Performance Comparison of Orthogonal and Quasi-orthogonal Codes in Quasi-Synchronous Cellular CDMA Communication

Sujit Jos[1], Preetam Kumar[2], and Saswat Chakrabarti[2]

[1] Dept. of Electronics and Electrical Communication Engineering
[2] G. S. Sanayal School of Telecommunications
Indian Institute of Technology, Kharagpur
sujit@gssst.iitkgp.ernet.in, preetam@gssst.iitkgp.ernet.in,
saswat@ece.iitkgp.ernet.in

Abstract. Orthogonal and quasi-orthogonal codes are integral part of any DS-CDMA based cellular systems. Orthogonal codes are ideal for use in perfectly synchronous scenario like downlink cellular communication. Quasi-orthogonal codes are preferred over orthogonal codes in the uplink communication where perfect synchronization cannot be achieved. In this paper, we attempt to compare orthogonal and quasi-orthogonal codes in presence of timing synchronization error. This will give insight into the synchronization demands in DS-CDMA systems employing the two classes of sequences. The synchronization error considered is smaller than chip duration. Monte-Carlo simulations have been carried out to verify the analytical and numerical results.

1 Introduction

Direct Sequence Code Division multiple Access (DS-CDMA) is considered to be a promising technology for broadband deployment of next generation cellular networks. The performance of DS-CDMA system relies largely on the correlation properties of the codes employed for the spreading of the user signal [1]. Synchronous CDMA(S-CDMA) has been proposed in [2], whereby Multiple Access Interference (MAI) could be completely eliminated by the use of orthogonal spreading codes. Hence, orthogonal codes are the obvious choice when perfect synchronization is maintained. This perfect synchronization is difficult to implement in the uplink and the performance will degrade in the presence of access timing error. When the synchronization error present in the system is less than a chip period, the DS-CDMA system is usually referred to as Quasi-Synchronous CDMA or QS-CDMA system [3].

Behavior of orthogonal codes in quasi-synchronous environment has been well addressed in [4]-[6]. In QS-CDMA system, quasi-orthogonal codes provide superior performance compared to orthogonal codes. They are extensively employed in uplink communication where different users are misaligned in time. Hence, quasi-orthogonal sequences attract a great deal of attention in systems where synchronizations errors are inevitable. Performance analysis of quasi-orthogonal codes in QS-CDMA has been presented in [6]-[8]. But, a quantitative comparison between the two classes of codes has not been addressed sufficiently.

C. Wang (Ed.): AccessNets, LNICST 6, pp. 78–88, 2009.

In this work, orthogonal and quasi-orthogonal codes have been compared with respect to the maximum tolerable synchronization error that can be tolerated so that complete error-free despreading is guaranteed. BER analysis of QS-CDMA system is also presented in presence of timing synchronization error. The analysis for maximum tolerable synchronization error for Walsh Hadamard codes has been done in [4].

In the next section we will describe the system model of QS-CDMA with BPSK modulation. We compare orthogonal and quasi-orthogonal codes in section 3. In section 4, we present the BER analysis in presence of noise. Simulation results are presented in section 5. Finally we conclude the paper.

2 System Model for QS-CDMA with BPSK

The DS-CDMA system is modeled as in [4]. The system consists of K users simultaneously signaling over a common transmission channel. With BPSK transmission, the signal of k 'th user can be written as

$$u_k(t) = A_k \sum_{l=-\infty}^{\infty} b_k^l s_k(t - lT_b), k = 0, ..., K-1 \tag{1}$$

with amplitude A_k, symbol duration $T_b, b_k^l \in \{-1, 1\}$ and spreading code $s_k(t)$, which is given by

$$s_k(t) = \sum_{j=0}^{N-1} s_k^j p_c(t - jT_c) \tag{2}$$

with $T_b = NT_c$ and $s_k^j = \pm 1$ the elements of the k 'th codeword with chip duration T_c and length N. The function $p_c(t) = 1$ if $0 < t < T_c$, and $p_c(t) = 0$ otherwise. The received signal $r(t)$ is the summation of all K signals

$$r(t) = \sum_{k=0}^{K-1} u_k(t - \tau_k) \tag{3}$$

where $\tau_k \in [-T_c, T_c]$ is the time shift between the transmitted and the received signal of user k. Since noise is common to all systems, we have ignored the effect of AWGN in equation (3). Analysis in presence of noise will be resumed in section 4.

If a correlation receiver is used to detect the desired symbol b_i^n, the received signal $r(t)$ has to be multiplied with the desired user code $s_i(t)$ and integrated over the symbol duration.

$$\tilde{b}_i^{n} = \frac{1}{T_b} \int_0^{T_b} s_i(t) r(t + nT_b) dt$$

$$= \frac{1}{T_b} \int_0^{T_b} s_i(t) \left[\sum_{k=0}^{K-1} A_k \sum_{l=-\infty}^{\infty} b_k^l s_k(t + (n-l)T_b - \tau_k) \right] dt \tag{4}$$

Due to the time shift τ_k between the spreading and despreading code of user k, the integration not only contains the cross correlation from b_k^n but also from $b_k^{n\pm1}$. This minor contribution can be neglected when $\tau_k < T_c$ [5]. Hence the received symbol is given as:

$$\tilde{b}_i^{n} = \frac{1}{T_b}\int_0^{T_b} s_i(t)\left[\sum_{k=0}^{K-1} A_k b_k^n s_k(t-\tau_k)\right]dt$$

$$= A_i b_i^n \varphi_{ii}(\tau_i) + \sum_{k=0,k\neq i}^{K-1} A_k b_k^n \varphi_{ik}(\tau_k) \qquad (5)$$

Where,

$$\varphi_{ik}(\tau_k) = \frac{1}{NT_c}\int_0^{NT_c} s_i(t)s_k(t-\tau_k)dt \qquad (6)$$

is the correlation between codeword $s_i(t)$ and codeword $s_k(t)$ delayed by τ_k. From (5), we observe that the amplitude of the desired user is degraded by the autocorrelation coefficient which is unity in the case of perfect synchronization. The nonunity autocorrelation coefficient results in reduced signal strength in the despreaded signal. Synchronization errors generally results in increased MAI because of the higher value of crosscorrelation compared to the zero phase crosscorrelation. The original user data symbol b_i^n can be deduced from the received signal as long as

$$A_i\varphi_{ii}(\tau_i) + \sum_{k=0,k\neq i}^{K-1} A_k\left(\frac{b_k^n}{b_i^n}\right)\varphi_{ik}(\tau_k) > 0 \qquad (7)$$

Assuming all user amplitudes to be equal, i.e., $A_i = A_k$ for all k, we find the worst-case situation under which (7) is guaranteed to be fulfilled so that the reception is completely error-free. In the worst case scenario, (7) is transformed into

$$\varphi_{ii}(\tau_i) - \sum_{k=0,k\neq i}^{K-1} \left|\varphi_{ik}(\tau_k)\right| > 0 \qquad (8)$$

It follows from (8) that whenever the autocorrelation value is larger than the sum of absolute crosscorrelation values, it is possible to recover the desired data stream b_n^i. Hence, the strict condition for error-free despreading requires that synchronization errors for all users are smaller than the maximum tolerable synchronization error ΔT, which would be derived subsequently. The autocorrelation function for i'th user may be expressed as

$$\varphi_{ii}(\tau_i) = \left(\frac{\varphi_{ii}(T_c) - \varphi_{ii}(0)}{T_c - 0}\right)\tau_i + \varphi_{ii}(0) \qquad (9)$$

Similarly, the crosscorrelation function can be written as

$$\varphi_{ik}(\tau_k) = \frac{\varphi_{ik}(T_c)}{T_c}\tau_k + \varphi_{ik}(0), 0 \le \tau_k \le T_c \tag{10}$$

with $\varphi_{ik}(0) = 0$ for orthogonal code and $\varphi_{ik}(0) = -\frac{1}{N}$ for quasi-orthogonal codes. We first derive the expression for maximum tolerable synchronization error for orthogonal codes. Substitution of (9) and (10) into (8) gives

$$\frac{\varphi_{ii}(T_c)-1}{T_c}\tau_i + 1 - \frac{\sum_{k=0,k\ne i}^{K-1}\left|\varphi_{ik}(T_c)\right|}{T_c}\tau_k > 0, 0 \le \tau_i, \tau_k \le T_c \tag{11}$$

The maximum tolerable synchronization error $\Delta\mathrm{T}$ can be incorporated into (11) with the following equality

$$\frac{\varphi_{ii}(T_c)-1}{T_c}\Delta\mathrm{T} + 1 = \frac{\sum_{k=0,k\ne i}^{K-1}\left|\varphi_{ik}(T_c)\right|}{T_c}\Delta\mathrm{T} \tag{12}$$

Equation (12) can be written as:

$$\Delta\mathrm{T} = \frac{T_c}{\sum_{k=0,k\ne i}^{K-1}\left|\varphi_{ik}(T_c)\right| - \varphi_{ik}(T_c) + 1} \tag{13}$$

with $\Delta\mathrm{T}$ upper bounded by T_c. The minimum of $\Delta\mathrm{T}$ over all users gives the maximum tolerable synchronization error. Hence, the maximum tolerable synchronization error for orthogonal codes is obtained as

$$\Delta\mathrm{T} = \min_i \left\{ \frac{T_c}{\sum_{k=0,k\ne i}^{K-1}\left|\varphi_{ik}(T_c)\right| - \varphi_{ii}(T_c) + 1} \right\} \tag{14}$$

Now we proceed to obtain a similar expression for quasi-orthogonal codes. The autocorrelation function remains same as in the case of orthogonal codes. The crosscorrelation function with appropriate substitution in (10) is given by

$$\varphi_{ik}(\tau_k) = \frac{\varphi_{ik}(T_c)}{T_c}\tau_k - \frac{1}{N}, 0 \le \tau_k \le T_c \tag{15}$$

Substitution of (9) and (15) into (8) gives

$$\frac{\varphi_{ii}(T_c)-1}{T_c}\tau_i + 1 - \sum_{k=0,k\ne i}^{K-1}\left|\frac{\varphi_{ik}(T_c)\tau_k}{T_c} - \frac{1}{N}\right| > 0, 0 \le \tau_i, \tau_k \le T_c \tag{16}$$

With a small approximation, equation (16) can now be written as

$$\frac{\varphi_{ii}(T_c)-1}{T_c}\tau_i+1-\frac{\sum_{k=0,k\neq i}^{K-1}\left|\varphi_{ik}(T_c)\right|\tau_k}{T_c}+\frac{1}{N}>0 \tag{17}$$

Since $\left|\varphi_{ik}(T_c)\right|-\left|\varphi_{ik}(0)\right|\leq\left|\varphi_{ik}(T_c)-\varphi_{ik}(0)\right|$, the inequality of equation (17) still holds good. From (17), it follows that

$$\frac{\varphi_{ii}(T_c)-1}{T_c}\Delta\mathrm{T}+1=\frac{\sum_{k=0,k\neq i}^{K-1}\left|\varphi_{ik}(T_c)\right|}{T_c}\Delta\mathrm{T}-\frac{1}{N} \tag{18}$$

which can be written as:

$$\Delta\mathrm{T}=\frac{\left(1+\frac{1}{N}\right)T_c}{\sum_{k=0,k\neq i}^{K-1}\left|\varphi_{ik}(T_c)\right|-\varphi_{ii}(T_c)+1} \tag{19}$$

Taking minimum over all codewords, we have $\Delta\mathrm{T}$ for quasi-orthogonal codes as

$$\Delta\mathrm{T}=\min_i\left\{\frac{\left(1+\frac{1}{N}\right)T_c}{\sum_{k=0,k\neq i}^{K-1}\left|\varphi_{ik}(T_c)\right|-\varphi_{ii}(T_c)+1}\right\} \tag{20}$$

3 Comparison between Orthogonal and Quasi-orthogonal Codes

An accurate comparison of the two types of codes is not possible since orthogonal codes have even length while quasi-orthogonal codes have odd length. But looking at equations (14) and (20), we can make an approximate comparison of the two classes of codes. From the two equations we can observe that quasi-orthogonal can tolerate a synchronization error which is $\left(1+\frac{1}{N}\right)$ times that tolerated by orthogonal sequences for a given code length N. Hence, this justifies the usage of quasi-orthogonal codes in the uplink communication from user to base station, where perfect synchronization cannot be maintained between different user signals.

The maximum tolerable synchronization error is numerically evaluated for some of the commonly employed orthogonal and quasi-orthogonal codes. The orthogonal codes considered are Walsh-Hadamard, Orthogonal Gold and Quadratic-Residue (QR) orthogonal codes, while m-sequences and balanced gold codes are considered for the evaluation of quasi-orthogonal codes. Both the quasi-orthogonal sequences have normalized crosscorrelation value of $-1/N$. The value of ΔT as a fraction of chip duration for different orthogonal codes are given Table 1. The spreading code length N is 8, 16, 32, 64 and 128. Table 2 shows the values of ΔT for quasi-orthogonal sequences. The spreading lengths considered are 7,15,31,63 and 127 respectively.

Table 1. Maximum Tolerable synchronizatrion error of orthogonal codes for different values of N

N	*Walsh Code*	*QR Code*	*Orthogonal Gold Code*
8	0.3333	0.3333	0.3333
16	0.2857	0.2857	0.2857
32	0.2353	0.2667	0.1739
64	0.2051	0.2581	0.1600
128	0.1798	0.2540	0.1019

Table 2. Maximum Tolerable synchronizatrion error of Quasi-orthogonal codes for different values of N

N	*m-sequence*	*Gold Code*
7	0.4000	0.4000
15	0.3636	0.3636
31	0.3478	0.1882
63	0.3404	0.1768
127	0.3368	0.1061

As seen from tables 1 and 2, quasi-orthogonal codes have better immunity towards synchronization error. Hence, quasi-orthogonal codes are often employed in uplink communication where perfect orthogonality cannot be maintained due to timing synchronization errors.

4 BER Analysis in Presence of Noise

In this section, we derive the BER expression for individual users in a QS-CDMA system when BPSK signaling is employed. In presence of noise, (7) is modified as

$$A_i \varphi_{ii}(\tau_i) + \sum_{k=0,k\neq i}^{K-1} A_k \left(\frac{b_k^n}{b_i^n}\right) \varphi_{ik}(\tau_k) + z > 0 \tag{21}$$

where,

$$z = \frac{1}{T_b}\int_0^{T_b} s_i(t)n(t)dt \tag{22}$$

and $n(t)$ is the additive white Gaussian noise process with two sided power spectral density $\frac{N_0}{2}$. An error is made in the detection process of b_i^n when I_i exceeds $A_i\varphi_{ii}(\tau_i)$. To determine the probability of error we need the distribution of MAI_i which depends on the crosscorrelation of the codewords. Since, we assume that data symbols are equiprobable, MAI_i approaches a Gaussian random variable as the number of interferers increases. Under the Gaussian approximation of MAI_i, we can model the interference noise I_i as Gaussian. With the Gaussian approximation the probability of error or BER of i'th user is given by

$$BER_i = \frac{1}{\sqrt{2\pi\sigma}}\int_{A_i\varphi_{ii}(\tau_i)}^{\infty} e^{-I_i^2/2\sigma^2} dI_i \tag{23}$$

Let

$$x = I_i/2\sigma \tag{24}$$

Equation (23) now takes the form

$$BER_i = \sqrt{2}\sigma \times \frac{1}{\sqrt{2\pi\sigma}}\int_{\frac{A_i\varphi_{ii}(\tau_i)}{\sqrt{2}\sigma}}^{\infty} e^{x^2} dx \tag{25}$$

$$= \frac{1}{2} erfc\left(\frac{A_i\varphi_{ii}(\tau_i)}{\sqrt{2}\sigma}\right) \tag{26}$$

$$= Q\left(\sqrt{\frac{A_i^2\varphi_{ii}{}^2(\tau_i)}{\sigma^2}}\right) \tag{27}$$

where

$$\sigma^2 = \sigma_z^2 + \sigma_{MAI_i}^2 \tag{28}$$

Here, $\sigma_z^2 = \frac{N_0}{2}$ and the variance of MAI, $\sigma_{MAI_i}^2$ can be calculated as

$$\sigma_{MAI_i}^2 = E\{MAI_i\}$$

$$= \sum_{k=0,k\neq i}^{K-1} A_k^2\varphi^2{}_{ik}(\tau_k) \tag{29}$$

From (27) and (29), we have the final equation for the BER of the i 'th user as

$$BER_i = Q\left(\sqrt{\frac{A_i^2 \varphi_{ii}{}^2(\tau_i)}{\frac{N_0}{2} + \sum\limits_{k=0,k\neq i}^{K-1} A_k^2 \varphi_{ik}^2(\tau_k)}}\right) \tag{30}$$

5 Simulation Results

Monte-Carlo simulations have been carried out to verify the analytical and numerical results presented in the previous sections. Fig. 1 shows the BER curves of 5th and 20th codeword of Walsh Hadamard codeset in presence of synchronization error of 0.3Tc when the number of users and the spreading length of the code is 64. The analytical curve is also shown to support the BER analysis presented in the earlier section. Gaussian approximation for MAI seems to be more valid for lower values of Eb/No.

Fig. 2 shows the BER performance of 1st codeword of balanced Gold code in presence of synchronization error of 0.3Tc when the number of users and the spreading length of the code is 63. Quasi-orthogonal spreading sequences like balanced gold codes and m-sequences which are generated from shift register sequences exhibit a greater degree of randomness. Hence, the Gaussian approximation for MAI becomes more valid for quasi-orthogonal sequences. This fact could be observed from figures 1 and 2.

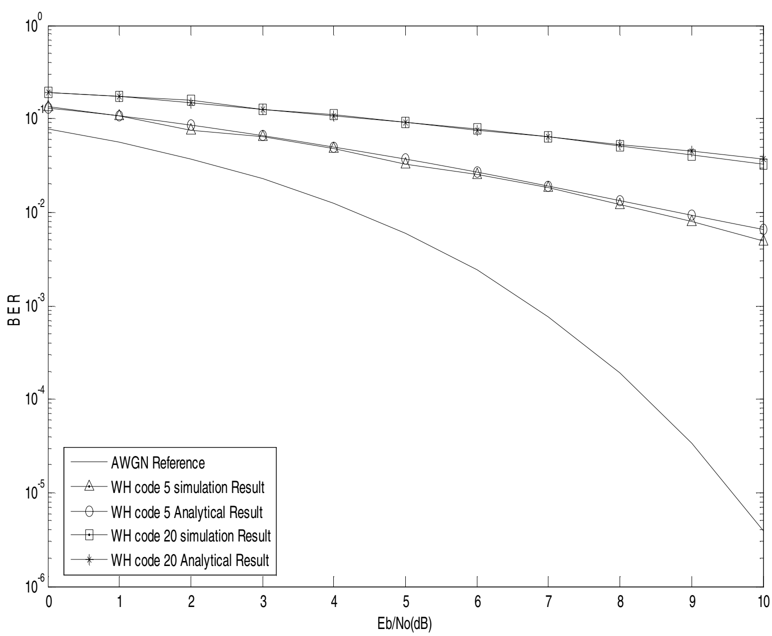

Fig. 1. Analytical and simulation plots of BER performance of 5th and 20th codes from WH matrix of order 64 at synchronization error of 0.3Tc when K=N=64

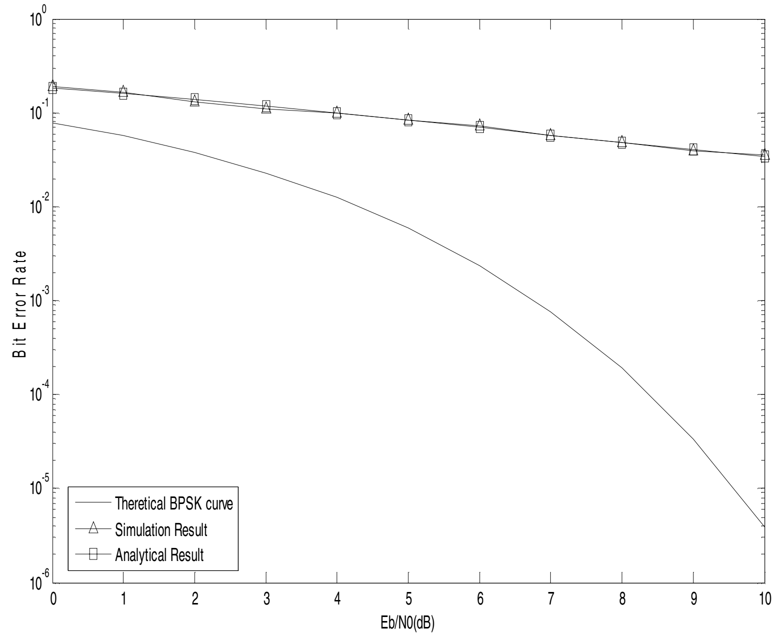

Fig. 2. Analytical and simulation plots of BER performance of 1st code from balanced gold set at synchronization error of 0.3Tc when K=N=63

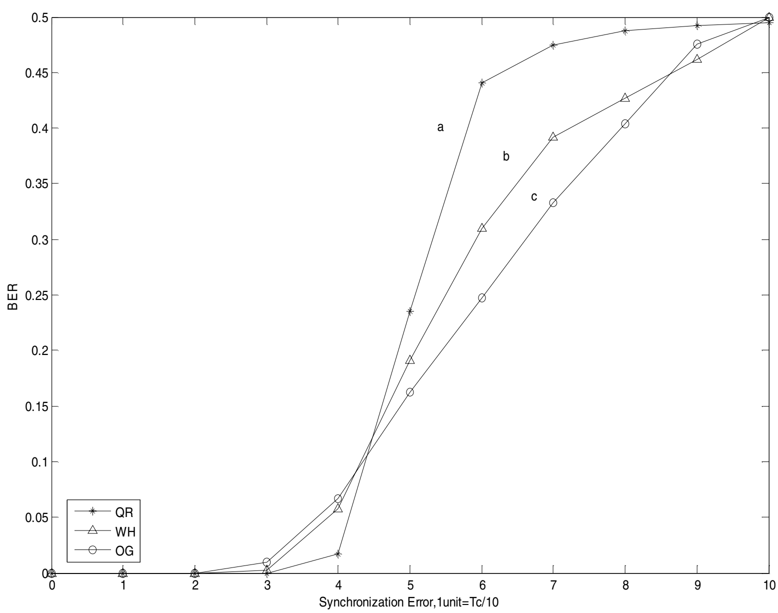

Fig. 3. BER variation with synchronization error when number of users, K=N and AWGN =0 for the following: a) QR Code; b) WH code and c) OG code

Fig. 3 shows the BER plots for the three orthogonal codes at different synchronization errors. As observed from the figure, QR codes have maximum tolerance to synchronization errors. The BER is very less till 0.4Tc for QR codes but it increases rapidly from this value and it gives higher BER as compared to WH and OG codes. In contrast orthogonal Gold codes provide very small BER till 0.3Tc and beyond this value, it generally gives less error. For Walsh Hadamard codes there is no despreading error till 0.2Tc, but it increases rapidly and gives error performance between QR and OG codes.

Fig. 4 shows the variation of BER of two quasi-orthogonal codes for different synchronization errors. It is observed from Fig 4 that the BER is very less for m-sequences till 0.4Tc and increases rapidly beyond this point while balanced gold codes give very less errors till 0.3Tc and moderate errors beyond this point. The precision in the plots is limited to 0.1Tc, hence finer insight is not obtained from the plots. These results seem to in agreement with the analytical expression for the maximum tolerable synchronization error for free despreading.

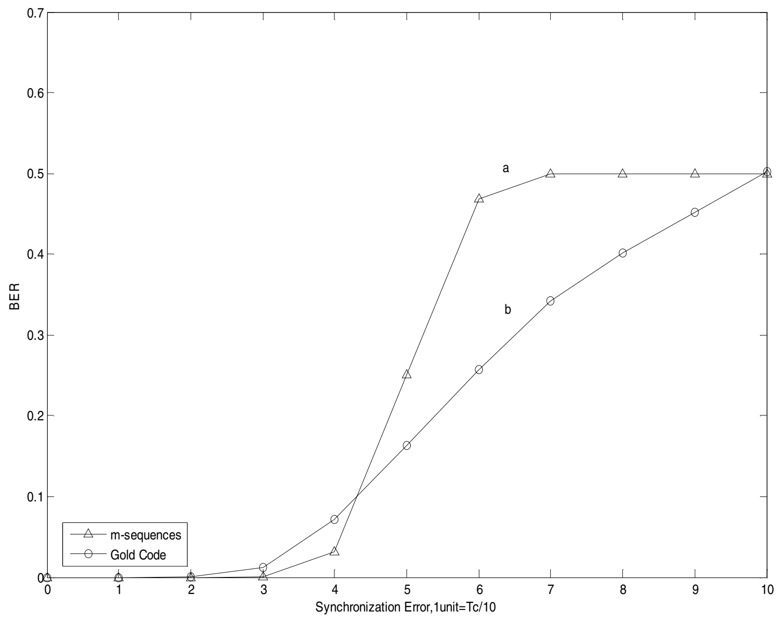

Fig. 4. BER versus timing synchronization error of a QS-CDMA system, with number of users, K=N=63 and AWGN =0 for the following quasi-orthogonal codes : a)m-sequence and b)Gold codes

6 Conclusion

Orthogonal and quasi-orthogonal codes have been compared in terms of the maximum tolerable synchronization error for error-free despreading. It has been shown that quasi-orthogonal codes are more immune to synchronization errors compared to their orthogonal counterparts. The orthogonal codes considered were Walsh-Hadamard, Orthogonal Gold and Quadratic-Residue(QR) codes, while m-sequences and balanced gold codes were considered for the performance evaluation. BER analysis in presence of noise has also been presented. Monte-Carlo simulation was carried out to verify the analytical and numerical results. Among the orthogonal codes considered, it is observed that QR orthogonal codes have better immunity towards synchronization errors as compared to Walsh Hadamard and Orthogonal Gold codes while m-sequences perform better in case of quasi-orthogonal codes.

References

1. Pursley, M.: Performance Evaluation for Phase-Coded Spread-Spectrum Multiple-Access Communication–Part I: System Analysis. IEEE Trans. Commn 25, 795–799 (1977)
2. Zhu, J., Nomura, T., Yamada, T.: Performance of Spread Spectrum Synchronous access Communication system by Orthogonal Sequences. IEICE Trans. Commn 68, 319–326 (1985)
3. Lin, X.D.: Optimal PN sequence design for quasi-synchronous CDMA communication systems. IEEE Trans. Commn 45, 221–226 (1997)
4. Houtum, W.J.V.: Quasi-synchronous code-division multiple access with high-order modulation. IEEE Trans. Commn. 49, 1240–1249 (2001)
5. Dasilva, V.M.V., Sousa, E.S.: Multicarrier Orthogonal CDMA signals for quasi-synchronous communication systems. IEEE JSAC 12, 842–852 (1994)
6. Khairy, M.M., Geraniotis, E.: Effect of time-jitter on CDMA networks with orthogonal and quasi-orthogonal sequences. In: Proceedings of Symposium on Computers and Communications, pp. 260–264 (1997)
7. Matolak, D.W., Manchiraju, D.: Evaluation of pseudorandom sequences for third generation spread spectrum systems. In: Proceedings of Southeastern Symposium on system theory 2003, pp. 288–290 (2003)
8. Sunay, M.O., McLane, P.J.: Sensitivity of a DS CDMA system with long PN sequences to synchronization errors. In: Proceedings of ICC 1995, pp. 1029–1035 (1995)