Quantum-Like Computations Using Coupled Nano-scale Oscillators

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Abstract. In this paper we consider possibilities to mimic quantum-like computations with classical nano-scale devices. In particular, we study dynamics of coupled oscillators arrays and propose a method to imitate basic one-qubit and two-qubit operations using coupled oscillator networks.

1 Introduction

The concept of quantum computations is based on principles of quantum mechanics and stimulated extensive research activities. It is expected that quantum computers could extend class of solvable problems by utilizing the following features: (i) state of n-bit quantum register $|\psi\rangle = |\psi_1\psi_2...\psi_n\rangle$ is a linear superposition of $N = 2^n$ classical states with complex coefficients c_k, $|\psi\rangle = \sum_{k=1}^{N} c_k|k\rangle$; (ii) existence of entangled (not separable) states; (iii) computation as the unitary evolution (rotations or reflections in Hilbert space); (iv) measurement to obtain a result of computations from a high-dimensional superposition.

In general quantum computations may be described as algorithms that operate on vectors in Hilbert space and make use of math features of quantum mechanics. However, considering math description of quantum computations, it seems irrelevant how Hilbert space is physically implemented [1][2]. For example, quantum-like system may be based on optical fibers which obey equations formally identical to the Schrödinger equation (SE), but role of time is replaced by a space coordinate [3].

Another example is computations using networks of coupled oscillators [4]-[6]: its dynamics may be described in a formalism mathematically indistinguishable from one used in quantum computing.[1] On the other hand, recent progress in nano-technology allows to build large scale oscillatory arrays, e.g., based on NEMS. These facts motivate us to search for new computation architectures based on nano-scale devices with computing principles borrowed from quantum physics. In this paper we consider arrays of classical oscillators with controlled

[1] With the exception that it does not include quantum property of non-locality.

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coupling to model (mimic) behavior of basic quantum gates in quantum computations. The paper is organized as follows: Section 2 outlines similarities of quantum dynamics and unitary evolution of classical oscillators. Quantum-like computations using coupled oscillators are suggested in Section 3 including possible implementations of basic quantum gates using nano-resonators.

2 Classical Formalism for Quantum Dynamics

Schrödinger equation ($\hbar = 1$) for wave function $\psi(q,t)$ of a 1-dim particle may be written as $\dot{\psi}(q,t) = -iH\psi(q,t)$. Formal solution for N-level system with a time-invariant Hamiltonian H describes evolution of quantum state

$$|\psi(t)\rangle = \exp(-iHt)|\psi(t_0)\rangle = U(t)|\psi(t_0)\rangle \tag{1}$$

where $|\psi(t)\rangle = |\psi_1(t)\psi_2(t)...\psi_n(t)\rangle$ is complex-valued state vector of length $N = 2^n$, H is a Hermitian matrix presenting Hamiltonian of the system, $U(t)$ is a unitary operator (matrix). For Hermitian matrix $H = A^\dagger DA$ we may write

$$\exp(-iHt) = \exp(-iA^\dagger DAt) = A^\dagger e^{-iDt}A = A^\dagger D_1(t)A \tag{2}$$

where A is a unitary matrix, D is a diagonal matrix $D = \mathrm{diag}\{\omega_1, \omega_2, ..., \omega_\mathrm{N}\}$, $D_1(t) = \mathrm{diag}\{e^{-i\omega_1 t}, e^{-i\omega_2 t}, ..., e^{-i\omega_N t}\}$. Therefore, evolution of the system may be presented as unitary evolution of N oscillators with frequencies $\{\omega_1, \omega_2, ..., \omega_N\}$.

On the other hand, vector field for a finite quantum system is equivalent (via canonical transformations) to the vector field of classical harmonic oscillators [1]. For example, for two coupled classical oscillators described by 4 real-valued functions (coordinates and momenta) we may associate 2-dimensional Hilbert space which formally is equivalent to the artificial spin-1/2 states. Similarly, state space of $2n$ independent linear oscillators is defined over $R_1^2 \otimes R_2^2 \otimes ... \otimes R_{2n}^2 = \bigotimes_{k=1}^{2n} R_k^2 = R^{4^n}$ and formally corresponds to Hilbert space $C^{2^n} = \bigotimes_{k=1}^{n} C_k^2$ of n-qubit register.

2.1 Quantum Gates

To implement computations a desired unitary matrix $U(2^n)$ is first to be decomposed into a sequence of elementary operations. For example, 1-qubit $\{\sigma_x, \sigma_y, \sigma_z\}$ and 2-qubit Control-NOT operations form a set of "universal" quantum gates sufficient to implement arbitrary quantum computations. A general 2-qubit Hamiltonian may be described as

$$H = \sum_{i=1}^{3} \alpha_i \sigma_i^{(1)} \otimes I_2 + \sum_{j=1}^{3} \beta_j I_2 \otimes \sigma_j^{(2)} + \sum_{i=1}^{3}\sum_{j=1}^{3} \gamma_{ij} \sigma_i^{(1)} \otimes \sigma_j^{(2)} = H_1 + H_2 + H_{12}$$

where $\sigma_{\{1,2,3\}}^{(k)} = \sigma_{\{x,y,z\}}^{(k)}$ are the Pauli operators acting on k-th qubit; α,β and γ are elements of two real vectors and a real matrix, respectively. Note that H_1 and H_2 describe dynamics of isolated qubits, while H_{12} corresponds to coupling between qubits. From this perspective, to design a quantum gate we need to specify corresponding coupling H_{12}.

Direct implementation of coupling Hamiltonian for a required quantum gate, e.g., U_{CNOT}, is not always possible. In practice, the required Hamiltonian H is approximated using some available physical system accompanied by steering implemented by either fast control (pulses) or adiabatic (infinitesimal) evolution, such that at time moment t_f the total Hamiltonian realizes the required unitary transform $U(t_f) = e^{-iHt_f} = e^{-i(H_{\text{drift}}+H_{\text{ctr}})t_f}$ [7]-[10].

3 Quantum-Like Computations with Classical Oscillators

3.1 Quantum-Like Qubit (QLB)

Let's consider complex Hilbert space formed by complex functions

$$\psi(t) = q_1(t) + iq_2(t) = c_0(t)\cos(\omega t) + i\,c_1(t)\sin(\omega t) \qquad t \in [0, n\pi], \tag{3}$$

where $q_0(t)$ and $q_1(t)$ are oscillators with complex amplitudes $c_k(t) \in \mathbb{C}$ and initial conditions $c_k(t_0)$ $(k = 0, 1)$; $|c_k(t_n)|^2$ is instant power of the k-th oscillator measured at time moment t_n with normalization $\sum_k |c_k(t_n)|^2 = 1$.

Definition 1. *The ordered set of two (identical) oscillators $Q_{LB} \doteq \{q_0(t), q_1(t)\}$ is called as quantum-like bit (QLB) such that*

- QLB pure (or number) states $|0\rangle$ and $|1\rangle$ correspond to configurations when one of oscillators is switched off, while another one is running with max amplitude $|c_k| = 1$ and $\varphi_k(t_0) = 0$, i.e., $|0\rangle \Leftrightarrow (|c_0(t_n)|, |c_1(t_n)|)^T = (1, 0)^T$; $|1\rangle \Leftrightarrow (|c_0(t_n)|, |c_1(t_n)|)^T = (0, 1)^T$.
- QLB mixed state at time t_n is described by complex-valued vector $Q_{LB}(t_n) = (c_0(t_n), c_1(t_n))^T$ and corresponds to a superposition of pure states $|\psi\rangle = \sum_k c_k|k\rangle$. Note that (similar to quantum mechanics) $|\psi\rangle$ has a form of the wave function with number states $|k\rangle$ and probability amplitudes c_k.

Recall that unitary transformations U (acting from time t_{n-1} to time instant t_n) may be presented by a set of canonical transformations resulting in interference of complex amplitudes. Since oscillators $q_k(t)$ evolve in continuous time, there is a transition period T between time instants t_{n-1} and $t_n = t_{n-1} + T$ for which transformations are defined. These transformations have a simple probabilistic interpretation. For example, let's set QLB initial conditions as $(c_0(t_0,)c_1(t_0))^T = (1, 0)^T \Leftrightarrow |0\rangle$. After transformation U acting during $[t_0, t_1]$ at time t_1 we get $U(c_0(t_0)c_1(t_0))^T = (c_0(t_1)c_1(t_1))^T$. Probability that QLB is in the number state $|k\rangle$ is proportional to energy of k-th oscillator $|c_k(t_1)|^2$ at time t_1.

3.2 Single-QLB Operations

To implement arbitrary QLB transforms we define the following operations:

(i) $S_k^{(m)}(\varphi)$ is phase φ rotation of a single k-th oscillator $(k = 0, 1)$ within m-th QLB; corresponds to the canonical transform where $u_{kk} = e^{i\varphi}$, $u_{ij} = \delta_{ij}$.

(ii) $D_{k_m,k_j}(\varphi)$ as joint rotation (coupling) of k_m-th oscillator of m-th QLB with k_j-th oscillator of j-th QLB.

We may realize these operations with two types of adiabatic perturbations:

(a) perturbation as an external control to drive a given state $q(t_0)$ of one of oscillators into a target state $q(t_T)$

$$\ddot{q}_k + w^2 q_k + \delta w^2(t) q_k = 0$$

to implement U, where $u_{kk} = e^{i\varphi}$, $u_{ij} = \delta_{ij}$; $\varphi = \int_{t_1}^{t_2} \delta w^2(t)dt/(2w)$.

(b) perturbation as a controlled coupling between two oscillators (switched on/off with a certain timing) which couples dynamics of two systems

$$\ddot{q}_k + w^2 q_k + \delta v^2(t) q_l = 0 \qquad \ddot{q}_l + w^2 q_l + \delta v^2(t) q_k = 0 \tag{4}$$

to implement the canonical transform U, where

$$u_{kk} = u_{ll} = \cos\varphi,\ u_{kl} = u_{lk} = i\sin\varphi;\ \varphi = \int_{t_1}^{t_2} \delta v^2(t)dt/(2w).$$

Then Pauli operators and Hadamard transform for single-QLB are obtained as

$$\sigma_z = S_1(\pi)S_0(0) \qquad \sigma_x = S_1(\pi)D_{01}(-\pi/2) \qquad \sigma_y = -i\sigma_z\sigma_x \tag{5}$$

$$U_H = \tfrac{1}{\sqrt{2}}(\sigma_x + \sigma_z) = S_1(3\pi/2)D_{01}(\pi/4)S_1(3\pi/2) \tag{6}$$

3.3 Two-QLB Operations

Two non-interacting QLBs with labels m and j may be presented by 4 oscillators; their common state is the tensor product of two corresponding states $|k_j k_m\rangle$ with four configurations. Each configuration corresponds to a single oscillator with amplitude $c_{k_j k_m} = c_{k_j} c_{k_m}$ (written in the following in the binary notation). Any 2-QLBs state may be presented as a superposition of states of four numbered oscillators

$$|\psi\rangle = \sum_{k_j=0}^{1} \sum_{k_m=0}^{1} c_{k_j} c_{k_m} |k_j k_m\rangle \quad \text{where } (k_j k_m) \in \{00, 01, 10, 11\}. \tag{7}$$

to which we apply the needed transformations. To implement arbitrary 2-QLBs gates we may use four single phase rotations and six controlled coupling connections between pairs single oscillators. As an example, CNOT operation may be realized by applying joint rotation $D_{10,11}(\pi/2)$, as if they belong to the same QLB, and then applying controlled single-oscillator rotations as follows

$$U_{CNOT} = S_{10}(-\pi/2)S_{11}(-\pi/2)D_{10,11}(\pi/2) \tag{8}$$

A possible implementation of two-QLB gates in a form of 4 oscillators with controlled coupling is depicted at Fig.1, left. Control inputs `Ctr_Osc` are used to set initial conditions and together with `Ctr_Q` provide adiabatic control to implement

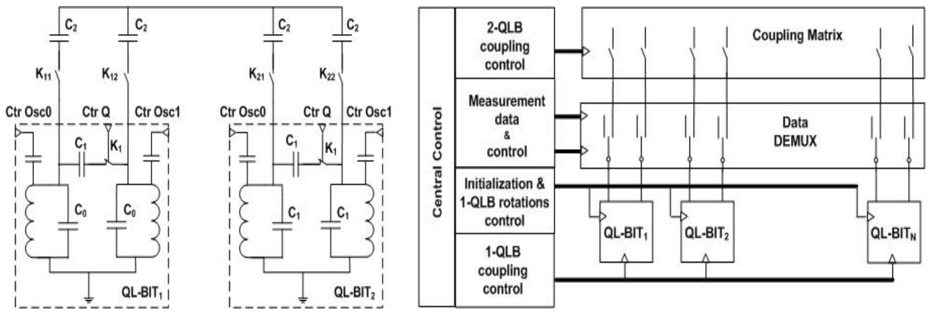

Fig. 1. Schematics to model 2-QLBs (left) and quantum-like computations (right)

phase rotations for single oscillators. Controlled switches $\mathtt{K}_{\mathtt{ij}}$ allow to implement joint rotation by coupling different oscillators. Possible NEMS implementation of coupled oscillators is outlined in [11]. Oscillator array (Fig.1, right) may be used to realize quantum-like computations by sequentially activating single-QLB and two-QLB operations for different qubits by means of `Data DEMUX` and `Central Control`. Detailed description of quantum-like gates to appear elsewhere.

4 Conclusions

In this paper we consider modeling quantum computations by classical oscillators with controlled coupling. In particular, it seems possible to mimic behavior of basic quantum gates by using quantum-like qubits formed by coupled oscillators, e.g., implemented as nano-resonator arrays.

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