texture map, with satisfactory results. It may be noted that, only for such an artistic finish of the digital pottery, floating-point computations have been resorted to. Fig. 5 demonstrates the quad decomposition of a wheel-thrown vase and the resultant texture mapping. To create realistic potteries (one shown in the front page) having thick walls, we need to supply (connected and irreducible) digital generatrices having two parallel segments — one for the outer wall and another for the inner wall including the rim top.

3 Experiments and Results

We have developed the software in Java3D^{TM} API, version 1.5.2. Snapshots of a few potteries are given in Fig. 5 and in the front page, and some statistical figures and CPU times are presented in Table 1. The total CPU time required to create a digital pottery is for constructing the digital surface $\mathcal{S}_{\mathcal{G}}$ from a given digital generatrix, $\mathcal{G}$, and for quad-decomposition followed by texture mapping coupled with necessary rendering. The number of voxels constituting $\mathcal{S}_{\mathcal{G}}$ not only increases with the length of $\mathcal{G}$ and the distance of $\mathcal{G}$ from the axis of revolution, α, but it also depends on the shape of $\mathcal{G}$. For, if the each pair of consecutive points comprising $\mathcal{G}$ has the radius difference unity, then the number of missing voxels increases, which, in turn, increases the total number of voxels defining $\mathcal{S}_{\mathcal{G}}$.

Table 1. Some statistical figures and CPU times for some digital potteries

♯Control Points	♯Points in original generatrix	♯Points in approximate generatrix	♯Sides of polygon	♯Quads	Time (millisecs.)
4	103	26	163	4075	475
7	227	57	185	10360	805
10	508	127	87	10962	792
15	599	150	113	16837	1009
20	592	148	186	27342	1624

4 Conclusion and Future Possibilities

We have shown how wheel-throwing can be done using certain efficient techniques of digital geometry for creating various digital potteries, which satisfy both mathematical and aesthetic criteria. Future prospects include generating irregular/bumpy surfaces and sub-surfaces of revolution, and in defining various morphological/set-theoretic operations on digital surfaces of revolution so as to generate more interesting potteries out of the "digital wheel".

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