Modeling of Channel Allocation in Broadband Powerline Communications Access Networks as a Multi-Criteria Optimization Problem

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Abstract. The planning process of the Broadband Powerline communications access networks contains two main problem parts: the *Generalized Base Station Placement* (GBSP) problem and the *PLC Channel Allocation Problem* (P-CAP). The GBSP is investigated/solved in our previous works. In this paper, we focus on the P-CAP. The task of the P-CAP consists in allocating a sub-set of channels from an available set of PLC channels to each base station in the B-PLC site. Two optimization objectives are considered for the solution of this problem; namely the maximization of the resource reuse and the minimization of the generated interferences in the site. These objectives are conflicting, since the optimization of one of them results in the deterioration of the other. Therefore, this problem is modeled as a Multi-objective (or multi-criteria) Optimization Problem (MOP). Three variants of Pareto-based multi-objective algorithms, using evolutionary search, are used to solve it. Their performances are evaluated on four problem instances.

Keywords: Channel allocation, broadband powerline communications, access network planning, multi-criteria optimization, evolutionary algorithms.

1 Introduction

Broadband PowerLine Communications (B-PLC) access network uses low-voltage supply networks as a transmission medium to provide high data rates to the end-users. Current B-PLC systems are reaching a raw bit rate of 200Mbps, which allows the realization of a large platform of services. The B-PLC presents an alternative solution for the realization of broadband access networks. Because of the high costs and the liberalization of the telecommunications market, the access area is very important for new network providers, who are trying to overcome their position, which is disadvantageous, compared to incumbent network providers. The direct access to the customers can be realized by building new networks or by use of existing infrastructure; e.g. CATV (Cable TV) or electrical power supply infrastructure. Installing new networks can realize high bit rates; such Passive Optical Network (PON) with up to Gbps, but it is economically not yet feasible. Another alternative is wireless solutions, in form of WiMAX, even if this reaches a maximum shared bit

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rate of 70Mbps. PLC access networks use the low-voltage networks to connect a number of subscribers within a geographically small area (~ few hundred meters), covering the "last meters" communications areas, and represent a cost-effective solution. This is achieved by installing a base station (BS, called also Head-End -HE), which builds a kind of bridge between the telecommunications world (i.e. backbone network) and low-voltage network; as depicted in Figure 1.

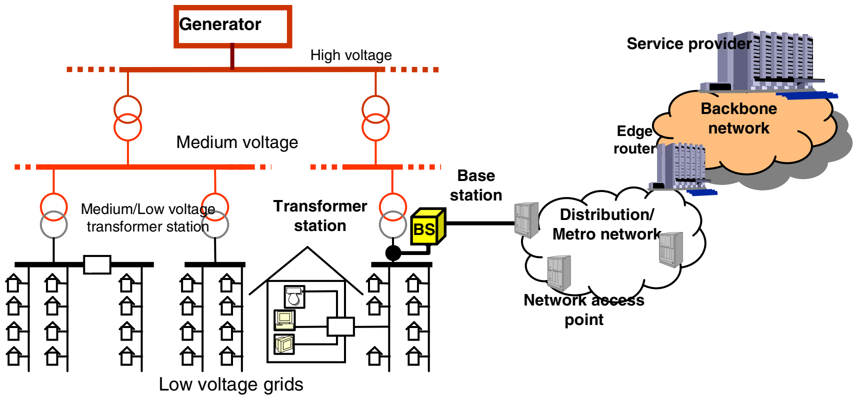

Fig. 1. Typical structure of a Broadband Powerline Communications (B-PLC) access networks

The power supply networks are not designed for communications and they do not present a favorable transmission medium. Therefore, the PLC transmission channel is characterized by a strong attenuation, changing impedance and fading, as well as a strong influence of noise caused by various devices usually connected to the supply networks. This could require the installation of PLC repeaters to allow the extension of the system coverage/reachability. Additionally, PLC networks providing higher data rates have to operate in a frequency spectrum up to 30 MHz, which is also used by other radio services. Therefore, the regulatory bodies specify very strong limits regarding the electro-magnetic emission from PLC networks to the environment. As consequence, an optimal use of the resources (i.e. the available frequencies) is decisive in the design and the management of this access network. This paper investigates the resources (channels) allocation to the different installed bases stations in the low-voltage grid. The goal of these investigations is the maximization of the resource reuse in the network; however, this maximization must not lead to (high level) interferences. In this paper, the PLC Channel Allocation Problem (P-CAP) is modeled as a Multi-criteria Optimization Problem (MOP). This problem is described, mathematically formulated and solved by evolutionary search-based metaheuristics.

The paper is organized as follows: An overview on the design of the B-PLC access network is given in the next section. Also the design of B-PCL and wireless network is done here. In the third section, the P-CAP is described in more details, its input information is analyzed and the optimization objectives are mathematically formulated. Short overview on the optimization algorithmic is given in the fourth section. Section five is reserved for the experiments and the evaluation of the results.

2 Planning Process of B-PLC Access Networks

2.1 Design of B-PLC Sites

The realization of Broadband Power Line Communications (B-PLC) access network consists of placing an element called Base Station (BS), which allows the connection of the Low-Voltage Network (LVN) and the backbone network. This BS uses a part of the power line spectrum to communicate with the PLC modem of the end-user. The planning process of the B-PLC access networks can be subdivided into two main problems; [1]. The first one is called *Generalized Base Station Placement* (GBSP) Problem, where an optimum number of base stations have to be installed to serve all the PLC subscribers. Each BS gets allocated a subset of users. If a user is located above a certain coverage distance from his BS, one or several PLC repeaters will be used. Adequate locations have to be determined, where such repeaters can be installed. The installed base stations have to be connected to the backbone/metro network over some network access points. Such access points can be for example an Optical Line Termination (OLT) or a Passive Optical Network (PON) node that could be available in the neighbourhood of the low-voltage grid. Solutions for these tasks must be realized with minimum network costs and a good quality of service. The PLC systems considered in this paper are assumed to use Orthogonal Frequency Division Multiplexing (OFDM) modulation, which has been chosen as a standard within the OPERA consortium [2]. In OFDM systems, the spectrum is subdivided into small frequency bands called subcarriers. If two base stations, which are too close to each other, use the same subcarrier, then interferences between them will occur. Therefore, a kind of subcarrier (i.e. frequency) allocation has to be realized, so that each BS gets enough frequencies to serve its users, but without interfering with its neighbours. To simplify the problem, we assume that a subset of the available subcarriers is virtually bundled to build an entity called *PLC channel*, with capacity K_{Ch}=*64kbps*, as illustrated in Figure 2. In this case, the problem of allocating frequencies to BSs can be called *PLC Channel Allocation Problem* (P-CAP). This problem is analysed in this paper and solutions are proposed and evaluated. The P-CAP has similarities with CAP in mobile networks. Therefore, similar analysis could be used to model the P-CAP. Such similarities may lead to the assumption that P-CAP is also a NP-hard optimization problem like wireless CAP; [3].

The different subtasks of the design of the broadband powerline communications access networks can be organized under two main subproblems; a GBSP and a P-CAP.

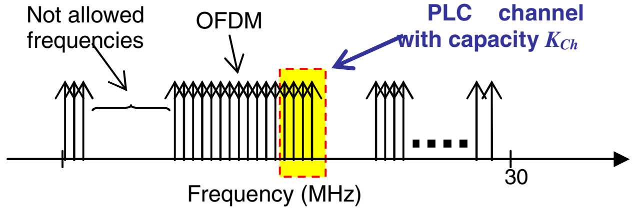

Fig. 2. Organization of powerline spectrum utilization

The entity built by the quintet: BS *j*, its user sub-set $|U^{(j)}{}_S|$, its set of repeaters, its WAP and the channel sub-set $F^{(j)}{}_S$ builds a *PLC cell*. A set of PLC cells of a LVN builds a *PLC site*, as shown in Figure 3 using a German LVN.

2.2 Comparison with Wireless Networks

As already mentioned above, the design process of B-PLC access networks has a strong similarity to the design of wireless networks. However, the B-PLC design has the following characteristics:

Complex cells and neighbourhood structures: PLC uses a physical medium that is having a certain wiring topology, and not like the wireless network that is using the free space. Because of that, a PLC cell *j* can have two types of neighbourhoods (or neighbours), which are called *in-line* and *in-space neighbourhood*. Thus, if a cell *j'* is belonging to in-space neighbourhood $\mathcal{N}^{(j)}{}_{in\text{-}space}$ of cell j, this means that one or more segments building the wiring of cell *j'* are located too near to a line segment belonging to cell *j*. This is the case of cell 7 and cell 2 in Figure 4. If cell *j'* is belonging to the in-line neighbourhood $\mathcal{N}^{(j)}{}_{in\text{-}line}$ of cell *j*, then these cells have one or more common line segments; like cells 3 and 2 in Figure 4. Therefore, the standard wireless model, where a cell is represented as a hexagon having at maximum six neighbouring cells is not valid here. A *neighbourhood matrix* ($\boldsymbol{M}_{\mathcal{N}}$) is used to define the neighbourhood of cells in PLC site. An element $\eta_{jj'}$ of this matrix is one if the cells *j* and *j'* are neighbours; and zero otherwise. General qualitative comparison between wireless cells/site and B-PLC cells/site is illustrated in Figure 3;

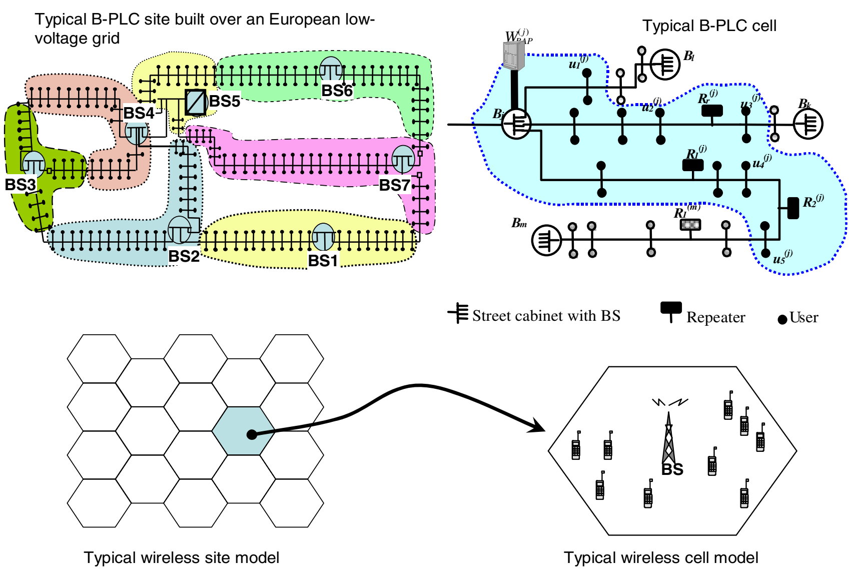

Fig. 3. Analogy between B-PLC sites/cells and wireless sites/cells

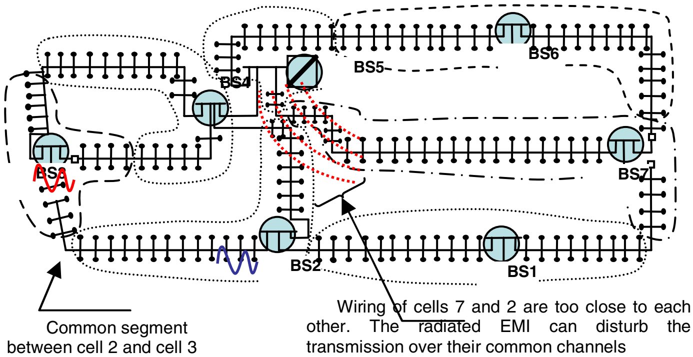

Fig. 4. Types of interferences in a B-PLC site

Two types of interferences: A PLC cell j can have two types of neighbours (in-line and in-space ones); therefore, there are two types of interferences in PLC site. On one hand, the in-space interferences that are Electro-Magnetic Interferences (EMI) and are similar to those encountered in wireless sites. On the other hand, there are the in-line interferences that are conducted interferences. These occur between cells that are in-line neighbours. Interference occurrence is illustrated in Figure 4;

Use of wired medium requires recovery mechanism: the B-PLC uses an existing wired topology, which limits the number of potential locations, where networks elements could be placed. Also each BS and its users must be in-sight; [1]. Furthermore, a physical medium is a subject of failures (e.g. cuts in the line). In this case, recovery mechanisms are needed to overcome this problem;

Use of repeaters: because of the usage of PLC repeaters, for extending BS coverage, the BS and its repeaters have to be multiplexed either in time or in frequency domain or both. In all cases, the effective bit rate of a cell j is lower than the row bit rate offered by the allocated channels. Thus, quality of service (delay, bit rate, etc.) can be different from a user to another according to his distance from BS.

3 Analysis of PLC Channel Allocation Problem

3.1 Problem Definition

For solving a P-CAP for a given B-PLC problem instance, some information about the GBSP solution is necessary. It is necessary to know the number of installed BSs (i.e. cells) in the site (referred by $|B_S|$), the number of users allocated to each BS and the characteristic of the traffic generated by them, and the neighbourhood matrix ($\boldsymbol{M}_{\mathcal{N}}$). This allows knowing the neighbourhood of each cell and the calculation of the elements of the interference distance matrix ($\boldsymbol{M}_{ID}$). An element $\delta_{jj'}$ of this matrix defines the minimum channel distance separation between the channels allocated to cell j and those allocated to cell j', such that no interferences occur. A P-CAP solution without interferences is a solution where the distances between the channels of cells are equal or larger than the values specified by $\boldsymbol{M}_{ID}$. Due to some possible technical or regulatory restrictions, some channels may not be available in some PLC environments. Such channels are said to be *locally blocked*, and build the set $F^{(j)}_{block}$, which has to be specified for every cell *j*, in order to be avoided during the allocation.

$$\mathbf{M}_{CA} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,f} & \cdots & a_{1,|F_S|} \\ \vdots & \ddots & & & \vdots \\ a_{j,1} & & a_{j,f} & & a_{j,|F_S|} \\ \vdots & & & \ddots & \vdots \\ a_{|B_S|,1} & \cdots & a_{|B_S|,f} & \cdots & a_{|B_S|,|F_S|} \end{pmatrix} \quad \text{such that } a_{jf} = \begin{cases} 1; & if\ j\ uses\ f \\ 0; & otherwise \end{cases} \qquad (1)$$

The task of any P-CAP solver is to allocate to each cell *j* a subset $F^{(j)}_{alloc}$ from the set of the PLC system channels F_S. Each cell should get enough capacity to serve its users adequately. A P-CAP solution can be expressed in the form of a ($|B_S| \times |F_S|$) matrix is called *channel allocation matrix* ($\boldsymbol{M}_{CA}$) as represented in (1). This matrix is built by the *allocation decision variables* a_{jf}, which values must be optimally chosen from set {0;1}. The problem investigated here is Fixed P-CAP and not dynamic, since each cell gets a fix channel subset.

Generally, the different mathematical models that are met in wireless CAP could be applied for P-CAP. Examples of such models are: *the maximum service* proposed in [4], *minimum block* in [5], *minimum order* (MO-FAP) in [6], *minimum span* (MS-FAP) in [6], and *minimum interferences* (MI-FAP) in [7], etc. In this work, maximum service (referred here as maximum resource reuse and synonym to *maximum network throughput per cell*) and minimum interferences are considered. They are two conflicting objectives, and this makes the P-CAP a MOO problem.

The P-CAP can generally be defined as an optimization problem by the following:

Given: Neighbour matrix $\boldsymbol{M}_{\mathcal{N}}$; interference distance matrix $\boldsymbol{M}_{ID}$; set of available PLC channels (F_S); traffic demand; set of locally blocked channels for each cell j ($F^{(j)}_{blocked}$);

Tasks: Allocate to each cell j sub-set of channel $F^{(j)}_{alloc}$;

Objectives: *maximize* resource reuse, *minimize* interferences

3.2 Traffic Demand in B-PLC Sites

Each BS *j* serves a set of users $U^{(j)}_S$. Each user *i* generates a traffic demand $A_i^{(j;UL)}$ in uplink (UL) and a traffic $A_i^{(j;DL)}$ in downlink (DL) direction, with:

$$A_i^{(j,UL)} = \frac{\lambda_i^{(j,UL)} / \mu_i^{(j,UL)}}{K_{Ch}} \text{ and } A_i^{(j,UL)} = \frac{\lambda_i^{(j,UL)} / \mu_i^{(j,UL)}}{K_{Ch}}$$

with $\lambda_i^{(j,UL)}$ and $\lambda_i^{(j,UL)}$ are the mean inter-arrival rate of the data packets of user *i* in UL and DL; respectively. The arrival processes are assumed to be a Poisson process. The average packet length of this user in UL and DL are $1/\mu_i^{(j,UL)}$ and $1/\mu_i^{(j,DL)}$; respectively. The constant K_{Ch} is the channel capacity. Let us assume $\lambda_i^{(j,UL)} = \lambda_{UL}$, $1/\mu_i^{(j,UL)} = 1/\mu_{UL}$, $\lambda_i^{(j,DL)} = \lambda_{DL}$ and $1/\mu_i^{(j,DL)} = 1/\mu_{DL}$ for all users *i* and all cells *j*. Thus, the total traffic demand in cell *j* has the following form:

$$A_{Total}^{(j)} = \left|U_S^{(j)}\right| \cdot \left(\frac{\lambda_{UL} / \mu_{UL}}{K_{Ch}} + \frac{\lambda_{DL} / \mu_{DL}}{K_{Ch}} \right) = \left|U_S^{(j)}\right| . A_{User} \tag{2}$$

Thus, the number of channels needed by BS j is $F_{demand}^{(j)}$ that can be formulated by, where "$\lceil . \rceil$" is the rounding operator:

$$F_{demand}^{(j)} = \left\lceil A_{Total}^{(j)} \right\rceil = \left| \left|U_S^{(j)}\right| . A_{User} \right| \tag{3}$$

3.3 Resource Reuse and Network Throughput

The main goal of the P-CAP is to allocate from the available channels as much as possible to each cell. This means to maximize the value the elements a_{jf} of the channel allocation matrix ($\boldsymbol{M}_{CA}$), and this can be formulated as follows:

$$maximize \quad f_{RR} = \frac{1}{\left|B_S\right|} \sum_{j \in B_S} \frac{\left|F_{alloc}^{(j)}\right|}{F_{demand}^{(j)}} \tag{4}$$

Such that:

$$\left|F_{alloc}^{(j)}\right| = \sum_{f \in F_S} a_{j,f} \qquad \forall j \in B_S \tag{5}$$

$$a_{j,f} \in \{0;1\} \qquad \forall j \in B_S ; \forall f \in F_S \tag{6}$$

$$a_{j,f} = 0 \qquad \forall j \in B_S ; \forall f \in F_S \wedge f \in F_{block}^{(j)} \tag{7}$$

The resource reuse measure (f_{RR}) supplies a normalized metric, which simplies the theoretical analysis. However, network operators could be interested in having a practical measure of network performance, such as network throughput. In our previous investigations, we considered the *Cell Throughput* ρ_{Cell} as practical metric; [8]. This was derived from f_{RR} and represents the average capacity (or bit rates) of the cells. The maximization of the network throughput has the same constraints.

3.4 Modelling of Interferences in B-PLC Sites

The interference between two different cells of the site is called *co-site interferences* in wireless networks. In such a network, the co-site interferences can be treated in different ways. On one hand, the co-site interference is considered as a hard constraint. This means, any candidate solution that generates any co-site interference (i.e. the distance $\delta_{jj'}$ is not respected at least once) is considered as infeasible solution rejected. This approach has been used in [9] [10]. On the other hand, in [11] co-site interferences occurrence is accepted; however, they must be as low as possible. This means, the distance $\delta_{jj'}$ can be not always fulfilled. In this case, the interference minimization in CAP becomes similar to the *Constraint Satisfaction Problem* (CSP). In CSP, if no solution exists that satisfies all constraints, the solver tries to find a solution that minimizes the number of constraints violations, or to minimize some function(s) of the costs incurred to the violated constraints. This is used in this work.

In B-PLC network, two types of co-site interferences that require frequency separation can occur:

In-line interferences: These occur in the case illustrated in Figure 4, where the disturbance source and the victim are in-line neighbours. The coupling path is in this case the power line segment that is common between the cells;

In-space interferences: the source and the victim are in-space neighbours, and the coupling path is the space separating their cables. In this case, the signal from a cell j generates an electromagnetic field (seen by a cell *j'* as radiated disturbance). Such example is illustrated in Figure 4. This means that some of the cells wires are geographically very close to each others, so that interfering signal is strong enough to disturb the original signal. The extreme case of in-space neighbouring is the case where one or more segments from both cells are in the same bundle or duct.

For simplification reasons, the in-space interferences are not considered. Therefore, a frequency separation is needed between the channels of two cells, only if those cells are in-line neighbours. Therefore, the elements $\delta_{jj'}$ can be defined as follows, where $\Delta F_{in\text{-}line}$ is a constant:

$$\delta_{jj'} = \begin{cases} \Delta F_{in-line}; & if \quad j' \in \aleph_j^{(in-line)} \\ 0; & otherwise \end{cases} \tag{8}$$

The objective of P-CAP optimization is the minimization of interferences by minimizing number of interference distance violations. This is formulated as follows:

$$minimize \quad f_I = \sum_{j \in B_S} \sum_{j' \in \aleph_j^{(in-line)}} \sum_{g \in F_S^{(j)}} \sum_{g' \in F_S^{(j')}} \kappa_{gg'}^{(jj')} \tag{9}$$

Such that: $$\kappa_{gg'}^{jj'} = \begin{cases} I_{in-line}^{(Interf)}; & if \quad |g - g'| < \delta_{jj'}; \\ 0; & otherwise \end{cases} \qquad \forall j, j' \in B_S; \forall g \in F_S^{(j)}; \forall g' \in F_S^{(j')} \tag{10}$$

Finally, the P-CAP can be formulated as a MOP as follows:

$$minimize \quad \mathbf{f}_{P\text{-}CAP} = [-f_{RR}, f_I] \tag{11}$$

such that: (4), (5), (6), (7), (9) and (10).

4 Optimization Approaches and Used Metaheuristics

The classical method for solving the MOP problems consists in aggregating the different objectives into a single general objective function, which is then optimized by the means of the traditional Single-Objective Optimization (SOO) algorithms. This method is called “scaling method” that combines linearly the objectives using weighting factors w_i’s, $\sum w_k=1$. In the last years, the Multi-Objective Optimization (MOO) (called also Multi-Criteria Optimization -MCO) approach has been intensively investigated, in order to optimize the conflicting objectives quasi-independently. Different MOO Algorithms (MOA) have been developed, in order to solve the MOPs without using the conversion into SOO. A MOA explores the solution space by eliminating dominated (i.e. the worst) solutions and keeping

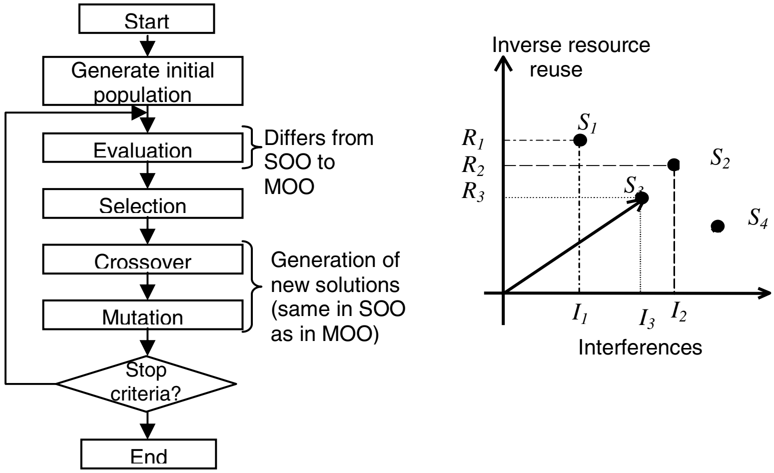

Fig. 5. General flowchart of evolutionary search-based algorithm (*left*) and principles of the multi-criteria optimization in case of minimization of both objectives (*right*)

non-dominated (i.e. the relative good) ones to build the so-called *Approximation Set* ***A*** (called also *Front* or *Pareto Front* ***A***). An example of the solutions comparison in a two-dimensional optimization space is depicted in Figure 5. In this example, two objectives are minimized: the interferences and the inverse of resource reuse (i.e. maximization of reuse). The comparison of solutions S_1 and S_2 shows that the first one is better than the second, because $I_1<I_2$ and $R_1<R_2$. In this case, S_1 dominates S_2; therefore, S_2 is seen as bad (i.e. dominated) solution and has to be eliminated. The comparison of S1 and S3 shows that S1 is better than S3 in the interference dimension ($I_1<I_3$), but worst in the other dimension ($R_1>R_3$). In this case, S_1 and S_3 are called to be indifferent, also non-dominated. At the end of the algorithm iterations, the non-dominated solutions build the approximation set. In the example, the approximation set is built by S_1, S_3 and S_4, i.e. $\boldsymbol{A}=\{ S_1,S_3,S_4\}$.

In these investigations, Metaheuristics have been used, because they deliver good (but not always the best) solution to the problem in relative short computation time, in comparison to exact algorithms. The class of Metaheuristics is very large. From this class, the metaheuristics using the evolutionary search have been selected, because of their successful application in different engineering application, especially in network planning. In the numerical experiments, the SOO uses the genetic algorithm; [12]. For MOO both versions of Non-dominated Sorting Genetic Algorithm (NSGA and NSGA-II) ([13]) and Strength Pareto Evolutionary Algorithms (SPEA) ([14]) are applied. The evolutionary search consists in sampling the search space in a controlled random way. To realize this search, an initial population (i.e. set of solutions/individuals) is randomly generated. Iteratively, this population is evaluated and some solutions (generally the fittest/best) to generate new individuals/solutions until a break criteria is filled. The new solutions are generated using by means of crossover and mutation operations, according the flowchart shown in Figure 5; [12].

These genetic operators have a decisive effect on the algorithms performances, as we will see in the section of numerical results.

5 Numerical Experiments and Results Analysis

5.1 Problem Instances and System Characteristics

The experiment results have to be realized for different problem instances, in order to evaluate the optimization algorithm in different levels of problem complexity, such different number of cells, different number of users per cells, etc. For the generation of the problem instances for P-Cap, a low-voltage network with 14 locations for the BS/repeaters placement and 943 users. Through the solution of the generalized bases stations placement problem with different heuristics, two solution of B-PLC have been selected as instances. On one hand, a B-PLC site is constituted by 9 cells, which use the PLC system#1 in one case and a system#2 in the other case. These systems are also used to build the other remaining two P-CAP instances, which site contains 12 cells. This results in four P-CAP instances; as presented in Figure 6.

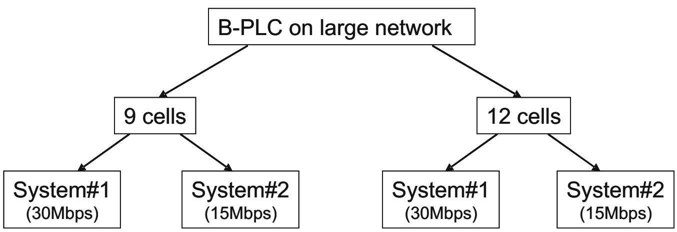

Fig. 6. Problem instances used in the numerical experiments

The four P-CAP instances are modeled by the neighborhoods matrices and the number of users per each cell. Users' distribution in cells is shown in Tables 1 and 2. Two PLC systems are considered, so that system#2 has $|F_S|$=235 (which characterizes a system with 15Mbps) and system#1 with $|F_S|$=469 (equivalent to 30Mbps).

For the interference calculation, we take $I^{in\text{-}line}{}_{interf}$=1 (i.e. no penalization function, only constraints violations counting), $\delta_{jj'}$=1. The users traffic demand is characterized by A_{User}=3, which represents an average of 128kbps in the downlink and 64kbps for the uplink. The statistical results are generated from 21 experiment runs.

Table 1. Number of users per cell inside the 9 cell-based site

BS	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$\lvert U^{(j)}{}_S \rvert$	137	137	99	70	None	None	173
BS	**8**	**9**	**10**	**11**	**12**	**13**	**14**
$\lvert U^{(j)}{}_S \rvert$	None	156	90	None	None	42	30

Table 2. Number of users per cell in case of 12cell-based site

BS	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$\lvert U^{(j)}_S \rvert$	105	80	62	149	None	47	54
BS	**8**	**9**	**10**	**11**	**12**	**13**	**14**
$\lvert U^{(j)}_S \rvert$	None	109	88	58	30	68	84

5.2 Solution Encoding and Evolutionary Operators

The encoding of P-CAP solution is realized by the means of a binary matrix that builds a solution (or an individual) in the form of an allocation matrix. The columns are corresponding to the available PLC channels, while the rows are corresponding to the installed BSs. The bit in the j-th row and f-th column corresponds to the element $a_{j,f}$ of the allocation matrix. This is equal to 1 if the channel f is allocated to the cell j in the considered solution (i.e. individual). Otherwise, this bit is a zero. There are different ways to realize the crossover of two matrices or individuals. In this paper, four schemes are considered and evaluated. These crossover schemes are illustrated in Figure 7. The first crossover scheme is referred to as 1-Point Simple Crossover (1-PSC) (Figure 7-a, 1st row-left). In this scheme, the parent solutions are cut vertically at one and the same point for all rows (i.e. cells). The same is realized also in the 2-Point Simple Crossover (2-PSC) (Figure 7-b, 1st row-right). But in this scheme the matrix is cut in two different points by 2-PSC instead of only one point like 1-PSC. The third crossover operator is called 1-Point Multiple Crossover (1-PMC) (Figure 7-c, 2nd row-left). In this case, each row of the matrix is cut exactly at one point, but this point differs from a row (i.e. cell) to another. Each row can be also cut at two different locations and in this case we have 2-Point Multiple Crossover (2-PMC) (Figure 7-d, 2nd row-right). All the four crossover schemes are tested later and compared by the numerical results, in order to choose the efficient one for the generation of the final numerical results that will be used for the evaluation of the algorithms performance. The mutation operator can be built also in different ways. Among the possible schemes, two variants have been selected for test and evaluation in this section; namely the 1-Bit Mutation (1-BM) and the 1-Cell Bit Mutation (1-CBM), as represented in Figure 8-*left*. In the 1-BM, exactly one bit is chosen randomly in the whole matrix and its values is switched by a given probability (mutation probability -p_m). In the 1-CBM, one bit is randomly selected from each matrix row (i.e. cell) and its value is switched by a given mutation probability.

5.3 Parameters Setting

The genetic operators (crossover and mutation) are decisive factors for the evolutionary algorithm performance. Because of that, ten simulation runs have been executed, in order to check the effect of the different operators' realizations on the evolutionary search. More intention has been given to the effect of those schemes in the multi-objective optimization paradigms. For such initial experiments, the following parameters have been used: 100 generations, 100 individuals in the

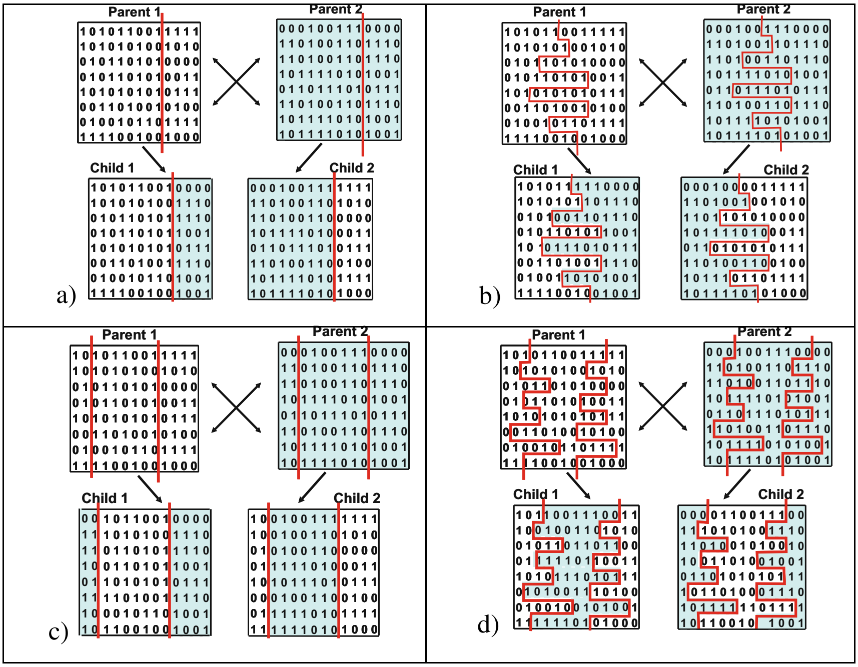

Fig. 7. Different schemes of crossover operator that can be used for P-CAP evolutionary search

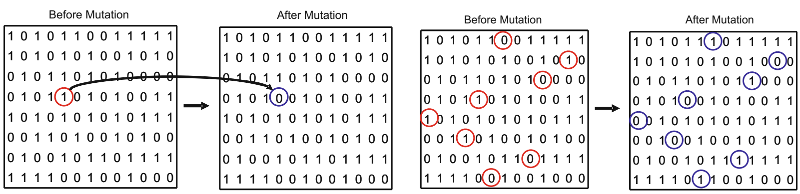

Fig. 8. Mutation operators for the evolutionary search

population, crossover probability is 1 and mutation probability is 0.1 using 1-Bit Mutation for crossover schemes testing. All the runs have issued similar results. A sample of the resulted approximation sets is shown in Figure 9 for crossover. For the crossover, the 2-PMC supplies better front, which converges better (i.e. faster) than the other three schemes; for of all three MOEAs. Such behavior is expected, because this scheme allows the realization of larger springs in the search space. A large spring is represented in large difference between parents' solutions and their children. The difference considered here is in the representation (i.e. coding) space. These wide springs are the main characteristics of the global search, such as the evolutionary search or any other population search-based algorithm. In other words, the 2-PMC crossover scheme allows the population to keep a higher diversity. Such diversity prevents the search form getting trapped in local optima. A possible drawback of very high diversity in the population could result in a bad convergence, because in such case the search becomes more oriented toward exploration of the search space, but with a bad exploitation (i.e. learning process).

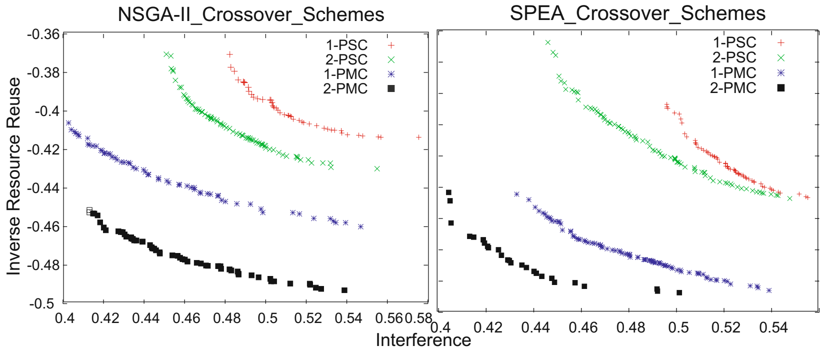

Fig. 9. Approximation sets of diff. crossover schemes using NSGA-II (*left*) and SPEA (*right*)

Similar remarks and argumentation are valid also for the mutation operators. The 1-CBM mutation method has shown a clear dominance in comparison to the 1-BM method. For the final numerical experiments, the number of generations has been set to 300 generations, while the population size is 150 individuals. With higher number of iterations, the drawback of the 2-MPC scheme becomes clearer, in comparison to the 1-PSC for the case of SPEA and NSGA-II. The very high diversity lets the individuals be spread out in the objective space and only a few individuals remain not dominated. This is reflected by the low cardinality of the approximation set. A very high diversity becomes destructive for the convergence of the population. The scheme with 1-PSC allows a progressive convergence of population individuals. This leads to dense non-dominated set. Furthermore, a slower convergence (or small springs) allows a balanced convergence. This means, that the non-dominated solutions are almost equally distributed over the front spread in both optimization objective dimensions. The relative behavior of 2-PMC and 1-PSC in this case is similar to the relative behavior of genetic algorithms (which is a global search) and the simulated annealing (which is local search) in the SOO. In fact, the GA converges early and more quickly than the SA. But if the optimization algorithm has enough time for the convergence (i.e. higher number of iterations), then the SA will converge later, but to better solution than GA, which early convergence leads to less good solution. However, 2-MPC has been kept for numerical experiments, in spite of this drawback, because 2-MPC could be strongly preferred for its higher resource reuse.

From the discussion above and from different test runs, the following parameters are set for the numerical experiments: Number of generations: 300; Population size: 150; Crossover probability: 1; Mutation probability: 0.1; Crossover method: 2-PMC; and Mutation method: 1-CBM.

5.4 Evaluation of Pareto-Based MOEAs

The evaluation of the MOEAs in the P-CAP is based on the coverage of two sets. This metric measures the percentage of solutions in front $\boldsymbol{A_1}$ that are dominated by at least one solution from front $\boldsymbol{A_2}$. It shows how good is the solutions convergence towards the optimal front in comparison to solutions of the other front. The coverage of two sets shows also a constant dominance behavior between the algorithms over all four

Table 3. Coverage of two sets for MOEAs in case of 9cells-based site

9 Cells-based site – System#1			
	NSGA	NSGA-II	SPEA
NSGA	#	0.07±0.01	0.55±0.19
NSGA-II	0.84±0.11	#	0.93±0.06
SPEA	0.33±0.15	0.04±0.06	#
9 Cells-based site – System#2			
	NSGA	NSGA-II	SPEA
NSGA	#	0.09±0.14	0.61±0.21
NSGA-II	0.88±0.1	#	0.9±0.14
SPEA	0.28±0.25	0.05±0.08	#

Table 4. Coverage of two sets for MOEAs in case of 12cells-based site

12 Cells-based site – System#1			
	NSGA	NSGA-II	SPEA
NSGA	#	0.06±0.06	0.38±0.26
NSGA-II	0.9±0.06	#	0.72±0.27
SPEA	0.54±0.25	0.14±0.13	#
12 Cells-based site – System#2			
	NSGA	NSGA-II	SPEA
NSGA	#	0.15±0.12	0.54±0.18
NSGA-II	0.71±0.14	#	0.8±0.08
SPEA	0.38±0.24	0.11±0.09	#

considered P-CAP instances. It can be generally concluded from the coverage of two sets shown in Tables 3 and 4 that NSGA-II outperforms the SPEA, while the SPEA is dominating the NSGA. For example, in case of 9 Cells with System#1 instance, the SPEA dominates about 86\% of all NSGA solutions, while only 8% of them are dominated. The NSGA is also outperformed by its extension (i.e. NSGA-II). In the same instance example, the NSGA-II dominates about 90% of NSGA solutions, while only 5% of its solutions are dominated. The difference in the coverage of two sets metric between NSGA and NSGA-II is clear in P-CAP, as can be observed in Figure 10. A reason of the relative low convergence is the possible losses of the good

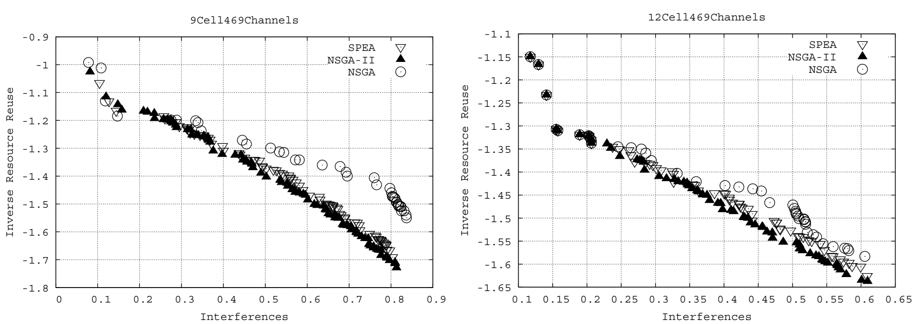

Fig. 10. Samples of approximation sets achieved by the Pareto-based MOEAs in case of 9Cell- and 12Cells-based B-PLC sites using system#1

solutions found during the search, either through the destruction through genetic operators (crossover and mutation) or through the randomness in the selection procedure. The destruction of the good solutions by the crossover operators is more probable with 2-PMC mechanism, because it introduces strong changes on the resulting offsprings compared to their parents. Such destructive effect is less present in other crossover schemes, especially with 1-PSC. This problem is avoided by NSGA-II and SPEA by using the elitism.

During the parameter setting for the numerical experiments, a big influence of the used crossover scheme was observed, as discussed previously. The results discussed above have been realized with 2-PMC crossover scheme. Results with 1-PSC have been also realized and a part of them is given in the appendices for two problem instances only. Three observations have been made concerning the MOEAs results: *(i)* the very large cardinalities of the approximation sets (for all three variants); *(ii)* this higher number of solutions performs finer numerical results for the coverage of two sets metric (with very small confidence intervals). This exactitude comes also from the fact of the stability on the convergence, where the MOEA explores the whole Pareto fronts, on the contrary to the 2-PMC MOEAs that converge to a narrow part of the front; and *(iii)* the NSGA-II shows also in this scenario with 1-PSC best performances, followed by SPEA and then NSGA (on the contrary to the 2-PMC where NSGA has light advantage compared to SPEA).

6 Conclusions

This paper deals with the design of Broadband Power Line Communications access networks, with focus on the PLC Channel Allocation Problem (P-CAP). In the P-CAP, each installed BS (or cell) has to get allocated a subset from the set of the available B-PLC channels. This problem is a Multi-objective Optimization Problem (MOP), where the resource reuse has to be maximized, while the interferences have to be kept as minimal as possible. The interferences in a B-PLC have been categorized into two main classes; the in-line and the in-space interferences. In the application section only the in-line interferences have been considered, while the in-space interference that are more complex to model have been avoided.

For the solution of the P-CAP, two optimization paradigms have been used: the single objective optimization (SOO) where the MOP is converted into a single-objective problem and the Multi-objective Optimization (MOO), where the objectives are optimized quasi-independently from each other. In the numerical experiments two problem instance sizes have been used (9 cell- and 12 cell-based sites) with two possible B-PLC systems (system#1 and system#2), which differ in the costs and performances. Four crossover schemes have been proposed and tested. Among those schemes, the two-point multi-crossover (2-PMC) has shown the best convergence. The comparison of MOEA variants with each other is sometimes not fine, because of the large realized confidence intervals. Main cause of this effect is relative low cardinality of approximation sets. This is clear in P-CAP with 2-PMC crossover. However, P-CAP using 1-PSC supplies precise results. In spite of this, it can be seen that the NSGA-II is more successful for P-CAP followed by NSGA and then SPEA.

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