

# Reliable Communications Using Multi-layer Transmission

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**Abstract.** In this paper, we propose a MIMO approach for packet combining in hybrid automatic repeat request (HARQ) protocols using single-carrier multi-layer transmission over block fading channels. Based on this model, the problem of the optimization of the linear superposition coefficients is briefly addressed.

**Keywords:** Hybrid-ARQ, superposition coding.

## 1 Introduction

Multi-layer transmission is an efficient technique to improve data throughput when no channel state information (CSI) is available at the transmitter [1]. This is a promising technique for future extensions of the third generation mobile systems 3GPP-LTE (Long-Term-Evolution) standard [2]. In multi-layer transmission, multiple coded packets, each of which is referred to as a *layer*, are simultaneously transmitted using linear superposition of the modulated packets. Each layer is allocated a transmission rate and a transmitting power under the constraint of a fixed average total power per transmission. The performance of a multi-layered transmission system depends on the efficiency of the receiver in separating the different layers taking into account the effect of the channel on the transmitted signal. Reliable data communication systems usually implement HARQ protocols in order to combat against errors introduced by the communication channel. The design of HARQ protocols for multi-layer transmission must take in the account layered structure of the signal in order to help the receiver in layer separation and decoding. In this paper,<sup>1</sup> we address the receiver structure and the design of the HARQ protocols for better system performance.

## 2 Multi-layer Transmissions

We consider a multi-layered transmission where the transmitted signal  $\mathbf{x}$  is formed by linear superposition of  $K$  modulated and interleaved packets  $\mathbf{s}^{(k)}$ ,

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$$\mathbf{x} = \sum_{k=1}^K a^{(k)} \mathbf{s}^{(k)}, \quad (1)$$

with  $a^{(k)} = \rho_k \exp(j\theta_k)$  where  $\rho_k$  is a scaling factor and  $\theta_k$  is a phase-shift ( $\theta_k \in [0, 2\pi]$ ). The scaling parameters  $\rho_k$  determine the allocated power to each layer under the constraint of a unit average transmitted power, i.e.  $\sum_k (\rho_k)^2 = 1$ . Whereas, the phase-shift angles  $\theta_k$  determine the shape of the combined constellation. In each layer, an information data packet  $\mathbf{d}^{(k)}$ , including CRC bits for error detection, is first encoded by a forward error correction code (FEC) to obtain the coded sequence  $\mathbf{c}^{(k)}$  having  $2N$  coded bits. Different coding rates may be used for each layer. However, we assume in this paper that the same code is used for all layers. This simplifies the system complexity by using the same encoder and decoder for all layers. The coded sequence is then interleaved using a pseudo-random interleaver  $\Pi^{(k)}$  and mapped to a sequence  $\mathbf{s}^{(k)}$  of  $N$  complex symbols using a Gray-mapped QPSK modulation.

We consider a single-input single-output transmission system through a flat fading channel. The received signal at the instant  $t$  is given by

$$\mathbf{y}_t = \sqrt{\gamma} h_t \mathbf{x}_t + \mathbf{n}_t, \quad (2)$$

where  $\sqrt{\gamma}$  is average transmitted power,  $h_t$  is the complex channel gain and  $\mathbf{n}_t$  is the noise vector, with elements that are i.i.d. complex Gaussian random variables with zero mean and unit variance. We assume that the channel gain  $h$  remains constant during the period of one block transmission and may change from one block to another depending on the channel model. After the decoding of the received signal by the receiver, an ACK signal is returned to the transmitter for each correctly decoded layer through an error free feedback channel, whereas a NACK signal is returned for layers in error. In the case of successful decoding of all layers, the transmitter sends another block containing new  $K$  packets. Otherwise, the transmitter responds resending a block of  $K$  packets including the erroneous packets and eventually new packets on the correctly decoded layers. This is the main difference with the rateless coding [3] problem where the retransmission contains retransmitted layers only. The retransmission continues for each packet until the correct reception or a maximum number  $M$  of transmissions per packet have been reached. In the latter case, the packet in error is dropped out from the transmission buffer and an error is declared.

### 3 Equivalent MIMO Channel Model

At the time  $t$ , the received block contains a maximum of  $K$  layers consisting of a combination of retransmitted and new packets. Let  $M_{t,k}$  be the number of transmissions of the  $k$ -th layer at the instant  $t$ . Naturally, for new transmitted layers, we have  $M_{t,k} = 1$ . One can see the multi-layer transmission as a multiple-input single-output (MISO) system with  $K$  transmitting antennas,

$$y_t = \sqrt{\gamma} \sum_{k=1}^K \tilde{h}_t^{(k)} s_t^{(k)}, \quad (3)$$

where  $\tilde{h}_t^{(k)} = a_t^{(k)} h_t$  is the equivalent channel for the  $k$ -th layer by considering the linear coefficients  $\{a^{(k)}\}$  as part of the channel. The received block is initially stored in a buffer of size  $M$  blocks. The buffer contains the current received block and the  $M-1$  previously received signal after removing the contribution of correctly decoded packets in previous transmissions. Thus, the buffered blocks contain undecoded layers only. The buffered block is referred to by the variable  $\underline{\mathbf{y}}_t$ . In fact, we distinguish between three types of undecoded packets: the new packets ( $M_{t,k} = 1$ ), the active packets which have not yet reached the maximum number of transmissions ( $1 < M_{t,k} \leq M$ ), and the dropped packets which had been expired the maximum number of transmissions. We denote by  $K_n$ ,  $K_a$ , and  $K_d$  the number of new, active and dropped packets respectively. Note that  $K_n + K_a = K$ . We regroup the undecoded packets in the same matrix  $\mathbf{S}_t = [\mathbf{s}_1, \dots, \mathbf{s}_{K_u}]^T$ , where  $K_u = K_n + K_a + K_d$ . The first  $K_a$  lines include the active packets, next the  $K_n$  new packets, and then the  $K_d$  dropped packets. The equivalent MIMO model between the undecoded packets and the buffered signals can be written as

$$\underline{\mathbf{Y}}_t = \sqrt{\gamma} \mathbf{H}_t \mathbf{S}_t + \mathbf{N}_t, \quad (4)$$

where  $\underline{\mathbf{Y}}_t = [\underline{\mathbf{y}}_t, \dots, \underline{\mathbf{y}}_{t-M+1}]^T$ ,  $\mathbf{N}_t = [\mathbf{n}_t, \dots, \mathbf{n}_{t-M+1}]^T$ ,  $\mathbf{H}_t$  is the  $M \times K_u$  equivalent channel response for the undecoded packets defined by their elements as

$$[\mathbf{H}]_{i,j} = \varepsilon_{i,j} \tilde{h}_{t-i+1}^{(k_j)}, \quad (5)$$

for  $1 \leq i \leq M$ , and  $1 \leq j \leq K_u$ , where  $k_j$  is the number of layer used to transmit the  $j$ -th packet.  $\varepsilon_{i,j} = 1$  if an the undecoded packet  $\mathbf{s}_j$  was transmitted at the time  $(t - i + 1)$ , and  $\varepsilon_{i,j} = 0$  otherwise. Since we are interested in the decoding of active layers only, the dropped packets in our MIMO model plays the role of additional noisy transmitting antennas.

Now, having determined the equivalent MIMO channel model for multiple HARQ layered transmissions, we can apply classical methods for MIMO detection in order to separate layers as in [4]. For successive interference cancellation (SIC), the receiver performs a layer by layer detection and decoding in the descending order of the received power per layer. Under the minimum mean square error (MMSE) detection, the  $j$ -th active packet is given by

$$\hat{\mathbf{s}}_j = \mathbf{w}_j^H \underline{\mathbf{Y}}_t, \text{ with } \mathbf{w}_j^H = \sqrt{\gamma} \mathbf{h}_j^H (\mathbf{I}_M + \gamma \mathbf{H}_t \mathbf{H}_t^H)^{-1}, \quad (6)$$

where  $(\cdot)^H$  denotes the complex conjugate transpose and  $\mathbf{h}_j$  is the  $j$ -th column of the matrix  $\mathbf{H}_t$ .

## 4 Linear Layer-Time Coding

In order to determine the best choice for the linear coefficients at the current transmission, we maximize the instantaneous channel capacity  $C_t$  assuming Gaussian source distribution  $C_t = \log_2(\det[\mathbf{I}_M + \gamma \mathbf{H}_t \mathbf{H}_t^H]) = \log_2(\det[\Gamma_t])$ . Applying Hadamard's inequality [5] to  $\Gamma_t$ , we find that the determinant of  $\Gamma_t$  is maximized when the lines of the channel response  $\mathbf{H}_t$  are orthogonal. Since the current transmission does not contain any

dropped packet, and the new transmitted packets are only contained in the current transmission, the orthogonality condition reduces to the orthogonality within the retransmitted packets only

$$\psi_i = h_t h_{t-i}^* \sum_{j=1}^{K_a} \epsilon_{i+1,j} a_t^{(k_j)} (a_{t-i}^{(k_j)})^* = \delta_i , \quad (7)$$

for  $i = 0, \dots, M-1$ . This condition can not be satisfied always depending on the values of  $\epsilon_{i+1,j}$ . However, minimizing the total squared correlation  $\sum_i |\psi_i|^2$  leads to an optimal solution. In addition, the condition (7) does not specify the power repartition between the active packets and the new packets. When the channel gain is unknown to the transmitter, this problem is subject to some trade-off between the dropping rate and the data throughput in the system. For example, when the total available power is allocated to the active packets, i.e. no new packets are transmitted until the complete decoding of the active packets, this would reduce the frame error rate in the system at the expense of reduced throughput for high signal to noise ratio. This point remains subject for future works.

## 5 Conclusions

We presented in this paper a MIMO approach for packet combining in HARQ protocols using multi-layer transmission. This model takes into the account the colored nature of the noise for a SIC receiver and the effect of the previously dropped packets on the current decoding. Moreover, this approach allows optimizing the linear superposition coefficients for better HARQ performance.

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