Analysing the Orbital Movement and Trajectory of LEO (Low Earth Orbit) Satellite Relative to Earth Rotation*

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Abstract. Next generation of wireless Internet scenarios include LEOs (Low Earth Orbit Satellites). Lower altitudes of LEO constellations could allow global coverage while offering: low end-to-end propagation delay, low power consumption, and effective frequency usage both for the users and the satellite network. LEOs rotate asynchronously to the earth rotation. Fast movement of LEOs makes it necessary to include efficient mobility management. In past few years mobility patterns have been proposed by considering the full earth coverage constellation whereby, the rotation of earth was often assumed too negligible to be taken into account. The prime objective of this study is to provide facts and figures that show LEOs traverse relative to the rotation of earth. In order to analyse the orbital movement and trajectory of LEOs relative to earth rotation mathematical analysis have been done and justification have been made through equations.

Keywords: Orbital movement, Trajectory, LEOs.

1 Introduction

With the advancement of communication technology in general and wireless communication in particular, the use of small size and hand held devices such as laptops, PDAs (Personal Digital Assistance) etc. is increasing day by day. The use of such devices has created a challenging task for researchers in order to provide scalable communication networks. LEO networks are gaining popularity because of the advantage that satellite constellation covering most of the surface of earth could provide connectivity in isolated areas such as rural or transit ways. The advantage of using LEOs lie in their low end-to-end propagation delay, and low power consumption while effectively re-uses the available bandwidth. As far as civil, military, and personal global communication is concerned, LEO satellites will play an

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important role in next generation of Internet. Integrating LEO satellite in the Internet [[Sun 05], [Woo 03]] remains closely dependable on an effective mobility management which accommodates short connectivity windows with any single point on earth due to satellite's movement. The dynamic nature of satellite (high rotational speed) has been dealt with by increasing the number of satellites in a constellation in order to provide coverage across the globe.

A LEO satellite network [[Woo 03], [UAYY 99]] comprises a constellation of satellites circling at near polar inclination (their angle of inclination lies between the equatorial plane and the satellite orbital plane where as a true polar orbit has an inclination of $90°$) are launched at an altitude between $500-1500\ (Km)$. According to [[Sun 05], [Yeo 03]] Kepler's law of planetary motion, a satellite's orbit is an ellipse and the body it orbits is at one focus. The speed of LEO satellite increases with decreasing altitude (height of the satellite) and the dynamic nature [[AUB 99], [MRER 91]] makes the problem concerning mobility more challenging. Mobility models such as ATCR (Adaptive Time-Based Channel Reservation Algorithm), TCRA (Time-Based Channel Reservation Algorithm), GH (Guaranteed Handover) [[BBGP 03], [BGP 02], [MRRFG 98]] have been proposed for LEO satellites in past few years. However, in all the said strategies, the movement of earth and that of user is being neglected because of the fact that LEOs traverse with high speed relative to the rotation of earth and that of user.

Mobility models presented by [[Yeo 03], [NOT 02]] are designed by taken into consideration the rotation of earth. First model [NOT 02] is designed by taken into consideration that the earth is rotating with a constant linear speed of $1670\ (Km/hr)$, which is not true. Fact is that the rotational speed of earth decreases with cosine of the latitude. The speed $1670\ (Km/hr)$ is the maximum rotational speed of earth at the equator ($0°$ latitude). Second model [Yeo 03] is proposed by taken into consideration that the earth is rotating at an angle of $23.5°$ which is also not true because $23.5°$ is an axial tilt which is constant and is responsible for the change of seasons through out the year.

The earth is tilted at $23.5°$ after every 3 months and that is why the change in seasons occurs. This axial tilt has nothing to do with the orbital movement of LEOs as such satellites are not responsible for bringing change in season. However, the rotational speed of earth decreases with increasing latitude which affects both the orbital movement and trajectory of LEO satellites.

The aim of this study is to analyse mathematically the orbital movement and trajectory of LEOs relative to the rotation of earth. The study is based [[Heindl 05], [FRGT 98], [WJLB 95], [UWE]] upon the orbital parameters of IRIDIUM and UWE–1 because of the fact that both are launched at an altitude between $700-800\ (Km)$ with minimum angle of elevation $\alpha_{\min}$ of $10°$ and an inclination angle of $86.4°$ and $98.190°$ respectively as shown in Table 1. An inclination angle [Sun 05] measured in degrees, is an angle which determines how much a satellite's orbit is tilted with respect to the equatorial plane of earth. However, in order to provide coverage to the users sufficiently high angle of elevation α of a satellite is required. An angle of elevation α in degrees is an angle between the centre of satellite beam and the surface of earth.

- Elevation angle affects satellite's coverage area (area on the earth's surface where satellite is seen). Ideally an elevation angle of $0°$ is required so that the transmission beam reaches the horizon visible to the satellite in all directions.
- However, because of the environmental factors like: objects blocking the transmission, atmospheric attenuation, and earth's electrical background noise there is a minimum elevation angle $\alpha_{\min}$ of earth stations.
- Minimum angle of elevation $\alpha_{\min}$ in degrees is required to communicate with the satellite.

Table 1. Orbital Parameters

Attributes	IRIDIUM Constellation	UWE – 1 (Single satellite)
No. of satellites	66	1
No. of orbital planes	6	1
Satellites per plane	11	1
Altitude (Km)	780	800
Inclination (degrees)	86.4°	98.190°
Min. angle of elevation $\alpha_{\min}$ (degrees)	10°	10°
Orbital period T_{sat} (min)	Approx.102 min	Approx.102 min

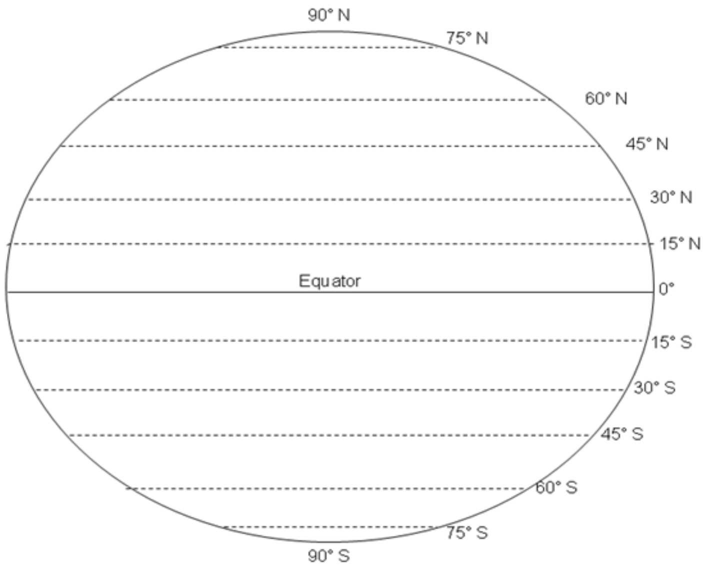
CubeSat [UWE] UWE–1 was built by the engineers at University of Wuerzburg, Germany and is launched in October 2005. The study of orbital movement of LEOs relative to earth's rotation is based upon realistic and accurate facts.

The rest of the paper is organised as follows. Section II gives the mathematical analysis of the rotation of earth and depending upon that analysis the orbital movement and trajectory of LEO satellite has been studied in section III. Conclusion and future work is given in section IV.

2 Orbital Rotation of Earth

LEOs rotate around their orbital plane faster than the rotation of earth and hence continuously circulate around the surface of earth. As is described in [Sun 05] a satellite orbit plane and a point on the orbit can define the satellite trajectory. Satellite orbits lie in planes and hence bisect the orbiting body because of the fact that earth rotates continuously. It can be said that if earth is not rotating continuously then the orbiting satellite would have passed over the same point on earth crossing the equator repeatedly at the same longitude. It is therefore, very important to consider the rotation of earth while designing the mobility model of LEOs.

Continuous rotation of earth has been effective both upon the movement and trajectory of LEOs and in order to determine the length of LEO satellite's orbit around the earth it is necessary to calculate earth's circumference and how it rotates throughout the day. Above mentioned factors must be kept in consideration when orbital movement of LEO satellite is taken into account.



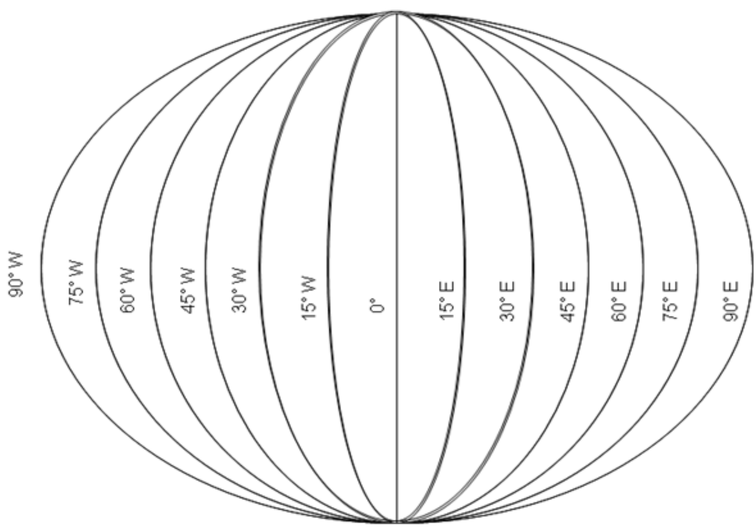


Fig. 1. (a) Latitude lines **(b)** Longitude lines

Consider Fig.1 which shows latitude and longitude lines. From Fig.1 (a), moving from $0°$ latitude (equator) towards $90°$ (North or South pole) then the circumference of circle defined by that latitude line will decrease in direct proportion to the cosine of the angle of latitude or simply it can be said that the circumference of earth decreases with increasing latitude. Hence, it is quite obvious to calculate the maximum circumference at equator where latitude is $0°$.

$$C = 2\pi R_E * (Cos\,\phi) \quad (1)$$

Where C is the circumference of earth, R_E is the mean radius of earth $(6378Km)$ and ϕ is the latitude in degrees ($0°$ at the equator) and $Cos\,\phi = Cos\,0° = 1$. From Eq. (1) maximum value of the earth's circumference C at equator would be $40.075\,9(Km)$. Angular speed of earth $\omega_{earth} = 2\pi f$ or $\omega_{earth} = 2\pi / T$ as $T = 1/f$ is constant irrespective of the latitude, where $T = T_{earth}$ (earth's orbital period $= 24hr$). The angular speed $\omega_{earth} = 2\pi / T_{earth}$ or $360°/24\ (hr) = 360°/\ (24 * 60)\ \ (min) = 0.25\ (deg/min)$. It is concluded that earth rotate $1°$ in 4 minutes. Linear velocity of earth $v_{earth} = d/t$ where $d = C$ (circumference of earth) and $t = T_{earth}$ is $1669.75\ (Km/hr)$ at the equator. Also, v_{earth} decreases with cosine of the latitude which gives the rotational speed $v_{rotation}$ of earth as is calculated in Eq. (2)

$$v_{rotation} = v_{earth} * Cos\,\phi \quad (2)$$

$If\,\phi = 10°$, then

$$v_{rotation} = 1669.75 * Cos\,10° = 1644.38(Km/hr)$$

From the above analysis it is clear that the rotational speed of earth decreases with increasing latitude and also earth rotate $1°$ in 4 minutes, therefore the mobility models proposed in [[Yeo 03], [NOT 02]] are not practical.

Keeping in view the above mentioned facts how the earth rotates throughout the day, next section will analyse the orbital movement and trajectory of LEOs relative to earth rotation.

3 Orbital Movement and Trajectory of LEOs Relative to Earth Rotation

A LEO network comprises a constellation of satellites such as [FRGT 98] IRIDIUM or for example; a single satellite such as [UWE] UWE–1, launched at an altitude between $700 - 800\ (Km)$ and are near polar orbiting satellites. One of the advantages [Wikipedia.org1] [daviddarling.info] of these satellites is that they move in sun–synchronous orbits which combine altitude and inclination and are typically low earth orbits with the range of altitudes mentioned above and period approximately equal to $102\ (min)$ as given in Table 1. Sun-synchronous orbit [crisp.edu.sg] is a near polar orbit and the sun–synchronous movement of the satellite causes it to pass through the equator and each degree of latitude exactly the same local solar time every day and hence passes overhead at the same solar time throughout the year. Therefore, in order to collect regular data, for example, weather forecast, remote sensing, etc. LEOs can be effectively used. Except for the seasonal variation (which takes place due to an axial tilt of $23.5°$ after every 3 months) same illumination conditions can be achieved for the images of a given location taken by the satellite and comparisons can easily be made.

Main purpose of this study is to mathematically analyse the orbital movement and trajectory of a LEO satellite relative to the rotation of earth with the help of orbital parameters of [[FRGT 98], [UWE]] IRRIDIUM and UWE–1. The orbital period [[Heindl 05], [FRGT 98], [WJLB 95]] T_{sat} as given in Table 1 for IRRIDIUM and UWE–1 is approximately $102\ (\min)$ and earth rotates $0.25\ (\deg/\min)$ hence, earth will be rotated at an angle of $25.5°\ (0.25 * 102)$ when LEO satellite completes one orbital rotation.

$$\lambda_{NR} \text{ (degrees longitude/rotation)} = 0.25 \ * T_{sat} \tag{3}$$

$$0.25° * 102 = 25.5 \text{ (degrees longitude /rotation).}$$

Consider Table 2 which summarizes the values for λ_{NR} after every T_{sat} for few N_R (number of rotations). It can be seen from Table 2 that if LEO satellite starts when $\lambda_{NR} = 0°$ longitude (west) with $T_{sat} = 00:00\ (\min)$ then it will again cross the equator after $51\ (\min)$ on one side of the earth and $102\ (\min)$ on the other side of earth at $25.5°$ longitude (west). For the next rotation the equator crossing on the same side will take place again after $102\ (\min)$ at $51°(25.5° + 25.5°)$ west longitude. Longitude lines are shown in Fig. 1(b).

It can be said that with an orbital period of $102\ (\min)$ the longitudinal increment is 25.5 degrees per rotation. Therefore, in order to cover the whole day (24hrs) or $(24 * 60 = 1440\text{min})$ the satellite will need $T_{earth} / T_{sat} = 1440/102 = 14.117$ rotations per day. It is concluded that at the end of the day the satellite will start its 15th rotation and has completed 14 rotations across the globe and at the same time the earth will also complete its rotation of $360°$.

From Table 2 and from the discussion given in Section II and Section III respectively, it is concluded that both the earth and the satellite are simultaneously

Table 2. Satellite crossing at the equator

N_R = No. of Rotation	$T_{sat}(\min) = 102(\min)$	λ_{NR}($0.25 * 102 = 25.5$) degrees longitude/rotation after every T_{sat})
$N_R = 0$	$T_o = 00:00$	$\lambda_o = 0°$
$N_R = 1$	$T_1 = T_o + T_{sat}$	$\lambda_1 = \lambda_o + 25.5°$
$N_R = 2$	$T_2 = T_1 + T_{sat}$	$\lambda_2 = \lambda_1 + 25.5°$
$N_R = 3$	$T_3 = T_2 + T_{sat}$	$\lambda_3 = \lambda_2 + 25.5°$
$N_R = n$ rotations, $n = 0,1,2,---$	$T_n = n * T_{sat}$, $n = 0,1,2,---$	$\lambda_n = n * 25.5°$ $n = 0,1,2,---$

changing their trajectories with the passage of time. Table 2 gives the new value after every rotation, whereas our aim is to find out the exact trajectory of the satellite relative to earth rotation which depends upon:

- D_{nad} (Nadir distance) is the distance in Km travelled by the nadir point on the surface of earth during one rotation. The Nadir point is a point [[HK 99], [wikipedia.org2]] on the surface of the earth directly between the satellite and the geo – centre (centre of earth), and is also referred as sub-satellite point, as shown in Fig. 2.
- t_u the time during which the signal is available to the users (time usable by the users).
- D_{rec} distance in Km from user's location to the points on earth over which the satellite will come into LOS (line of sight) and leave i.e. diameter of reception area or in other words coverage area of satellite as shown in Fig.3.

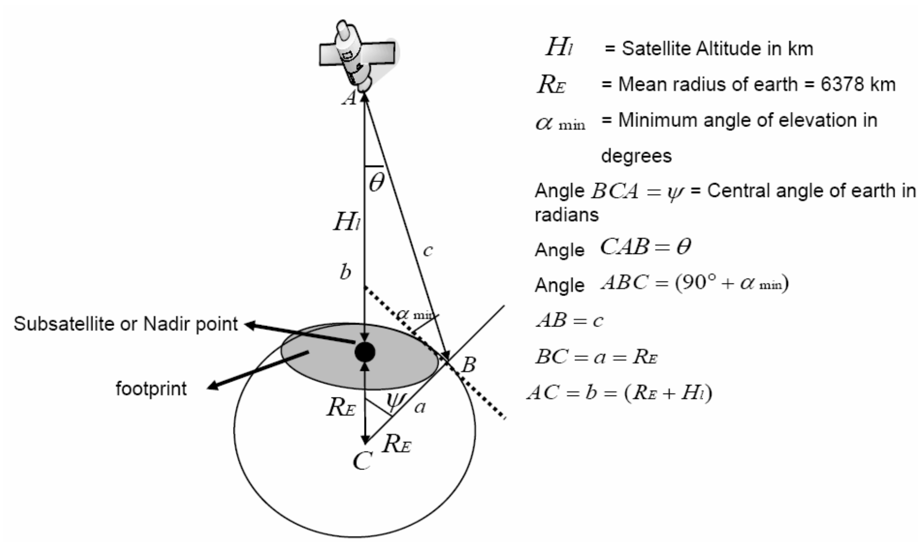


Fig. 2. Central angle of Earth

D_{nad} can be calculated with the help of the linear velocity V_{nad} of the nadir point by which the satellite traverses on the surface of earth. $V_{nad} = V_{sat}$ for LEO satellites [FRGT 98] relative to earth is given by Eq. (4) where R_g is GEO satellite orbit radius $(R_E + H_g)$, R_l is LEO satellite orbit radius $(R_E + H_l)$, H_g is altitude of GEO satellite, and H_l is altitude of LEO satellite respectively.

$$V_{nad} = \omega_{earth} * (\sqrt[3]{R_g} / \sqrt{R_l}) \quad (4)$$

D_{nad} is calculated in Eq.(5).

$V_{nad} = D_{nad} / T_{sat}$

$$D_{nad} = V_{nad} * T_{sat} \tag{5}$$

A LEO satellite completes at least 14.117 rotations per day; hence, total distance covered by the satellite D_{cov} during 24 (hr) or 1440 (min) is given by Eq. (6).

$$D_{\text{cov}} = D_{nad} * \text{No. of rotations per day} \tag{6}$$

D_{rec} is calculated using Fig.3. Consider Fig.3 where the inner circle represents the surface of earth while the outer circle represents a single satellite orbiting around the earth. The point L on the inner circle (earth surface) represent the user's location while a straight line through point L represents the horizon as it can be seen from point L (user's location). If it is assumed that users will only receive a signal when the satellite passes overhead, then mark two points A and B respectively where the horizon line intersects the outer circle (satellite orbit). Point A represents the point from where the satellite appears to be in LOS of the user whereas point B represents the point where the satellite appears out of LOS of the user. In the inner circle D_{rec} represents the diameter of reception area (coverage area of satellite) while R_{rec} is the radius of reception area.

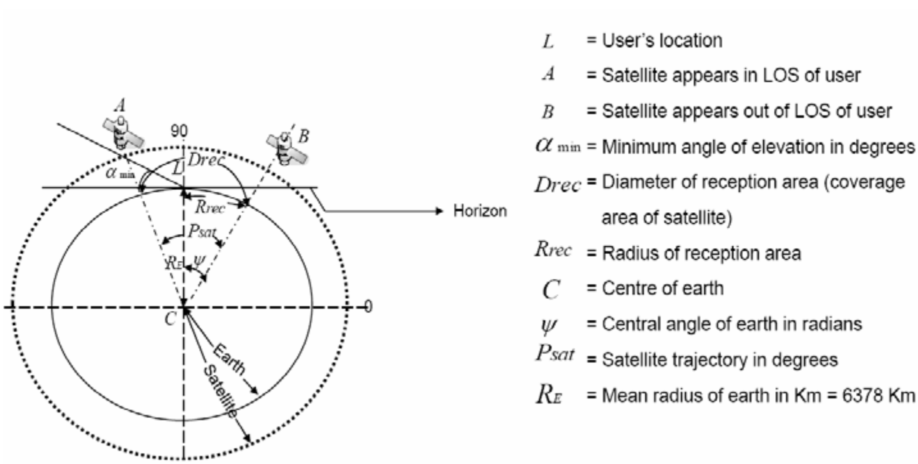


Fig. 3. Single satellite orbiting earth

We are interested in calculating the maximum distance from our location (point L) at which the satellite signal is expected to be received by the users. D_{rec} is actually the size of the instantaneous coverage of satellite as described in [MRER 91] and is given by Eq. (7).

$$D_{rec} = 2 R_E \psi \tag{7}$$

ψ as is described in [[WJBL 95], [MRER 91]] is the central angle of earth in radians as shown in Fig.2 and is calculated by Eq. (8), which depends upon H_l the altitude of LEOs in Km, $\alpha_{\min}$ the minimum angle of elevation in degrees and R_E the mean radius of earth in Km.

$$\psi = InvCos\ [R_E * Cos\,\alpha_{\min}/(R_E + R_l)] - \alpha_{\min} \tag{8}$$

The radius of reception area R_{rec} becomes:

$$R_{rec} = D_{rec} / 2 = R_E \psi \tag{9}$$

The distance D_{rec} is an important factor as it defines the geographical boundaries where a communication system can be used and is also helpful in determining the time interval during which the satellite signal is usable to the user. Exact time during which the satellite signal is usable to the user is calculated from Eq. (10).

$$D_{nad} / T_{sat} = D_{rec} / t_u \tag{10}$$

Where t_u is the time during which the signal is usable to user in minutes and is given by Eq. (11).

$$t_u = (T_{sat} * D_{rec}) / D_{nad} \tag{11}$$

From Eq. (11) it is concluded that the satellite will be out of sight of the user after t_u (min). t_u is an important factor as it helps in calculating the trajectory of the satellite relative to earth rotation in terms of degree. The trajectory of satellite P_{sat} can be obtained as a product of ω_{sat} angular speed of satellite and t_u.

$$P_{sat} = \omega_{sat} * t_u \tag{12}$$

Where $\omega_{sat} = 2\pi / T_{sat} = 360° / T_{sat}$

As mentioned in section II that the earth rotate 1° in 4 minutes and the earth will also be at a new position after every t_u. The trajectory of earth P_{earth} is calculated in Eq. (13).

$$P_{earth} = t_u * (1° / 4\text{min}) \tag{13}$$

t_u as calculated in Eq. (11) gives the time during which a single satellite for example, [UWE] UWE–1 is visible to the user. From the above discussion it is concluded that once the satellite will be out of sight from the 1st location it will be visible at another location on the surface of earth. In order to provide continuous coverage to the users a constellation of satellite such as [FRGT 98] IRIDIUM is required.

From Eq. (12) and Eq. (13) it is proved that both earth and satellite are changing their trajectories with the passage of time and hence, the models presented in [[Yeo 03], [NOT 02]] are not justified. In this study the prime objective is to calculate the trajectory of a satellite and how a satellite moves with respect to the rotation of earth. The issue how handover takes place if the rotation of earth is not neglected as is done in most of the available models is left as future work.

4 Conclusion

After this study it is concluded that it is unjustified to consider that the earth is rotating with a constant linear speed of $1670\ (Km/hr)$ as is considered in Nguyen et al's model because this means that the satellite will cross the equator again and again at the same longitude and hence not able to provide global coverage. Also, it is not reasonable to consider that the earth is rotating at an angle of $23.5°$, which actually is an axial tilt (earth is tilted at this angle after every 3 months), it would again mean that the satellite would not be able to complete its rotation of 360° during the orbital period of $102\ (min)$ and ultimately not able to move around the surface of earth when the day is finished. Therefore, no global coverage is achieved. From Eq. (12) and Eq. (13) it is also concluded that the trajectories of satellite and that of earth changes continuously with the passage of time. The prime objective of this study is to provide facts and figures that show LEOs traverse relative to the rotation of earth. How handover takes place when a constellation of satellite is taken into consideration by taken into account the rotation of earth is left as future work.

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