Auctions with Dynamic Populations: Efficiency and Revenue Maximization
(Extended Abstract)*

Maher Said

Department of Economics
Yale University
New Haven, CT, USA
maher.said@yale.edu

Abstract. We study a stochastic sequential allocation problem with a dynamic population of privately-informed buyers. We characterize the set of efficient allocation rules and show that a dynamic VCG mechanism is both efficient and periodic ex post incentive compatible; we also show that the revenue-maximizing direct mechanism is a pivot mechanism with a reserve price. We then consider sequential ascending auctions in this setting, both with and without a reserve price. We construct equilibrium bidding strategies in this indirect mechanism where bidders reveal their private information in *every* period, yielding the same outcomes as the direct mechanisms. Thus, the sequential ascending auction is a natural institution for achieving either efficient or optimal outcomes.

Keywords: Dynamic mechanism design, Random arrivals, Dynamic Vickrey-Clarke-Groves, Sequential ascending auctions.

In this paper, we study the problem of a seller faced with a stochastic sequential allocation problem. At each point in time, a random number of buyers and objects arrive to a market. Buyers are risk-neutral and patient, while objects are homogeneous and perishable. Moreover, each buyer desires a single unit of the good in question; however, valuations for the good vary across buyers. The mechanism designer must elicit the private information of these buyers in order to achieve a desirable allocation—one that is either efficient or revenue maximizing, depending on the designer's objective function.

We are concerned with two main questions. First, what outcomes are attainable in markets with dynamic populations of privately-informed buyers? In particular, can we achieve efficient or revenue-maximizing outcomes? And secondly, and equally importantly, can we achieve these outcomes using natural or simple "real-world" institutions?

It is well-known that the Vickrey-Clarke-Groves mechanism is efficient and dominant-strategy incentive compatible in static environments. By choosing a price for each buyer that equals the externality imposed by her report on other participants in the mechanism, the VCG mechanism aligns the incentives of

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the buyer with those of a welfare-maximizing social planner. In the dynamic environment we consider, the arrival of a new buyer imposes an externality on her competitors by reordering the (anticipated) schedule of allocations to those buyers currently present on the market, as well as to those buyers expected to arrive in the future. We show that by charging each agent, upon her arrival, a price equal to this *expected* externality, the buyer's incentives are aligned with those of the forward-looking planner. Therefore, this dynamic version of the VCG mechanism is efficient and periodic ex post incentive compatible.

This dynamic VCG mechanism is a direct revelation mechanism, requiring buyers to report their values to the mechanism upon their arrival to the market. As is well known, however, direct revelation mechanisms may be difficult to implement in practice. We therefore turn to the design of simple indirect mechanisms and consider the possibility of achieving efficient outcomes via a sequence of auctions. Despite the resemblance of the dynamic direct mechanism to its single-unit static counterpart, we find that this relationship does not hold for the corresponding auction format. In the canonical static auction environment, the "standard" analogue of the VCG mechanism is the second-price sealed-bid auction. In our dynamic environment, however, a sequence of such auctions cannot yield outcomes equivalent to those of the dynamic VCG mechanism. In a sequential auction, there is an "option value" associated with losing in a particular period, as buyers can participate in future auctions. The value of this option depends on the private information of all other competitors, as the expected price in the future will depend on their values—despite the assumption of independent private values, the market dynamics generate interdependence.

Therefore, a standard second-price sealed-bid auction does not reveal sufficient information for the correct determination of buyers' option values. In contrast, the ascending auction is an *open* auction format that allows for the gradual revelation of private information. We use this to construct bidding strategies that form an efficient equilibrium in a sequence of ascending auctions. In each period, buyers bid up to the price at which they are indifferent between winning an object and receiving their expected marginal contribution to the social welfare in the future. As buyers drop out of the auction, they reveal their private information to their competitors, who then condition their current-period bids on this information. This process of information revelation is repeated in *every* period, and is crucial for providing the appropriate incentives for new entrants to also reveal their private information. We show that these memoryless bidding strategies form a periodic ex post equilibrium that is outcome equivalent to the truth-telling equilibrium of the efficient direct mechanism.

In addition, we construct a revenue-maximizing direct mechanism for this setting. We show that the revenue-maximizing direct mechanism is the dynamic VCG mechanism applied to buyers' *virtual* values and the *virtual* surplus. When buyers' values are drawn from the same distribution, the sequential ascending auction with an optimal reserve price admits an equilibrium that is equivalent to truth-telling in the optimal direct mechanism. Thus, the sequential ascending auction is a natural institution for achieving either efficient or optimal outcomes.