Performance of Adaptive Sub-carrier and Bit Allocation Algorithms for Multi-user OFDM Wireless Systems

Xiangning Fan[1] and Chu Jintao[2]

[1] Institute of RF-&-OE-ICs, School of Information Science and Engineering,
Southeast University, Nanjing, 210096, China
[2] National Mobile Communications Research Laboratory,
Southeast University, Nanjing, 210096, China
xnfan@seu.edu.cn

Abstract. Multi-user orthogonal frequency division multiplexing (OFDM) with adaptive sub-carrier and bit allocation are considered in this paper. Assuming having the knowledge of the instantaneous users' channel gains, three typical dynamic sub-carrier and bit allocation algorithms are analyzed and compared. The goal of each algorithm is to minimize the total transmit power with the user's data rate constraint. Then, based on the comparison, an improvement method is proposed for Zhang algorithm. Finally, performance comparisons between adaptive schemes and static ones are given. The results prove that the performances of adaptive algorithms are much better than that of the traditional fixed sub-carrier allocation method.

Keywords: OFDM, OFDMA, bit loading, sub-carrier allocation, multi-user.

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a parallel high-speed data transmission scheme, in which the transmission bandwidth is divided into a number of parallel orthogonal narrowband sub-channels (known as sub-carriers). Its purpose is to convert a frequency-selective fading channel into several flat-fading sub-channels. Data symbols are transmitted in parallel on each sub-carrier with low symbol rates. As a result, inter-symbol interference (ISI) caused by the multi-path in frequency selective fading channel is reduced. So, OFDM has been adopted for many high data rate systems such as digital audio broadcasting (DAB), digital video broadcasting-terrestrial (DVB-T), broadband radio access networks (BRAN), and asymmetric digital sub-carrier lines (ADSL). Recently, OFDM has also been considered for future use in fourth generation mobile communication systems. For OFDM systems, when different sub-carriers experience different channel gains, adaptive bit loading is needed to improve the spectrum efficiency [1-4].

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Multi-user OFDM system is a system that extends OFDM to multiple access environments, which allowing more than one user to transmit data over an OFDM symbol. Therefore, a problem of allocating sub-carriers for different users arises. One solution for this problem is to use fixed resource allocation schemes, such as time division multiple access (TDMA) and frequency multiple access (FDMA). But these static resource allocation schemes, simply dividing sub-channels in time or frequency, are not optimal. Another approach is OFDMA in which adaptive sub-carrier allocation combining with bit loading are performed according to the multi-users' channel information.

This paper will focus on the analysis and comparison of several typical sub-carrier and bit allocation algorithms for multi-user OFDM systems. The organization of this paper is as follows: In Section 2, system model and formulation of optimal problem are given. In Section 3, three typical adaptive sub-carrier and bit allocation algorithms are discussed and compared. In Section 4, the simulation comparisons of performance among dynamic allocation algorithms and traditional static scheme are presented. Section 5 concludes the paper.

2 System Model

The system model of adaptive multi-user OFDM is shown in Fig. 1. Assume that the system has N sub-carriers and K users. The kth user has data rate R_k bits/OFDM symbol. The transmitter gets all the K users' channel information through channel estimation. Assume that the information of channel characteristics is available at the transmitter. Using this channel information, the sub-carrier allocation and bit loading algorithms are applied to assign the sub-carriers and determine the number of data bits on each sub-carrier. The information of sub-carrier and bit allocation needed by demodulation at the receiver is assumed to be transmitted through a dedicated control channel.

Let $c_{k,n}$ denote the number of bits allocated to the nth sub-carrier for the kth user. And $c_{k,n}$ take value from $D = \{0,1,2,3,\cdots,M\}$. Where, M denotes the maximum number of bits transmitted on one sub-carrier during an OFDM symbol interval, and 0 means the sub-channel is so bad that it can not be used to transmit data bits. Define the magnitude of channel gain of the nth sub-carrier of kth user be $\alpha_{k,n}$, and assume that the single-sided noise power spectral density level N_0 is equal to unity. Let $f_k(c)$ denote the required receive power in sub-carrier for reliable reception of c bits which satisfies a certain BER requirement for user k. The function $f_k(c)$ should meet the requirement: a) $f_k(0) = 0$, which means no power is needed when no bit is transmitted; and b) $f_k(c)$ is a convex, which means that the required additional power to transmit an additional bit increases with the number c. As we know, all MQAM, MPSK modulation schemes satisfy these conditions.

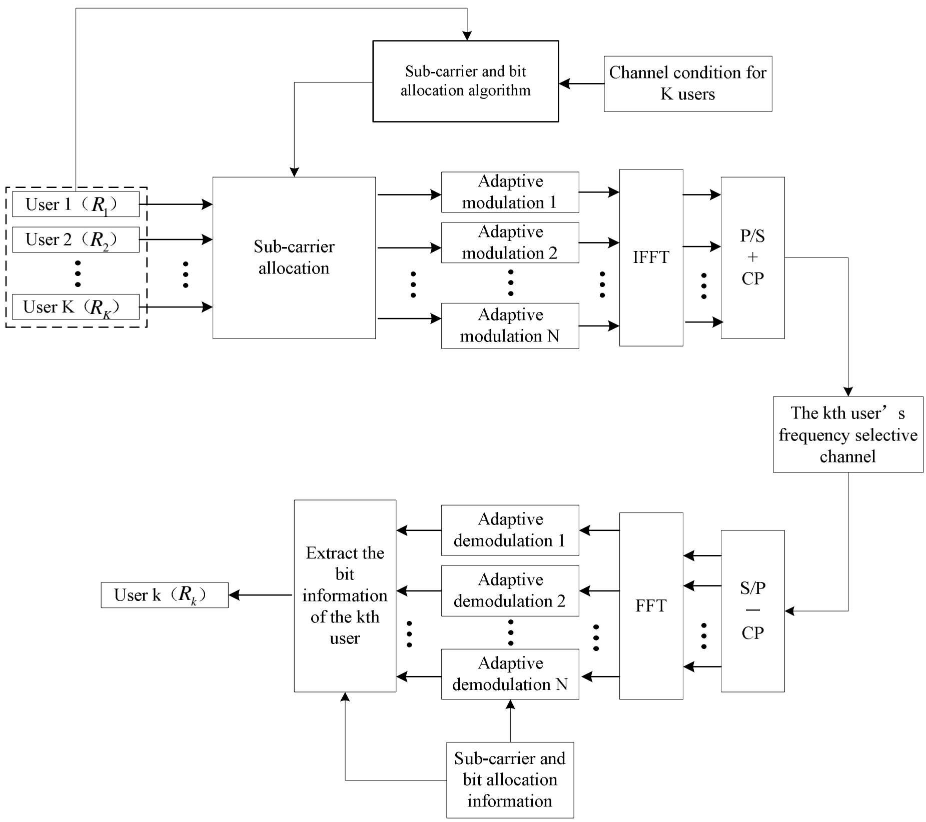

Fig. 1. Adaptive multi-user OFDM system model

Therefore, the transmitted power on sub-carrier n for user k is

$$P_{k,n} = \frac{f_k(c_{k,n})}{\alpha_{k,n}{}^2} \tag{1}$$

The total transmitted power for the system is

$$P_T = \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} = \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{f_k(c_{k,n})}{\alpha_{k,n}{}^2} \tag{2}$$

The margin adaptive (MA) optimization problem can then be formulated as:

$$P_T^* = \min \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} = \min \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{f_k(c_{k,n})}{\alpha_{k,n}{}^2} \tag{3}$$

and the minimization is subjected to the constraints:

$$\text{1) For all } k \in \{1, \cdots, K\} \quad R_k = \sum_{n=1}^{N} c_{k,n} \tag{4}$$

$$\text{2) For all } n \in \{1, \cdots, N\} \text{, if } c_{k',n} \neq 0 \text{, then } c_{k,n} = 0 \text{, } \forall k' \neq k \tag{5}$$

3 Algorithm Comparisons

3.1 Bit Allocation Algorithms for Single User

Before we solve the multi-user allocation problem, we first introduce bit loading algorithm for single user system, which will be applied in multi-user sub-carrier and bit allocation algorithm.

In single user environment, the margin adaptive optimization problem can be written as

$$P_T^* = \min \sum_{n=1}^{N} \frac{f(c_n)}{\alpha_n^2} \tag{6}$$

and the minimization is subjected to the constraint:

$$R_T = \sum_{n=1}^{N} c_n \tag{7}$$

A classical solution of this problem is greedy approach, which can be described as follows:

Step 1: Initialization. For all n, let $c_n = 0$, assume the modulation step length is Δc, then, compute the additional transmitted power $\Delta P_n = [f(\Delta c) - f(0)] / \alpha_n^2$

Step 2: Bit Allocation Iterations. Find the **minimum** ΔP_n, $\hat{n} = \arg\min_n \Delta P_n$, and add Δc bits on sub-carrier $\hat{n}$, $c_{\hat{n}} = c_{\hat{n}} + \Delta c$, then update the required additional transmitted power on sub-carrier $\hat{n}$, $\Delta P_{\hat{n}} = [f(c_{\hat{n}} + \Delta c) - f(c_{\hat{n}})] / \alpha_{\hat{n}}^2$

Step 3: Repeat. Repeat step 2) $R_T / \Delta c$ **times**, and $\{c_n\}_{n=1}^{N}$ is the final bit allocation solution.

3.2 Adaptive Sub-carrier and Bit Allocation Algorithms for Multi-users

In multi-user environment, as users can not share the same sub-carrier, allocating bits to a sub-carrier will prevent other users from using that sub-carrier. This makes the bit loading algorithm for single user could not be used directly for multi-users.

A number of papers have researched the sub-carrier and bit allocation algorithms for multi-user OFDM system. In [5], sub-carrier and bit allocation are done dynamically through the use of nonlinear optimization with integer variables. But an extremely computationally complex iteration is required to find the two Lagrangian multipliers which are used in the nonlinear optimization. So it may not be applied to practical system with high data rate transmission, and so we will not discuss this kind of algorithm further later in this paper.

In [6,7], C. Y. Wong et al present a method for real-time application to realize the sub-carrier and bit allocation, which separates the sub-carrier allocation from bit loading. After sub-carrier allocation is finished, bit loading is executed subsequently. As a result, the complexity declines. This scheme needs to convert all the user's data rate to user's sub-carrier demand, however, sometimes there may not all be matched. We denote this algorithm as **CYW algorithm** for brief in this paper.

In [8], a joint power and sub-carrier allocation scheme is proposed by E. Bakhtiari et al. This scheme can ensure each user's least sub-carrier demand. It executes in two steps: 1) In the first step, a simple solution is used to implement the initial sub-carrier allocation, which makes each user satisfy its lowest sub-carrier requirement. 2) An iterative improvement is then used to refine the solution. The iteration will stop when the reduction in total power is smaller than a threshold. We denote this algorithm as **EB algorithm** for brief in this paper.

And in [9], Guodong Zhang gives a new sub-carrier and bit allocation scheme. This scheme firstly assumes that all the sub-carriers can be used by all users, and bit loading algorithm is done for all the users. Then conflicting sub-carriers (shared by more than one user) are arbitrated to the user, which yields the lowest total reassignment power increase. Next, bits of other users are reassigned according to the reallocating solution and the conflicting sub-carrier list is updated. Then, repeat the arbitrating process of the conflicting sub-carriers until there are no conflicting sub-carriers. We call this algorithm as **Zhang algorithm**.

The flowchart of Zhang algorithm is shown in Fig. 2.

When dealing with the conflicting sub-carriers, **Zhang algorithm** arbitrates the conflicting sub-carriers to the user who brings the least increase of total transmit power. Viewed from another angle, it is equivalent to arbitrate the conflicting sub-carriers to the user who mostly needs to increase the transmit power if the conflicting sub-carriers are not given to the user. As a result, the computing complexity can be reduced to a certain amount.

However, there is a defect when actually using **Zhang algorithm**. If some user's magnitude of sub-channel gain is so small as compared to those of other users, most sub-carriers may be assigned to this user to minimize the total transmit power. Consequently, this user may occupy so many sub-carriers that other users have few sub-carriers, even fewer than the least sub-carriers requirement. **Zhang algorithm** will fail to carry on if this problem happens. So we improve this algorithm by adjusting the maximum sub-carriers that one user can occupy.

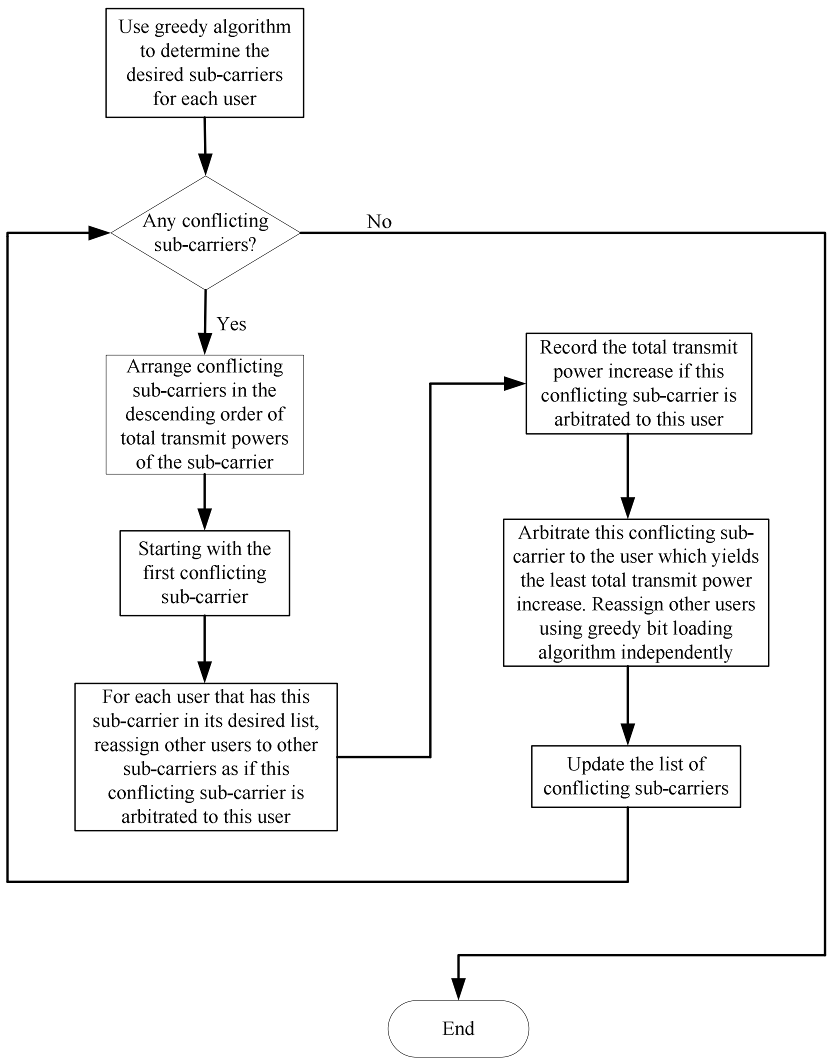

Fig. 2. The flowchart of **Zhang algorithm**

After adjusting, the maximum sub-carrier requirement of k^{th} user is

$$Th_k = \frac{T_k + N_{av}}{2}, \quad k \in \{1, 2, \cdots, K\} \tag{8}$$

where T_k is the maximum sub-carrier requirement of k^{th} user before adjusting, and $N_{av} = N/K$ is the average sub-carrier requirement of each user. So, T_k can be written as

$$T_k = N - \sum_{i=1, i \neq k}^{K} L_i , \quad k \in \{1, 2, \cdots, K\} \tag{9}$$

where L_k is the least sub-carrier requirement of user k, which can be defined as

$$L_k = ceil\left(\frac{R_k}{M} \right) \tag{10}$$

where, the function $ceil(\bullet)$ is rounded towards the nearest positive infinity. R_k and M have been defined in Section 2, being the data rate of user k and maximum available modulating level respectively. Some performance loss will happen after such adjustment to **Zhang algorithm**.

4 Simulation Results

Firstly, some assumptions are given here:

1) During an OFDM symbol, the channel fading is constant.
2) The channel estimation and synchronization are perfect.
3) The transmitter and receiver both know the channel information, and sub-carrier and bit allocation information.

And the simulation parameters are shown in Table 1.

For comparison purpose, some static sub-carrier allocation algorithms are briefly described here.

OFDM-FDMA with local sub-carrier allocation. Each user is allocated to a pre-determined sub-carrier group, which is composed of consecutive sub-carriers. Bit loading algorithm can be used to allocate bit of each user to sub-carriers. We call this algorithm as **Local OFDM-FDMA algorithm** in this paper.

Table 1. Simulation Parameters

Item	Parameters
Channel model	ITU VA
Number of users	4
Carrier frequency	2 GHz
Mobile speed of users	25 km/h
Doppler-shift	46.3 Hz
System bandwidth	5 MHz
Data rate of users	512 bit/OFDM symbol
Length of OFDM symbol	102.4 μs
Length of CP	25.6 μs
Sub-carrier requirements in **CYW algorithm**	128
Given BER	10^{-4}
Available modulation mode	No transmit, QPSK, 16QAM, 64QAM
Modulation mode in static allocation algorithms	16QAM

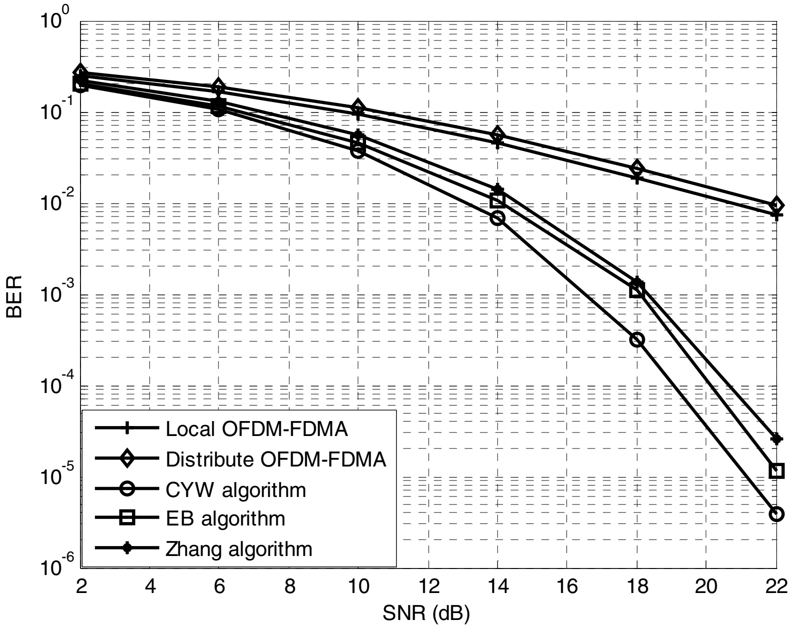

Fig. 3. Performance comparison for best user

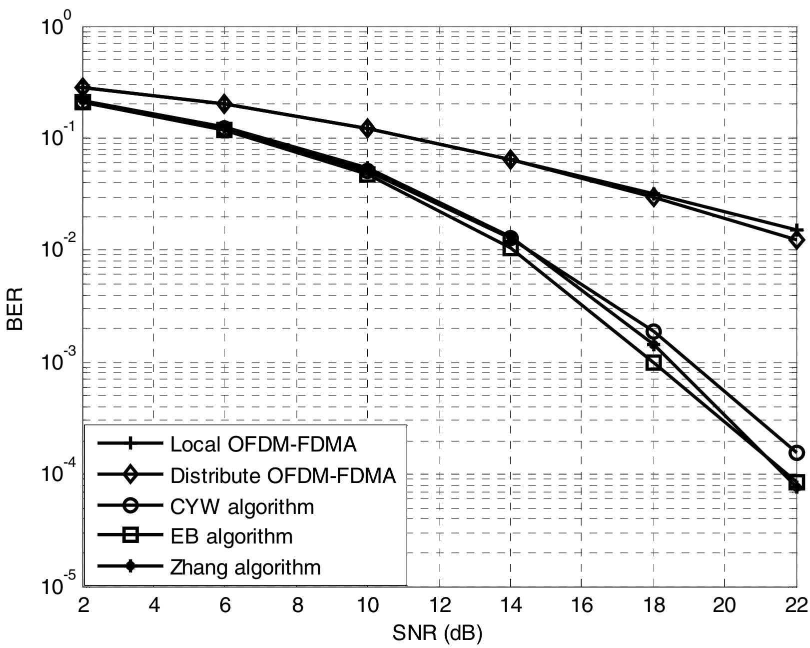

Fig. 4. Performance comparison for worst user

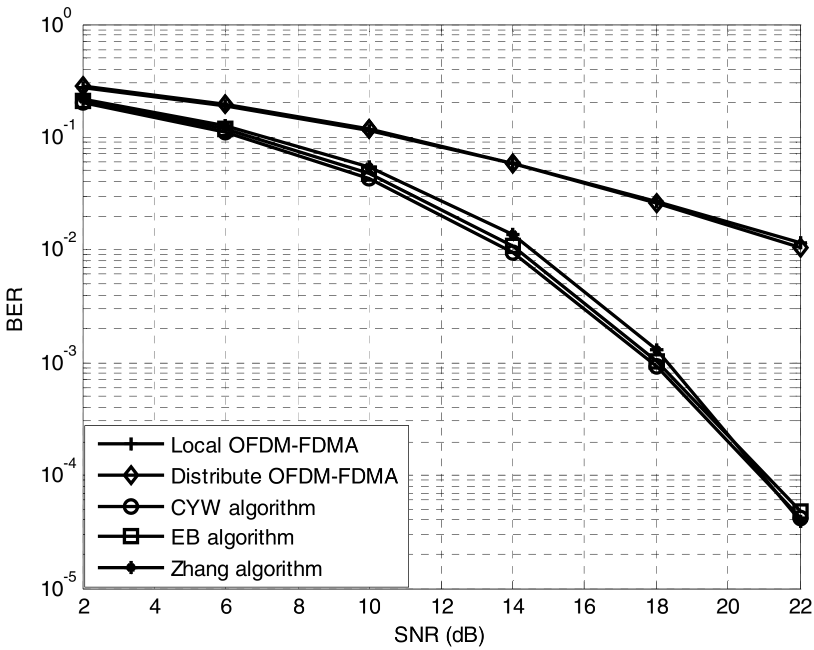

Fig. 5. Performance comparison for average user

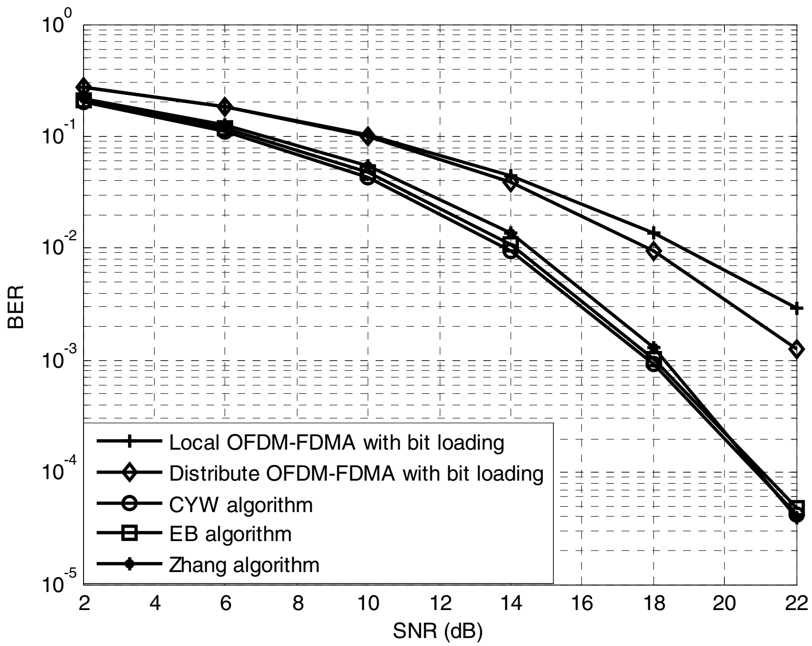

Fig. 6. Performance comparison with bit loading

OFDM-FDMA with distribute sub-carrier allocation. Each user is allocated to a pre-determined sub-carrier group, which is composed of comb sub-carriers. Bit loading algorithm can be used to allocate bit of each user to sub-carriers. We call this algorithm as **Distribute OFDM-FDMA algorithm** in this paper.

Simulation results are shown in Fig. 3, Fig. 4, and Fig. 5, which give the performance comparison of different algorithms for best user, worst user and average user cases respectively.

From these three simulation figures, we can conclude that the performance of OFDMA system with adaptive sub-carrier and bit allocation algorithms is much better than that of static OFDM-FDMA system. And for best user's performance, **CYW algorithm** is better than the other two adaptive algorithms. But **EB algorithm** and **Zhang algorithm**s' performances among their users are more stable than **CYW algorithm**. These three algorithms have similar performance for average user.

For further comparison, bit loading is applied to the static OFDM-FDMA systems. Fig. 6 presents the comparison results. Although after adding bit loading algorithm, static OFDM-FDMA systems get a large performance improvement, adaptive allocation algorithms still display an absolute superiority.

5 Conclusions

This paper discusses the principle of adaptive sub-carrier and bit allocation algorithms for multi-user OFDM systems. Three typical adaptive algorithms are analyzed and compared. The results show that these three adaptive algorithms have similar performance on average, and OFDMA systems with adaptive sub-carrier and bit allocation algorithms can get better BER performance than those of the static systems. Furthermore, among these three adaptive algorithms, **CYW algorithm** can get better performance in best users case, but **EB algorithm** and **Zhang algorithm** are superior in performance stability. Certainly, we have to pay more computing complexity and more time delay when we use these adaptive algorithms as compared with using a static system.

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References

1. Hughes-Hartogs, D.: Ensemble Modem Structure for Imperfect Transmission Media. U. S. Patents No. 4-679-22 (July 1987); 4-731-816 (March 1988); and 4-833-796 (May 1989)
2. Chow, P.S., Cioffi, J.M., Bingham, A.C.: A Practical Discrete Multitone Transceiver Loading Algorithm for Data Transmission over Spectrally Shaped Channels. IEEE Trans. on Comm. 43(4), 773–775 (1995)
3. Fischer, R.F.H., Huber, J.B.: A New Loading Algorithm for Discrete Multitone Transmission. IEEE Proc. Globecom 42(11), 724–728 (1996)
4. SHRCWC R1-051133.: DFT-S-GMC_GMC based SC-FDMA for 3GPP LTE uplink. San Diego, USA. 3GPP (2005)

5. Grüheild, R., Bolinth, E., Rohling, H.: A Blockwise Loading Algorithm for the Adaptive Modulation Technique in OFDM Systems. In: IEEE Vehicular Technology Conference, pp. 948–951 (October 2001)
6. Wong, C.Y., Cheng, R.S., Letaief, K.B., Murch, R.D.: Multiuser OFDM with Adaptive Subcarrier, Bit and Power Allocation. IEEE Journal on Selected Areas of Comm. 17(10), 1747–1758 (1999)
7. Wong, C.Y., Tsui, C.Y., Cheng, R.S., Letaief, K.B.: A Real-time Sub-carrier Allocation Scheme for Multiple Access Downlink OFDM Transmission. In: IEEE VTC 1999, pp. 1124–1128 (September 1999)
8. Ehsan, B., Babak, H.K.: A New Joint Power and Subcarrier Allocation Scheme for Multiuser OFDM Systems. In: IEEE PIMRC 2003, pp. 1959–1963 (2003)
9. Zhang, G.: Subcarrier and Bit Allocation for Real-time Services in Multiuser OFDM Systems. In: IEEE ICC, pp. 2985–2989 (June 2004)