Credit-Token Based Inter-cell Radio Resource Management: A Game Theoretic Approach

Chun-Han Ko and Hung-Yu Wei

Department of Electrical Engineering,
National Taiwan University, Taipei, Taiwan
b91901127@ntu.edu.tw,
hywei@cc.ee.ntu.edu.tw

Abstract. In this paper, a radio resource sharing scheme for wireless cellular network is investigated to achieve efficiency and fairness among base stations. We propose a credit-token based spectrum sharing algorithm. Game theory is utilized to formulate and analyze the proposed spectrum sharing algorithm. We first discuss the simplest two-base-station game through a graphical method to gain insights for the solution. Afterwards, the Nash Equilibrium of the n-base-station game is derived and the spectrum allocation at the Nash equilibrium is shown to be unique. Several desirable properties, including allocative efficiency, Pareto optimality, weighted max-min fairness, and weighted proportional fairness, are proved to be attained at the Nash equilibrium. Furthermore, we design a strategy-proof spectrum allocation mechanism based on the proposed spectrum sharing algorithm so that truthful declarations of spectrum demands maximize the performance in each cell.

Keywords: wireless cellular network, spectrum sharing, credit token, game theory.

1 Introduction

Dynamic spectrum sharing has been a promising approach to increase the efficiency of spectrum usage [1]. In the realm of dynamic spectrum sharing, many researchers are interested in introducing pricing mechanisms to further achieve efficient and fair spectrum utilization [2,3,4]. Credit token is one of such possible pricing solutions. The concept of credit token and and its utilization in dynamic spectrum sharing are first introduced in [4]. Credit token is similar to money except that credit token can be frozen but cannot be exchanged. In IEEE 802.22 standard, credit token is also used in the self-coexistence mechanism in the MAC protocol [5].

Recently, game theory has been applied to model dynamic spectrum sharing among BSs. S. Sengupta *et al.* applied minority game theory to investigate the problem that whether a BS should stay at the present channel or switch to another channel [6]. They showed a mixed strategy Nash equilibrium existed and the mixed strategy space performed better than the pure strategy space

N. Bartolini et al. (Eds.): QShine/AAA-IDEA 2009, LNICST 22, pp. 663–678, 2009.

in achieving optimal solution. D. Gao *et al.* modeled the dynamic renting and offering mechanism as a progressive second price auction [7]. The utilization of this auction mechanism had a major benefit that BSs would make their requests truthfully. D. Niyato *et al.* formulated the transaction of spectrum bands between licensed users and BSs by a sealed-bid double auction [8]. They also introduced a pricing mechanism to model the service between BSs and users. Nash equilibrium was found through a numerical method.

In this paper, we aim to find a game theoretic solution for inter-cell radio resource management. We propose a credit-token based spectrum sharing algorithm which comprises mechanisms of spectrum renting, offering, and contention. By applying game theory to formulate the spectrum sharing problem, we confirm that a Nash equilibrium always exists and the spectrum allocation at the Nash equilibrium is always unique. Several desirable properties, including allocative efficiency, Pareto optimality, weighted max-min fairness, and weighted proportional fairness, are attained at the Nash equilibrium. Finally, extended from the spectrum sharing algorithm, we devise a strategy-proof, efficient and fair spectrum allocation mechanism to adopt in the general case that BSs' max spectrum demands are private information.

2 Spectrum Sharing Scheme

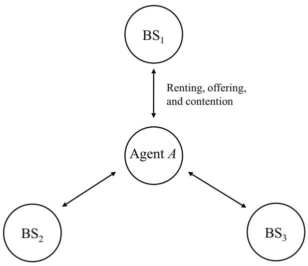
2.1 System Model

The system we consider consists of an agent, A, and n BSs, BS_i for $i = 1, 2, ..., n$. Agent A, serving as a marketplace, manages resource transactions among all BSs. Agent A also offers free spectrum using time O. (If O is less than zero, "offering O" means "retrieving $-O$.") Each BS_i has a single orthogonal spectrum band, spectrum using time T, a credit token budget B_i, and a max traffic demand x_i (in time) additional to T. All of these are assumed to be public information. Figure 1(a) is an illustration of a system of Agent A and three BSs. Figure 1(b) is the corresponding max additional traffic demands. The notations are summarized in Table 1. In the rest parts of the paper, we will use "spectrum" to denote spectrum using time for short.

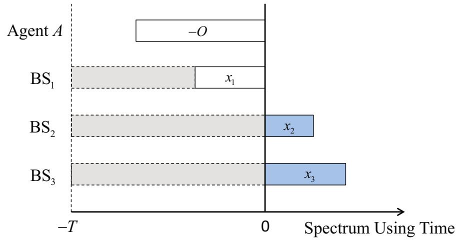
2.2 Credit-Token Based Spectrum Sharing Algorithm

We propose a credit-token based spectrum sharing algorithm. The algorithm has two phases: a spectrum renting-and-offering phase and a spectrum contention phase. We assume each BS_i will use credit tokens for spectrum acquisition and spectrum protection.

Initially, Agent A broadcasts that the renting-and-offering phase starts with spectrum O provided. After hearing the broadcasting, each BS_i will make an acquisition/offering request, y_i, which is the spectrum it claims to acquire if $y_i > 0$ or to offer if $y_i < 0$. Each BS_i is accordingly referred to as an acquirer or an offeror. As each BS_i makes its spectrum request, an assumption is adopted that



(a) System Diagram



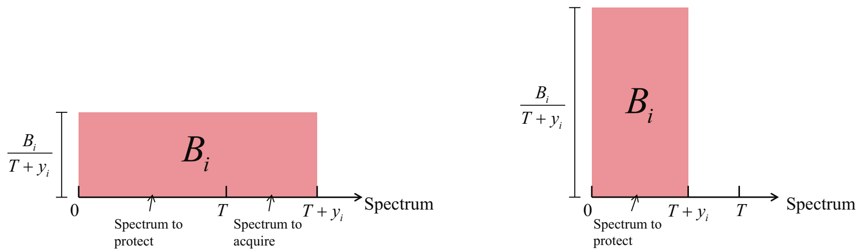
(b) Max Additional Traffic Demands

Fig. 1. System of Agent A and Three BSs

every unit of the spectrum BS_i wants to acquire, $[y_i]^+$, and of the spectrum BS_i wants to protect, $T - [-y_i]^+$, is equally important. Therefore the credit tokens should be fairly allocated. The unit spectrum acquisition price and the unit spectrum protection price are then both equal to $p_i\left(y_i\right) = \frac{B_i}{T+y_i}$. , as depicted in Figure 2. (The function $[\cdot]^+$ gives a non-negative value.) Alternatively, as Agent A receives the acquisition/offering requests from all BSs, it collects the offered spectrum from the offerors and then assigns the collected spectrum and O to the acquirers, in decreasing order of the unit acquisition price, for their requested amount until exhaustion. When multiple acquirers have the same unit acquisition price and there is not enough spectrum for them, the spectrum assigned to them is assumed proportional to their requested amounts.

If the acquirers cannot get enough spectrum in the renting-and-offering phase, the contention phase starts. In the contention phase, Agent A first collects each BS_i's spectrum to protect, $T - [-y_i]^+$. The collected $\left(T - [-y_i]^+\right)$s are then sorted in increasing order of the unit protection price. Afterwards Agent A assigns the sorted $\left(T - [-y_i]^+\right)$s to the acquirers for their inadequate amounts in decreasing order of the unit acquisition price. The assignment ends if the unit protection price is greater than or equal to the unit acquisition price. Finally, Agent A returns the unassigned spectrum back to original BSs. When multiple acquirers have the same acquisition price and there is not enough spectrum for them, we assume the spectrum assigned to them is proportional to their inadequate amounts. When multiple BSs have the same protection price and their spectrum is assigned to others, we assume the assigned spectrum is fairly afforded by these BSs.

After both renting-and-offering and contention phases finish, the credit tokens the acquirers spend for spectrum acquisition are frozen and data transmission begins. We show, in Table 1, the mathematical expressions of the spectrum BS_i acquires or offers in the renting-and-offering phase and the spectrum BS_i acquires or loses in the contention phase. The former is $min\left(y_i, r_i\right)$ and the latter is $min\left(\left[y_i - r_i\right]^+, c_i\right)$. The total spectrum BS_i acquires or loses in both phases is therefore $min\left(y_i, r_i\right) + min\left(\left[y_i - r_i\right]^+, c_i\right)$. However, we will use $min(y_i, t_i)$ to



(a) $y_i > 0$ (b) $y_i \leq 0$

Fig. 2. Unit Spectrum Acquisition and Protection Price of BS_i

Table 1. Notations

T	Spectrum owned by each BS.
O	Spectrum offered by Agent A.
B_i	Credit token budget of BS_i.
x_i	Max traffic demand additional to T of BS_i.
y_i	Spectrum acquisition/offering request of BS_i.
$p_i(y_i)$	Unit spectrum acquisition and protection price of BS_i. $p_i(y_i) = \frac{B_i}{T+y_i}$
$min\,(y_i, r_i)$	Spectrum BS_i acquires or offers in the renting-and-offering phase. $r_i(\mathbf{y}) = \frac{[y_i]^+}{\sum_{j;p_j=p_i} [y_j]^+} \left[O + \sum_{j=1}^{n} [-y_j]^+ - \sum_{j;p_j>p_i} [y_j]^+ \right]^+$
$min\left([y_i - r_i]^+, c_i\right)$	Spectrum BS_i acquires or loses in the contention phase. $c_i(\mathbf{y}) = \frac{[y_i - r_i]^+}{\sum_{j;p_j=p_i} [y_j - r_j]^+} \left[\sum_{j;p_j<p_i} \left(T - [-y_j]^+\right) - \sum_{j;p_j>p_i} [y_j - r_j]^+ \right]^+ - \left(T - [-y_i]^+\right) + \left[\left(T - [-y_i]^+\right) - \frac{T - [-y_i]^+}{\sum_{j;p_j=p_i} \left(T - [-y_j]^+\right)} \left[- \sum_{j;p_j<p_i} \left(T - [-y_j]^+\right) + \sum_{j;p_j>p_i} [y_j - r_j]^+ \right]^+ \right]^+$
$min\,(y_i, t_i)$	Spectrum BS_i acquires or loses in both phases. $min\,(y_i, t_i) = min\,(y_i, r_i) + min\left([y_i - r_i]^+, c_i\right)$ $t_i(\mathbf{y}) = \frac{[y_i]^+}{\sum_{j;p_j=p_i} [y_j]^+} \left[O + \sum_{j;p_j=p_i} [y_j]^+ - \sum_{j;p_j \geq p_i} y_j + \sum_{j;p_j<p_i} T \right]^+ - T + \left[\left(T - [-y_i]^+\right) - \frac{T - [-y_i]^+}{\sum_{j;p_j=p_i} \left(T - [-y_j]^+\right)} \left[-O - \sum_{j;p_j=p_i} [y_j]^+ + \sum_{j;p_j \geq p_i} y_j - \sum_{j;p_j<p_i} T \right]^+ \right]^+$
$P_i\,(\mathbf{y})$	Frozen credit tokens of BS_i. $P_i\,(\mathbf{y}) = p_i\,(y_i)\,[min(y_i, t_i)]^+$

represent the total spectrum BS_i acquires or loses in both phases for simplicity. (Due to lack of space, we skip the proof here.)

3 Game Formulation

The problem we want to study is as follows.

Problem. *Given that the original spectrum T, the credit token budget B_i, and the max traffic demand x_i of each BS_i are public information, if the acquisition/offering request y_i is such that $-T \le y_i \le x_i$, how does each BS_i make the acquisition/offering request to increase the spectrum?*

From each BS's perspective, spectrum sharing is intrinsically a game that each BS unitarily optimizes its performance by acquiring or offering spectrum. We utilize game theory to find if there is any steady state, Nash equilibrium, for this the spectrum sharing problem. Game theory is a set of mathematical tools for analyzing interactive decision processes [9]. Three primary components comprise a game: a player set N; a strategy space $S = \prod_{i \in N} S_i$ where S_i, $i \in N$ is player i strategy set; a utility-function set $U = \{u_i(\mathbf{s})\}$, where $u_i(\mathbf{s})$, $i \in N$ is player i's utility under a strategy profile $\mathbf{s} \in S$. In a game, a steady state where no player will unitarily deviate is called a Nash equilibrium [10].

Table 2. Spectrum Sharing Game Model

	$G = (N, Y, U, B, X)$
Player Set N	$N = \{1, 2, ..., n\}$. BSs are the players of the game.
Strategy Space Y	$Y = \prod_{i \in N} Y_i$ and $Y_i = \{y_i : -T \le y_i \le x_i\}$ $\forall i \in N$. We treat BS_i's acquisition/offering request y_i as the strategy.
Utility-function Set U	$U = \{u_i(\mathbf{y})\}$ and $u_i(\mathbf{y}) = min\,(y_i, t_i)$ $\forall i \in N$. Since each BS aims to increase its spectrum, it is reasonable to set the spectrum as the utility. We do not include any pricing term because each BS never receives credit tokens. (Recall that credit tokens can only be frozen.) We also ignore the constant term T for convenience. Each BS's utility is therefore the spectrum it acquires or loses from renting, offering, and contention.
Credit-token-budget Set B	$B = \{B_i\}$
Max Traffic Set X	$X = \{x_i\}$ with $\{p_i(x_i)\}$ in decreasing order. Without losing generality, we assume $\{p_i(x_i)\}$ is sorted in decreasing order.

Definition 1. *A strategy profile* $\mathbf{s}^* = \left(s_i^*, s_{-i}^*\right)$ *is a Nash equilibrium if*

$$u_i\left(\mathbf{s}^*\right) \geq u_i\left(s_i, s_{-i}^*\right) \quad \forall s_i \neq s_i^* \text{ and } \forall i \in N$$

The best response function of any player depicts his best (in term of highest utility) strategy given all possible s_{-i} from other players. A Nash equilibrium can also be defined by best response functions.

Definition 2. $BR_i\left(s_{-i}\right)$ *is the best response function of player* i *if*

$$BR_i\left(s_{-i}\right) = \left\{s_i : \ u_i\left(s_i, s_{-i}\right) \geq u_i\left(s_i', s_{-i}\right), \ \forall s_i' \neq s_i\right\}$$

Definition 3. *A strategy profile* $\mathbf{s}^* = \left(s_i^*, s_{-i}^*\right)$ *is a Nash equilibrium if*

$$s_i^* = BR_i\left(s_{-i}^*\right) \ \forall i \in N$$

By applying game theory, we construct a game model, denoted as G, for the spectrum sharing problem. The game model is shown in Table 2 with each BS's credit token budget and max traffic demand taken into account.

4 Graphical Analysis – Two Players with Same Budget

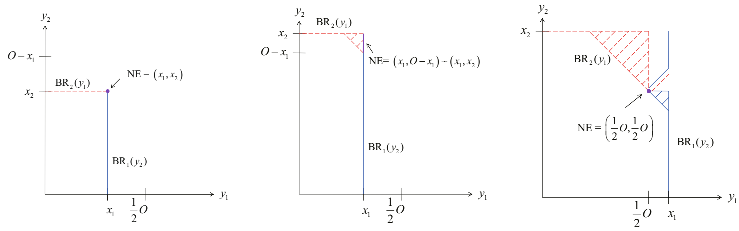
To gain insights for the solution of the general n-player game, we derive the Nash equilibrium in the simplest 2-same-budget-player game through a graphical method. We draw both players' best response functions together. The resulting intersection is the Nash equilibrium. Recall we assume $p_1\left(x_1\right) \geq p_2\left(x_2\right)$. When both players have the same credit token budget, this assumption reduces to $x_1 \leq x_2$. Accordingly, the system traffic demands can be clasified into three cases: $x_1 \leq \frac{O}{2}$ and $x_2 \leq O - x_1$; $x_1 \leq \frac{O}{2}$ and $x_2 > O - x_1$; $x_1 > \frac{O}{2}$ and $x_2 > \frac{O}{2}$.

4.1 Traffic Case 1 - $x_1 \leq \frac{O}{2}$ and $x_2 \leq O - x_1$

As illustrated in Figure 3(a), the best response function of player 1 is uniquely x_1. It means player 1 will always play the unique dominant strategy, $y_1 = x_1$. We call this strategy a dominant one since it always results in higher utility than all other strategies. Also, player 2's best response function is x_2. Player 2 plays the unique dominant strategy, $y_2 = x_2$. The intersection of two best response functions is (x_1, x_2), a unique Nash equilibrium. The corresponding utility profile is (x_1, x_2) as well.

4.2 Traffic Case 2 - $x_1 \leq \frac{O}{2}$ and $x_2 > O - x_1$

We already know player 1 plays the unique dominant strategy, $y_1 = x_1$, when $x_1 \leq \frac{O}{2}$. In Figure 3(b), player 2's best response function is $BR_2\left(y_1\right) = O - y_1 \sim x_2$ which implies that the strategy, $y_2 = x_2$, is player 2's unique dominant strategy. However, it is not meaningful to discuss the concept of dominant strategy



(a) Traffic Case 1 (b) Traffic Case 2 (c) Traffic Case 3

Fig. 3. Best Response Functions and Nash Equilibrium

for player 2 while it is like to play a single-player game. We explain why player 2 is like to play a single-player game. When $x_1 \le \frac{O}{2}$, player 1 plays the unique dominant strategy, $y_1 = x_1$, and acquires x_1 from O. (When x_1 is less than zero, "acquiring x_1" means "offering $-x_1$.") For player 2, it has $(O - x_1)$ remained to acquire without any other player. Therefore player 2 is like to play a single-player game and it can always acquire $(O - x_1)$ by playing y_2 such that $O - x_1 \le y_2 \le x_2$. This fact obviously results in multiple Nash equilibria. This can also be shown from the intersection of two best response functions, a line segment between $(x_1, O - x_1)$ and (x_1, x_2). It means multiple Nash equilibria, $(x_1, O - x_1 \sim x_2)$, exist. Though multiple Nash equilibria exist, the corresponding utility profile is uniquely $(x_1, O - x_1)$.

4.3 Traffic Case 3 - $x_1 > \frac{O}{2}$ and $x_2 > \frac{O}{2}$

In Figure 3(c), the best response function of player 1 is

$$BR_1(y_2) = \begin{cases} min\,(O - y_2, x_1) \sim x_1 & if\ y_2 \le \frac{O}{2} \\ y_2^- & if\ \frac{O}{2} < y_2 \le x_1 \\ x_1 & if\ x_1 < y_2 \end{cases}$$

and the best response function of player 2 is

$$BR_2(y_1) = \begin{cases} min\,(O - y_1, x_2) \sim x_2 & if\ y_1 \le \frac{O}{2} \\ y_1^- & if\ \frac{O}{2} < y_1 \end{cases}$$

We see neither player 1 nor player 2 has dominant strategy. The intersection of two best response functions is $\left(\frac{O}{2}, \frac{O}{2}\right)$, a unique equal-strategy Nash equilibrium. The corresponding utility profile is $\left(\frac{O}{2}, \frac{O}{2}\right)$.

We summarize the observations as follows. These observations, playing the essential roles in the two-same-budget-player game, can be extended in the general n-different-budget-player game.

1. Condition for unique dominant strategies: When $x_1 \le \frac{O}{2}$, player 1 plays the unique dominant strategy, $y_1 = x_1$. When $x_2 \le O - x_1$, player 2 plays the unique dominant strategy, $y_2 = x_2$.

2. Existence of a Nash equilibrium: A Nash equilibrium always exists in all cases.
3. Condition for multiple Nash equilibria: The only case where multiple Nash equilibria exist is $x_1 \leq \frac{O}{2}$ and $x_2 > O - x_1$. We have explained that because player 2 is like to play a single-player game with $(O - x_1)$ offered, it can always acquire $(O - x_1)$ by playing $O - x_1 \leq y_2 \leq x_2$. Multiple Nash equilibria, $(x_1, O - x_1 \sim x_2)$, hence exist.
4. Unique utility profile at the Nash equilibrium: Even in the multi-Nash-equilibrium case, the corresponding utility profile is unique.

5 Mathematical Analysis – n Players

In this section, we first extend the two-same-budget-player game to the general n-different-budget-player game, i.e. Game G. The extention is summarized in Table 3. Afterwards, we do formal derivations for the Nash equilibrium of Game G.

Table 3. Summary of Extension

	2-Same-Budget-Player Game	n-Different-Budget-Player Game
Traffic Threshold	$\{\frac{O}{2}, O - x_1\}$	$\{e_{j,-(j-1)}\}$
Traffic Case	$x_1 > \frac{O}{2}$ and $x_2 > \frac{O}{2}$; $x_1 \leq \frac{O}{2}$ and $x_2 > O - x_1$; $x_1 \leq \frac{O}{2}$ and $x_2 \leq O - x_1$	$x_j \leq e_{j,-(j-1)}\ \forall j \in \{1, ..., k\}$, $x_j > e_{j,-k}\ \forall j \in \{k+1, ..., n\}$ where $k \in \{0, N\}$
Nash Equilibrium	$(\frac{O}{2}, \frac{O}{2})$; $(x_1, O - x_1 \sim x_2)$; (x_1, x_2)	$\left(x_1, ..., x_k, e_{k+1}, ..., e_{n,-k}\right)$ if $k \neq n-1$; $\left(x_1, ..., x_{n-1}, e_{n,-(n-1)} \sim x_n\right)$ if $k = n-1$

5.1 Extension from Two-Player Game to n-Player Game

Recall that we have assumed the max traffic demands are such that $\{p_i(x_i)\}$ is ranged in decreasing order. In the two-same-budget-player game, we see there are two traffic thresholds, $\frac{O}{2}$ and $O = x_1$. Accordingly, the traffic can be categorized into three cases: $x_1 > \frac{O}{2}$ and $x_2 > \frac{O}{2}$; $x_1 \leq \frac{O}{2}$ and $x_2 > O - x_1$; $x_1 \leq \frac{O}{2}$ and $x_2 \leq O - x_1$. The corresponding Nash equilibrium is $\left(\frac{O}{2}, \frac{O}{2}\right)$, $(x_1, O - x_1 \sim x_2)$, and (x_1, x_2).

Extended from the two-same-budget-player game, it is reasonably to guess the n- same-budget-player game has the set of n traffic thresholds, $\left\{\frac{-\sum_{l=0}^{j-1} x_l}{n-j+1}\right\}$, where $x_0 = -O$. To further extend to the n-different-budget-player game, we must know what plays the same role as $\frac{-\sum_{l=0}^{k} x_l}{n-k}$ in the n-same-budget-player game.

Definition 4. *For Game G, we define*

$$e_{j,-k} \equiv \frac{B_j}{\frac{1}{n-k}\sum\limits_{l=k+1}^{n} B_l}\left(\frac{-\sum\limits_{l=0}^{k} x_l}{n-k}\right) + \left(\frac{B_j}{\frac{1}{n-k}\sum\limits_{l=k+1}^{n} B_l} - 1\right)T$$

$\forall j \in \{k+1,...,n\}$ *and* $\forall k \in \{0, N\}$*, where* $x_0 = -O$

$e_{j,-k}$ can be interpreted as weighted and translated $\frac{-\sum_{l=0}^{k} x_l}{n-k}$ with the weight $\frac{B_j}{\frac{1}{n-k}\sum_{l=k+1}^{n} B_l}$. The term $-k$ in the subscript indicates that player i, $i \in \{1,...,k\}$, which has already acquired x_i from O, is excluded. When $k = 0$, $e_{j,-0}$ is denoted as e_j for short. Following the definition, there is a corollary stating some properties of $e_{j,-k}$.

Corollary 1. *For Game G, the following statements about* $e_{j,-k}$ *are always true:*

1. $p_j\left(e_{j,-k}\right) = \frac{\frac{1}{n-k}\sum_{l=k+1}^{n} B_l}{T + \frac{-\sum_{l=0}^{k} x_l}{n-k}}$ $\forall j \in \{k+1,...,n\}$.
2. $\sum\limits_{j=1}^{k} x_j + \sum\limits_{j=k+1}^{n} e_{j,-k} = min\left(O, \sum\limits_{j=1}^{n} x_j\right)$ $\forall k \in \{0, N\}$.
3. $x_k \leq e_{k,-(k-1)} \Leftrightarrow x_j \leq e_{j,-(j-1)}$ $\forall j \in \{1,...,k\}$.
4. $x_{k+1} > e_{k+1,-k} \Leftrightarrow x_j > e_{j,-k}$ $\forall j \in \{k+1,...,n\}$.
5. $x_k \leq e_{k,-(k-1)} \Rightarrow p_k(e_{k,-(k-1)}) \geq p_j(e_{j,-k})$ $\forall j \in \{k+1,...,n\}$.

When $B_i = B_j$ $\forall i,j \in N$, the weights for all $e_{j,-k}$ become 1 and $e_{j,-k}$ reduces to $\frac{-\sum_{l=0}^{k} x_l}{n-k}$. It is intuitively to believe that $e_{j,-k}$ play the same roles as $\frac{-\sum_{l=0}^{k} x_l}{n-k}$ in the same-budget case. Hence Game G should have the set of n traffic thresholds, $\left\{e_{j,-(j-1)}\right\}$. Besides, we can classify the traffic into $(n+1)$ cases where the $(k+1)$-th case, $k \in \{0, N\}$, is $x_k \leq e_{k,-(k-1)}$ and $x_{k+1} > e_{k+1,-k}$. From Corollary 1.3 and 1.4, the $(k+1)$-th case can equivalently represented as $x_j \leq e_{j,-(j-1)}$ $\forall j \in \{1,...,k\}$ and $x_j > e_{j,-k}$ $\forall j \in \{k+1,...,n\}$.

Definition 5. *For Game G and* $\forall k \in \{0, N\}$*, we define*

$$\mathit{Traffic}_k \equiv x_j \leq e_{j,-(j-1)} \quad \forall j \in \{1,...,k\} \text{ and } x_j > e_{j,-k}\ \forall j \in \{k+1,...,n\}$$

The Nash equilibrium under $\mathit{Traffic}_k$ should be $(x_1,...,x_k,e_{k+1},...,e_{n,-k})$ if $k \neq n-1$, and $\left(x_1,...,x_{n-1},e_{n,-(n-1)} \sim x_n\right)$ if $k = n-1$.

5.2 *n*-Player Game

Before starting, we should mention that we will use u_i and t_i to express $u_i(\mathbf{y})$ and $t_i(\mathbf{y})$ at any given strategy profile $\mathbf{y}$ for short. If we need to compare the results between two different strategy profiles, say (y_i, y_{-i}) and (y_i', y_{-i}), we will distinguish by using u_i' and t_i' to express $u_i(y_i', y_{-i})$ and $t_i(y_i', y_{-i})$. Also, due to lack of space, we will only give proofs of important theorems.

First, Lemma 1 reveals the increasing property of utility functions with respect to strategies.

Lemma 1. *Given that Game G is under Traffic*$_k$*, $k \in \{0, N\}$, the following statements are always true:*

1. $u_i = y_i \ \forall i \in \{1, ..., k\}$.
2. *if* $y_i \leq e_{i,-k}$ *for some* $i \in \{k+1, ..., n\}$, $u_i = y_i$.

Lemma 1.1 shows that u_i, $i \in \{1, ..., k\}$, is an increasing function of y_i under *Traffic*$_k$. Therefore player i can always play $y_i = x_i$ to get the highest utility. In words, player i, $i \in \{1, ..., k\}$, plays the unique dominant strategy, $y_i = x_i$.

Theorem 1. *Given that Game G is under Traffic*$_k$*, $k \in \{0, N\}$, player i, $i \in \{1, ..., k\}$, plays the unique dominant strategy, $y_i = x_i$.*

Recall we have guessed the Nash equilibrium under *Traffic*$_k$ is $(x_1, ..., x_k, e_{k+1}, ..., e_{n,-k})$ if $k \neq n-1$ and $\left(x_1, ..., x_{n-1}, e_{n,-(n-1)} \sim x_n\right)$ if $k = n-1$. To verify our guess is correct, we prove that all other strategy profiles cannot be a Nash equilibrium. The proof is taken into two parts. The first part is to show that $y_i < x_i$ for any $i \in \{1, ..., k\}$ or $y_i < e_{i,-k}$ for any $i \in \{k+1, ..., n\}$ is not in any Nash equilibrium. The other part is to show that $y_i > e_{i,-k}$ for any $i \in \{k+1, ..., n\}$ is not in any Nash equilibrium.

Lemma 2. *For Game G under Traffic*$_k$*, $k \in \{0, N\}$, $y_i < x_i$ for any $i \in \{1, ..., k\}$ or $y_i < e_{i,-k}$ for any $i \in \{k+1, ..., n\}$ is not in any Nash equilibrium.*

Lemma 3. *For Game G under Traffic*$_k$*, $k \in \{0, N\}$ and $k \neq n-1$, $y_i > e_{i,-k}$ for any $i \in \{k+1, ..., n\}$ is not in any Nash equilibrium.*

Combining Lemma 2 and Lemma 3, we have verified that under *Traffic*$_k$, any strategy profile other than $(x_1, ..., x_k, e_{k+1}, ..., e_{n,-k})$ if $k \neq n-1$ and $\left(x_1, ..., x_{n-1}, e_{n,-(n-1)} \sim x_n\right)$ if $k = n-1$ cannot be a Nash equilibrium. In words, only $(x_1, ..., x_k, e_{k+1}, ..., e_{n,-k})$ if $k \neq n-1$ and $\left(x_1, ..., x_{n-1}, e_{n,-(n-1)} \sim x_n\right)$ if $k = n-1$ can be a Nash equilibrium. We therefore check its property and find it a Nash equilibrium.

Theorem 2. *Given Traffic*$_k$*, $k \in \{0, N\}$ and $k \neq n-1$, Game G has the unique Nash equilibrium, $NE_k = (x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$.*

Proof. Given $Traffic_k$, if Game G is at $(x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$, the corresponding utility profile is also $(x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$. For player i, $i \in \{1, ..., k\}$, if it plays $y_i' < x_i$, then $u_i' = y_i' < u_i$. For player i, $i \in \{k+1, ..., n\}$, if it plays $y_i' < e_{i,-k}$, $u_i' = y_i' < u_i$; if it plays, $y_i' > e_{i,-k}$, $u_i' = e_{i,-k}$. Consequently, $(x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$ meets the definition of Nash equilibrium. Since there is no other possible Nash equilibrium, Game G under $Traffic_k$, $k \in \{0, N\}$ and $k \neq n-1$, has the unique Nash equilibrium $(x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$. □

Theorem 3. *Given $Traffic_{n-1}$, Game G has multiple Nash equilibria, $NE_{n-1} = \big(x_1, ..., x_{n-1}, e_{n,-(n-1)} \sim x_n\big)$.*

Proof. When $k = n-1$, it is proved in Theorem 1 that player i, $i \in \{1, ..., n-1\}$, plays the unique dominant strategy $y_i = x_i$. For player n, it is like to play a single-player game with $-\sum_{j=1}^{n-1} x_j$, equivalently $e_{n,-(n-1)}$, offered. Player n can play $e_{n,-(n-1)} \leq y_n \leq x_n$ such that $u_n = e_{n,-(n-1)}$. Hence G has multiple Nash equilibria, $NE_{n-1} = \big(x_1, ..., x_{n-1}, e_{n,-(n-1)} \sim x_n\big)$. □

After deriving the Nash equilibrium, we can easily verify that the utility profile at the Nash equilibrium is always unique. This is drawn by substituting all NE_ks into the utility functions.

Theorem 4. *Given $Traffic_k$, $k \in \{0, N\}$, Game G has the unique utility profile, $U_k^* = (x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$, at the Nash equilibrium NE_k.*

Recall we have set spectrum as utilities for all BSs. In system meaning, the utility profile at the Nash equilibrium represents the spectrum allocation at the Nash equilibrium. Theorem 4, in words, reveals that our spectrum sharing algorithm always results in the unique traffic-dependent spectrum allocation at the Nash equilibrium, $AR^* = U_k^*$ given $Traffic_k$, $k \in \{0, N\}$.

6 Properties at Nash Equilibrium

After deriving the Nash equilibrium, we can proof that the spectrum allocation at the Nash equilibrium meets the criteria of allocative efficiency, Pareto optimality, weighted max-min fairness, and weighted proportional fairness.

Allocative efficiency [11] means that a resource allocation maximizes total utilities over all players. It is regarded as the most optimality since no other allocations can achieve greater social welfare. Pareto optimality [11] is defined as an allocation upon which no player can be made happier (in utility) without making at least one other player less happy. It is true that allocative efficiency always implies Perato optimality. The mathematical definitions of allocative efficiency and Pareto optimality are given as below. To conform with the expressions in our game, we use $\mathbf{y}$ and Y instead of $\mathbf{s}$ and S to represent the strategy profile and the strategy space respectively.

Definition 6. *A resource allocation game is allocatively efficient if the Nash equilibrium is a solution to the optimization problem*

$$max \sum_{i=1}^{n} u_i(\mathbf{y}) \quad s.t.\ \mathbf{y} \in Y$$

Definition 7. *A resource allocation game is Pareto optimal if the Nash equilibrium* $\mathbf{y}^*$ *satisfies*

$$\exists \mathbf{y}' \neq \mathbf{y}^*,\ u_i(\mathbf{y}') > u_i(\mathbf{y}^*) \ \Rightarrow\ \exists j \in N,\ u_j(\mathbf{y}') < u_j(\mathbf{y}^*)$$

An allocation satisfies weighted max-min fairness [12] if it is not possible to increase one player's weighted utility without simultaneously decreasing another player's weighted utility which is already smaller. An allocation exhibits weighted proportional fairness [12] if it maximizes the product of all players' utilities with weights in exponents.

Definition 8. *A resource allocation game is weighted max-min fair with the weights* $\{w_i\}$*, if the Nash equilibrium is a solution to the optimization problem*

$$max\ min \left(\frac{u_1(\mathbf{y})}{w_1}, ..., \frac{u_n(\mathbf{y})}{w_n} \right) \quad s.t.\ \mathbf{y} \in Y$$

Definition 9. *A resource allocation game is weighted proportional fair with the weights* $\{w_i\}$*, if the Nash equilibrium is a solution to the optimization problem*

$$max \prod_{i=1}^{n} u_i(\mathbf{y})^{w_i} \quad s.t.\ \mathbf{y} \in Y$$

Recall we ignore the constant term T when setting spectrum as utilities. While discussing weighted max-min fairness and weighted proportional fairness, we should replace u_i with $(T + u_i)$ $\forall i \in N$; otherwise, the objective functions will not be correctly characterized. We choose $\hat{B}_i = \frac{B_i}{\frac{1}{n}\sum_{j=1}^{n} B_j}$ as the weight for each player i. This is because B_i, mainly influencing player i's priority to acquire and to protect spectrum in system meaning, is the power to increase player i's utility. Also, by showing the range of utility functions in Lemma 4, we can transform the constraints of the optimization problems above from strategy domain into utility domain. Consequently, we can prove the properties by verifying that the utility profile at the Nash equilibrium is a solution to the corresponding optimization problems. We prove the properties of allocative efficiency and weighted max-min fairness here.

Lemma 4. *For game* G *and* $\forall \mathbf{y} \in Y$*, the following statements about utility functions are always true:*

1. $-T \leq u_i(\mathbf{y}) \leq x_i \ \forall i \in N$.
2. $-nT \leq \sum_{i=1}^{n} u_i(\mathbf{y}) \leq min\left(O, \sum_{i=1}^{n} x_i\right)$.

Theorem 5. *Game G is allocatively efficient. Equivalently, the utility profile at the Nash equilibrium is a solution to the optimization problem,*

$$max \sum_{i=1}^{n} u_i \quad s.t. \ -T \leq u_i \leq x_i \ \forall i \in N \ and \ -nT \leq \sum_{i=1}^{n} u_i \leq min\left(O, \sum_{i=1}^{n} x_i\right)$$

Proof. Recall in Theorem 4 that the utility profile under $Traffic_k$, $k \in \{0, N\}$, is $U_k^* = (x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$. Corollary 1.2 shows $\sum_{i=1}^{k} x_i + \sum_{i=k+1}^{n} e_{i,-k} = min\left(O, \sum_{i=1}^{n} x_i\right) \ \forall k \in \{0, N\}$. Therefore we know $\sum_{i=1}^{n} u_i$ is maximized by U_k^* $\forall k \in \{0, N\}$. Game G is allocatively efficient. □

Theorem 6. *Game G is weighted max-min fair with the weights $\left\{\hat{B}_i\right\}$. Equivalently, the utility profile at the Nash equilibrium is a solution to the optimization problem,*

$$max \, min\left(\frac{T+u_1}{\hat{B}_1}, ... \frac{T+u_n}{\hat{B}_n}\right)$$

$$s.t. \ -T \leq u_i \leq x_i \ \forall i \in N \ and \ -nT \leq \sum_{i=1}^{n} u_i \leq min\left(O, \sum_{i=1}^{n} x_i\right)$$

Proof. When Game G is under $Traffic_0$, by substituting U_0^* into the objective function and using Corollary 1.1, we derive

$$\frac{T+u_i}{\hat{B}_i} = \frac{T+e_i}{\hat{B}_i} = \frac{T+\frac{O}{n}}{\frac{1}{n}\sum_{l=1}^{n} B_l}\left(\frac{1}{n}\sum_{l=1}^{n} B_l\right) = T + \frac{O}{n} \quad \forall i \in N \tag{1}$$

$$min\left(\frac{T+u_1}{\hat{B}_1}, ..., \frac{T+u_n}{\hat{B}_n}\right) = min\left(T+\frac{O}{n}, ..., T+\frac{O}{n}\right) = T + \frac{O}{n} \tag{2}$$

Because $\sum_{i=1}^{n} e_i = O$, if $u_j > e_j$ for some player j, there must be some player m having $u_m < e_m$. Therefore we have

$$min\left(\frac{T+u_1}{\hat{B}_1}, ... \frac{T+u_n}{\hat{B}_n}\right) < min\left(..., \frac{T+e_m}{\hat{B}_m}, ...\right) \leq T + \frac{O}{n} \tag{3}$$

Equation (3) tells that $min\left(\frac{T+u_1}{\hat{B}_1}, ..., \frac{T+u_n}{\hat{B}_n}\right)$ is maximized by U_0^*.

When Game G is under $Traffic_k$ where $k \neq 0$, we know, from Corollary 1.5, $p_k(e_{k,-(k-1)}) \geq p_j(e_{j,-k})\ \forall j \in \{k+1, ..., n\}$. Then $p_1(x_1) \geq ... \geq p_k(x_k) \geq p_k(e_{k,-(k-1)}) \geq p_j(e_{j,-k})\ \forall j \in \{k+1, ..., n\}$. This is equivalent to

$$\frac{T+x_1}{\hat{B}_1} \leq ... \leq \frac{T+x_k}{\hat{B}_k} \leq \frac{T+e_{k,-(k-1)}}{\hat{B}_k} \leq \frac{T+e_{j,-k}}{\hat{B}_j} \quad \forall j \in \{k+1, ..., n\} \quad (4)$$

Equation (4) reveals that the max value of $min(\frac{T+u_1}{\hat{B}_1}, ..., \frac{T+u_n}{\hat{B}_n})$ is $\frac{T+x_1}{\hat{B}_1}$ and is reached at $u_1 = x_1$. Since $u_1 = x_1$ is implied by U_k^*, $k \neq 0$, $min(\frac{T+u_1}{\hat{B}_1}, ..., \frac{T+u_n}{\hat{B}_n})$ is maximized by U_k^* where $k \neq 0$.

In summary, $min\left(\frac{T+u_1}{\hat{B}_1}, ...\frac{T+u_n}{\hat{B}_n}\right)$ is maximized by the utility profile at the Nash equilibrium. Game G is weighted max-min fair. □

7 Strategy-Proof Mechanism – Max Traffic Declaration

In the previous content, we already show the Nash equilibrium, the spectrum allocation result, and the corresponding properties of our game. If we omit the process of players' making acquisition/offering requests and directly adopt the final spectrum allocation result, the proposed spectrum sharing algorithm can be simplified as the following spectrum allocation rule.

Definition 10. *Given that the spectrum T, the credit token budget B_i, and the max traffic demand x_i of each player i are public information and assuming that $\{p_i\,(x_i)\}$ is ranged in decreasing order without losing generality, the spectrum allocation is $AR^* = (x_1, ..., x_k, e_{k+1,-k}, ..., e_{n,-k})$ under $Traffic_k$, $k \in \{0, N\}$.*

According to this spectrum allocation rule, we can design a mechanism M to adopt in a more general case that all players' max traffic demands are private information. In Mechanism M, each player i declares its max traffic demand, x_i', which may be different from the true max traffic demand x_i. Given all players' declarations, Mechanism M applies the spectrum allocation rule to allocate spectrum. Since now each player possibly gains more spectrum than its true max traffic demand, it is reasonable to add the assumption that when a player has reached its true max traffic demand, its utility is the true max traffic. Mechanism M is strategy-proof [11,13], i.e. the truth-revelation of the max traffic is a dominant-strategy equilibrium.

Theorem 7. *Mechanism M is strategy-proof. Equivalently, the strategy profile, $(x_1, ..., x_n)$, is a dominant-strategy equilibrium.*

Proof. Given any x_{-i}', we want to prove $x_i' = x_i$ always results in the highest utility for every player i under all traffic cases.

Let $\underline{N} = \{\underline{i}\}$ be the sorted player set N such that $\left\{p_{\underline{i}}\left(x_{\underline{i}}'\right)\right\}$ is in decreasing order. Let $e_{\underline{j,-k}}$, $\forall \underline{k} \in \{0, \underline{N}\}$ and $\forall \underline{j} \in \{\underline{k+1}, ..., \underline{n}\}$, be the same as $e_{j,-k}$ in

Definition 4 with $\{x_i\}$ replaced by $\left\{x'_{\underline{i}}\right\}$. Also, let $Traffic_{\underline{k}}$, $\underline{k} \in \{0, \underline{N}\}$, denote $x'_{\underline{j}} \leq e_{\underline{j,-(j-1)}}$ $\forall \underline{j} \in \{\underline{1}, ..., \underline{k}\}$ and $x'_{\underline{j}} > e_{\underline{j,-k}}$ $\forall \underline{j} \in \{\underline{k+1}, ..., \underline{n}\}$.

Assume that player i now plays $x'_i = x_i$ and has the m-th priority, i.e. $i = \underline{m}$ and $x_i = x'_{\underline{m}}$. When $\underline{m} \leq \underline{k}$, we have $x_i = x'_{\underline{m}} \leq e_{\underline{m,-(m-1)}}$. Correspondingly, $u_i = x_i$ which is the highest utility player i can obtain. When $\underline{m} > \underline{k}$, we have $x_i = x'_{\underline{m}} > e_{\underline{m,-k}}$ and $u_i = e_{\underline{m,-k}}$. If player i plays $x'_i < e_{\underline{m,-k}}$, then $u'_i = x'_i < e_{\underline{m,-k}} = u_i$; if player i plays $x'_i \geq e_{\underline{m,-k}}$, then $u'_i = e_{\underline{m,-k}}$. In words, no other strategy results in higher utility. From the above, $x'_i = x_i$ results in the highest utility under all traffic cases and therefore is a dominant strategy of player i.

Because the derivation above is applicable $\forall i \in N$, $x'_i = x_i$ is a dominant strategy of every player i and the strategy profile, $(x_1, ..., x_n)$, is a dominant-strategy equilibrium. □

Given that Mechanism M is at $(x_1, ..., x_n)$, the spectrum allocation result is the same as that in Theorem 4. Efficiency and fairness thus hold.

8 Conclusions

In this paper, we propose an efficient and fair spectrum sharing scheme. We show all BSs always reach a Nash equilibrium where the spectrum allocation is unique. The proposed spectrum sharing algorithm is desirable because it achieves efficiency and fairness among all BSs. The spectrum allocation is efficient as allocative efficiency and Pareto optimality are achieved. It also meets both weighted max-min fair and weighted proportional fair criteria. By adopting this spectrum allocation result, a strategy-proof mechanism, ensuring efficiency and fairness at the truth-revealing dominant-strategy equilibrium, is designed to apply in the more general case that max traffic demands are private information.

References

1. Akyildiz, I., Lee, W., Vuran, M., Mohanty, S.: Next Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: a Survey. Elsevier Computer Networks 50, 2127–2159 (2006)
2. Ji, Z., Liu, K.J.R.: Dynamic Spectrum Sharing: a Game Theoretical Overview. IEEE Communications Magazine, 88–94 (May 2007)
3. Ileri, O., Samardzija, D., Mandayam, N.B.: Demand Responsive Pricing and Competitive Spectrum Allocation via Spectrum Server. In: IEEE DySPAN 2005, November 2005, pp. 194–202 (2005)
4. Grandblaise, D., Moessner, K., Vivier, G., Tafazolli, R.: Credit Token based Rental Protocol for Dynamic Channel Allocation. In: 1st International Conference on Cognitive Radio Oriented Wireless Networks and Communications (2006)
5. IEEE P802.22/D1.0 Draft Standard for Wireless Regional Area Networks Part22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Policies and Procedures for Operation in the TV Bands (April 2008)

6. Sengupta, S., Chandramouli, R., Brahma, S., Chatterjee, M.: A Game Theoretic Framework for Distributed Self-Coexistence Among IEEE 802.22 Networks. In: IEEE Global Telecommunications Conference, IEEE GLOBECOM 2008, November 2008, pp. 1–6 (2008)
7. Gao, D., Cai, J., Li, Z., Wei, X., Chen, R.: Credit Token Based Dynamic Resource Renting and Offering Mechanism for Cognitive Radio WRAN BS Spectrum Sharing. In: 22nd International Conference on Advanced Information Networking and Applications - Workshops, AINAW 2008, March 2008, pp. 296–300 (2008)
8. Niyato, D., Hossain, E., Han, Z.: Dynamic Spectrum Access in IEEE 802.22-Based Cognitive Wireless Networks: a Game Theoretic Model for Competitive Spectrum Bidding and Pricing. IEEE Wireless Communications 16(2), 16–23 (2009)
9. Osborne, M.J.: An Introduction to Game Theory. Oxford University Press, Oxford (2003)
10. Nash, J.F.: Equilibrium Points in n-Person Games. Proceedings of the National Academy of Sciences 36(1), 48–49 (1950)
11. Parkes, D.C.: Iterative Combinatorial Auctions. In: Cramton, P., Shoham, Y., Steinberg, R. (eds.) Combinatorial Auctions, ch. 2. MIT Press, Cambridge (2001)
12. Courcoubetis, C., Weber, R.: Pricing Communication Networks: Economics, Technology and Modelling, ch. 10. Wiley, Chichester (2003)
13. Mas-Colell, A., Whinston, M., Green, J.: Microeconomic Theory, ch. 23. Oxford University Press, New York (1995)