Performance Analysis of IEEE 802.15.4 Non-beacon Mode with Both Uplink and Downlink Traffic in Non-saturated Condition

Tae Ok Kim[1], Jin Soo Park[2], Kyung Jae Kim[1], and Bong Dae Choi[1,⋆]

[1] Department of Mathematics and Telecommunication Mathematics Research Center, Korea University, Seoul, Korea
{violetgl,kimkjae,queue}@korea.ac.kr
[2] USN Service Division, KT, Seoul, Korea
vtjinsoo@paran.com

Abstract. We analyze the MAC performance of the IEEE 802.15.4 LR-WPAN with non-beacon mode and non-saturated condition in a star topology. Our approach is to model stochastic behavior of one device with both uplink and downlink traffic as a discrete time Markov chain. First, we propose an analytical model of a device with only downlink traffic. Then, by combining the model of a device with only uplink traffic in [3] and one with downlink traffic in this paper, we obtain the performance measures such as throughput, packet delay, energy consumption and packet loss probability of a device with both uplink and downlink traffic. Our results can be used to find the optimal number of devices so as to satisfy QoS (quality of service) on delay and loss probability.

Keywords: IEEE 802.15.4, Medium Access Control(MAC) protocol, CSMA/CA, Markov Chain, performance analysis.

1 Introduction

IEEE 802.15.4[1,2] is a standard toward low complexity, low power consumption and low data rate wireless data connectivity. Therefore IEEE 802.15.4 will play a key role as a MAC protocol at WSN(Wireless Sensor Network) where energy consumption is an important factor.

IEEE 802.15.4 low rate WPAN (LR-WPAN) allows two network topologies: star and peer-to-peer. In a star topology, every sensor device must communicate with a PAN coordinator, while in a peer-to-peer topology, all devices can communicate each other. In a star topology, network uses two types of network channel

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access mechanism, non-beacon mode and beacon-enabled mode, depending on whether the network supports the transmission of beacons.

Diverse applications for wireless sensor network based on IEEE 802.15.4 have generated interests in analytical models of access mechanism based on CSMA/CA. Pollin *et al.*[6] and Park *et al.*[7] proposed analytic model for uplink traffic on IEEE 802.15.4 beacon-enabled mode under saturated condition where devices have always packets to send. In real environment, packets are generated in not too often, so that a device will have no packets to send or receive for most of time. Therefore we need to investigate non-saturated case where a device does not have packets to send or receive for some period of time. Misic *et al.*[8] analyzed performance of IEEE 802.15.4 with both uplink and downlink traffic in beacon mode under non-saturated condition by modeling of discrete-time Markov chains and the theory of M/G/1 queues. Also, Kim *et al.*[9] analyzed performance of uplink communication in non-beacon mode with unslotted CSMA/CA by busy cycle of M/G/1 queueing system.

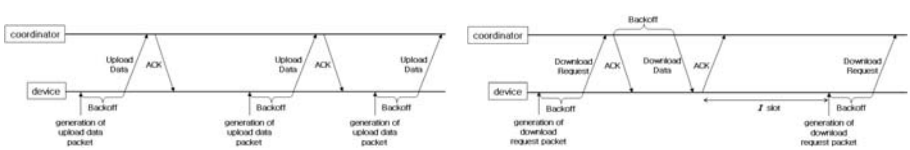
This paper attempts to analyze the MAC performance of star-shaped IEEE 802.15.4 network in non-saturated condition running under non-beacon mode. Unlike beacon mode operation where nodes periodically wake up to listen to the beacon frame, nodes in non-beacon mode need to wake up only when they have packets to upload or when they request to download. This way, the nodes can save energy unless otherwise spent on listening to the beacon frame. For example, in practical situation like the forest fire monitoring system, the traffic is quite rarely generated and it is unnecessary that sensor nodes wake up frequently. The simulation results in Section 5 show that the non-beacon mode reduces the energy consumption significantly compared with the beacon mode without too much degrading downlink delay (See Fig. 5). This provides our motivation to investigate the non-beacon mode. In this paper, we assume that all nodes are synchronized, which can be realized by following method. The PAN coordinator broadcasts periodic signal (e.g. sine signal) for synchronization through an extra channel. When device has packet to transmit, the device synchronized by sensing the extra channel. In the environment that time clocks of all devices are synchronized by this approaches, we adopt the non-beacon mode with slotted CSMA/CA as MAC transmission procedure.

We model the stochastic behavior of a device with both uplink traffic and downlink traffic as a discrete-time Markov chain. Note that our Markov chain model of IEEE 802.15.4 is different from one of IEEE 802.11 [5], since no freezing of backoff counter operates during transmission of other devices in IEEE 802.15.4. We analyzed the performances of station with uplink traffic[3] and downlink traffic[4] individually under unsaturation condition. However, the practical situation where uplink and downlink traffic coexist gives us the motivation of this paper. So, first, we propose the analytical model[4] of a device with only downlink traffic with some modification. Then, by combining the uplink model and downlink model, we can make the analytical model of a device with both uplink and downlink traffics. Finally, we obtain the performance measures such as throughput, average packet delay, packet loss probability and energy consumption for the network with both uplink and downlink traffic.

This paper is organized as follows. In Section 2, we describe the MAC procedure for uplink and downlink in IEEE 802.15.4 non-beacon mode. In the Section 3, we propose the analytic model of a device with only downlink traffic under non-saturated condition and obtain performance measures from our analysis. In the Section 4, we make the analytic model of a device with both uplink and downlink traffics by combining the model of a device with uplink traffic in [3] and one with downlink traffic in Section 3 and obtain performance measures. Numerical results for performance measures of the network with both uplink and downlink traffic are presented in Section 5.

2 MAC Procedure for Uplink and Downlink in Non-beacon Mode

When applying the slotted CSMA/CA, the MAC sublayer is delayed for a random number j of backoff slots (the chosen random number of backoff slots is called backoff counter) in the range 0 to $2^{BE} - 1$. BE is the backoff exponent, which is related to how many attempts (the number of attempts is called backoff stages) a device has tried to access the channel. BE shall be initialized to the value of $macMinBE$ (this corresponds to the 0th backoff stage). The backoff counter j is decreased at the boundary of each backoff slot. In IEEE 802.15.4, the backoff counter is decremented regardless of the channel status contrary to that of IEEE 802.11 DCF. When the backoff counter reaches to zero, CCA is performed twice before transmitting a packet. CCA(clear channel assessment) is the procedure that a device listens to the channel to make sure the channel clear before attempting to transmit a packet. When the channel is found to be idle during two CCA periods, the device shall transmit its packet. When the channel is found to be busy during either the first CCA or the second CCA, backoff stage is increased by 1 and BE is increased by 1 until BE reaches $macMaxBE$ (let N be the backoff stage that BE reaches $macMaxBE$), and then a random backoff is tried again. If the transmitted packet suffers collision, it will restart from the beginning of procedure. If one of two CCAs fails at the Mth backoff stage where M is the maximum retransmission number, the packet is discarded.



(a) Procedure of uplink (b) Procedure of downlink

Fig. 1. Download sequences in a non-beacon network

The communication sequences for uplink and downlink in a non-beacon network are shown in Fig. 1. When a device has a data packet to transmit, it simply transmits its uplink data packet, using the slotted CSMA/CA, to the PAN coordinator. The coordinator sends an acknowledgment packet which notifies the successful reception of the uplink data packet.

When the coordinator has a data packet to send to a certain device, it stores the downlink data packet and waits until the device requests downlink transmission. A device makes a contact by transmitting a downlink request packet, using the slotted CSMA/CA, periodically. The coordinator sends an acknowledgment packet which notifies the successful reception of the downlink request packet and existence of downlink data packet. If data are pending, the coordinator transmits the downlink data packet, using the slotted CSMA/CA, to the device. The standard allows the coordinator to send a downlink data packet without using the CSMA/CA after the acknowledgment packet for downlink request packet. Analysis of this method is studied by Kim *et. al.*[**?**]. After receiving the downlink data packet, the device sends an acknowledgment packet which notifies the successful reception of the downlink data packet.

3 Analysis for a Device with Only Downlink Traffic

Let n sensor devices be associated with the PAN coordinator. For downlink in IEEE 802.15.4 non-beacon mode, a device sends a downlink request packet to the PAN coordinator periodically to check whether there is a downlink data packet at the PAN coordinator. We assume that a device generates a downlink request packet after fixed number I of slots from the moment of the completion of the previous downlink procedure (See Fig. 1). We also assume that it takes exponential random time with mean $\frac{1}{\lambda_\mathrm{d}}$ that the PAN coordinator generates a downlink data packet destined to the tagged device after the previous packet is transmitted.

3.1 Mathematical Model

Let $s(t)$, $0 \leq s(t) \leq M$, be the backoff stage and $b(t)$ be the backoff counter. Let $(s(t), b(t))_\mathrm{r}$ and $(s(t), b(t))_\mathrm{d}$ denote the backoff stage and backoff counter for downlink request packet and downlink data packet, respectively. When the channel is idle at the first CCA, we define $b(t) = -1$. We assume that size of downlink request packet is fixed R in the unit of slots. Let $Tx_\mathrm{r}[k]$, $1 \leq k \leq R$, represent the state of the kth slot of downlink request packet transmission. Let Rx_d represent the state of downlink data packet in transmission. We assume that the length of data packet measured in slots is geometrically distributed with mean $\frac{1}{1-P_{\mathrm{Rx,d}}}$ where $P_{\mathrm{Rx,d}}$ is the probability that current transmission continues at the next slot. Let the duration for both waiting and receiving ACK be A slots (Default value of A is equal to 2). Let $(-1, k)$, $1 \leq k \leq A$, represent the state of the k^{th} slot of the duration for waiting and receiving ACK. Let $idle[k]$, $1 \leq k \leq I$, represent the state of the k^{th} slot from the start of duration of fixed length I for generating downlink request packet. Define $Y(t)$ at t by :

$$Y(t) = \begin{cases} idle[k], & \text{when a device is in the } k\text{th slot of state before generating downlink request packet} \\ (s(t), b(t))_{\mathrm{r}}, & \text{when a device is in the process of backoff for downlink request packet} \\ (s(t), -1)_{\mathrm{r}}, & \text{when channel is idle at the first CCA for downlink request packet} \\ Tx_{\mathrm{r}}[k], & \text{when a device is in the } k\text{th slot of a downlink request packet transmission} \\ (-1, k)_{\mathrm{r}}, & \text{when a device is in the } k\text{th slot of waiting and receiving ACK} \\ & \text{for downlink request packet} \\ (s(t), b(t))_{\mathrm{d}}, & \text{when PAN coordinator is in the process of backoff for downlink data packet} \\ (s(t), -1)_{\mathrm{d}}, & \text{when channel is idle at the first CCA for downlink data packet} \\ Rx_{\mathrm{d}} & \text{when a device receives a downlink data packet} \\ (-1, k)_{\mathrm{d}}, & \text{when PAN coordinator is in the } k\text{th slot of waiting and receiving ACK} \\ & \text{for downlink data packet} \end{cases} \quad (1)$$

Then $Y(t)$ is a discrete Markov chain with one-step transition probabilities described in Fig. 3 for downlink procedure. Let $\pi_{(i,j)_{\mathrm{r}}}$, $\pi_{(i,-1)_{\mathrm{r}}}$, $\pi_{(-1,k)_{\mathrm{r}}}$, $\pi_{Tx_{\mathrm{r}}[k]}$, $\pi_{(i,j)_{\mathrm{d}}}$, $\pi_{(i,-1)_{\mathrm{d}}}$, $\pi_{(-1,k)_{\mathrm{d}}}$, $\pi_{Rx_{\mathrm{d}}}$ and $\pi_{idle[k]}$ be the steady-state probability which can be obtained by solving the balance equations.

Next we will calculate the probability α of channel being busy at the first CCA, the probability β of channel being busy at the second CCA and the probability P_s of successful packet transmission. Note that these probabilities have the same values for downlink request packet and downlink data packet because these values are determined by the states of other $n-1$ devices. Since the probability of the channel being idle at the first CCA for the given device is equal to the probability that the all other $n-1$ devices are not in the states of $Tx_{\mathrm{r}}[k]$, $(-1,k)_{\mathrm{r}}$, Rx_{d} and $(-1,k)_{\mathrm{d}}$. Therefore α is given by :

$$\alpha = 1 - (1 - \pi_{\mathrm{d}})^{n-1} , \quad (2)$$

where

$$\pi_{\mathrm{d}} = \sum_{k=1}^{R} \pi_{Tx_{\mathrm{r}}[k]} + \pi_{Rx_{\mathrm{d}}} + \sum_{j=0}^{A} (\pi_{(-1,k)_{\mathrm{r}}} + \pi_{(-1,k)_{\mathrm{d}}}).$$

Note that in order to be eligible to sense the channel at the second CCA, the channel must be idle at the first CCA. So β is the probability that the channel is busy when the tagged device senses at the second CCA, given that the channel is idle at the first CCA, i.e,

$$\begin{aligned} 1-\beta &= P\{\text{channel is idle at the second CCA} \mid \text{channel is idle at the first CCA}\} \\ &= \frac{P\{\text{channel is idle at the first CCA, channel is idle at the second CCA}\}}{P\{\text{channel is idle at the first CCA}\}} \\ &= \frac{\left\{1 - \pi_{\mathrm{d}} - \sum_{i=0}^{M} \left(\pi_{(i,-1)_{\mathrm{r}}} + \pi_{(i,-1)_{\mathrm{d}}}\right)\right\}^{n-1}}{1-\alpha} \end{aligned} \quad (3)$$

The successful transmission probability, P_s, can be represented by :

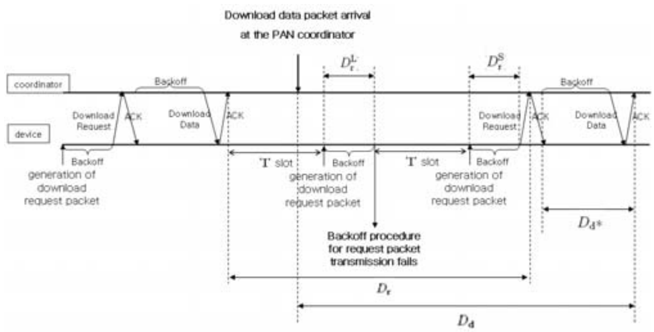


Fig. 2. Description of $D_{\rm r}$, $D_{\rm r}*$, $D_{\rm d}$, and $D_{\rm r}+I$

$$P_s = P\{\text{successful transmission} \mid \text{channel is idle at both the first CCA and the second CCA}\}$$
$$= \frac{\left\{1-\pi_{\rm d}-\sum_{i=0}^{M}\left(\pi_{(i,0)_{\rm r}}+\pi_{(i,-1)_{\rm r}}+\pi_{(i,0)_{\rm d}}+\pi_{(i,-1)_{\rm d}}\right)\right\}^{n-1}}{\left\{1-\pi_{\rm d}-\sum_{i=0}^{M}\left(\pi_{(i,-1)_{\rm r}}+\pi_{(i,-1)_{\rm d}}\right)\right\}^{n-1}} \tag{4}$$

Let $e_{\rm d}$ be the probability that there is a downlink data packet at the PAN coordinator when downlink request packet arrives at the PAN coordinator. This event occurs when downlink data packet arrives during the time duration, $D_{\rm r}$, from the completion of one downlink procedure to the next arrival of downlink request packet at the PAN coordinator (See Fig. 2). The expected delay $\mathrm{E}[D_{\rm r}]$ are calculated by :

$$\mathrm{E}[D_{\rm r}] = \sum_{k=0}^{\infty}(P_{\rm loss})^k(1-P_{\rm loss})\cdot\{k(I\cdot\sigma+\mathrm{E}[D_{\rm r}^{\rm L}])+(I\cdot\sigma+\mathrm{E}[D_{\rm r}^{\rm S}])\}+R\cdot\sigma \tag{5}$$

where $P_{\rm loss}$ is the probability of losing downlink request packet (given by (9)) and σ is the length of a slot. The expected delay $\mathrm{E}[D_{\rm r}^{\rm L}]$ from the moment of generation of downlink request packet to the moment of discarding the packet and the expected delay $\mathrm{E}[D_{\rm r}^{\rm S}]$ from the moment of generation of downlink request packet to the moment of beginning of downlink request packet transmission are calculated in Appendix 6. So, $e_{\rm d}$ is approximately calculated using $\mathrm{E}[D_{\rm r}]$.

$$e_{\rm d} \approx 1 - e^{-\lambda_{\rm d}\cdot\mathrm{E}[D_{\rm r}]} \tag{6}$$

To check the accuracy of the approximation (6), we simulated the system and it turns out that the approximation (6) is quite good (See Fig. 4).

Note that α, β, P_s and e_d in (2), (3), (4) and (6) express in terms of steady-state probability and vice versa. Therefore by solving nonlinear equation of (2), (3), (4), (6), balance equations of this Markov Chain and normalization condition, we obtain all necessary values such as steady-state probability, α and β.

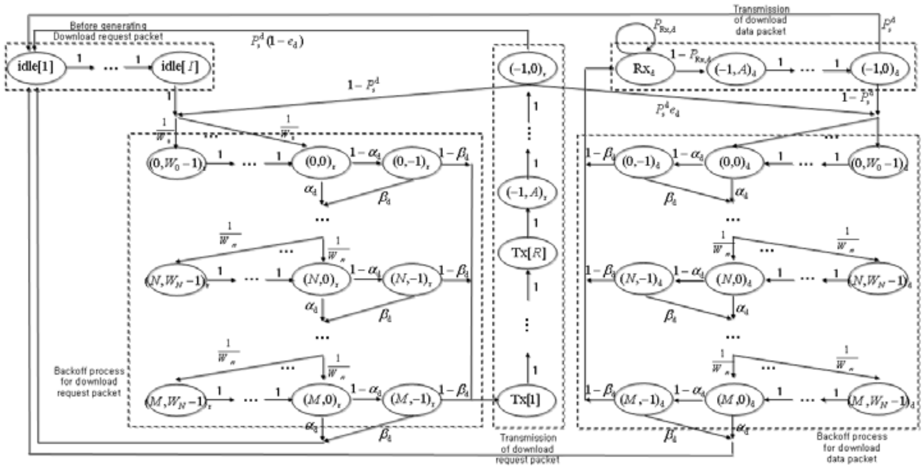


Fig. 3. Markov Chain for Downlink

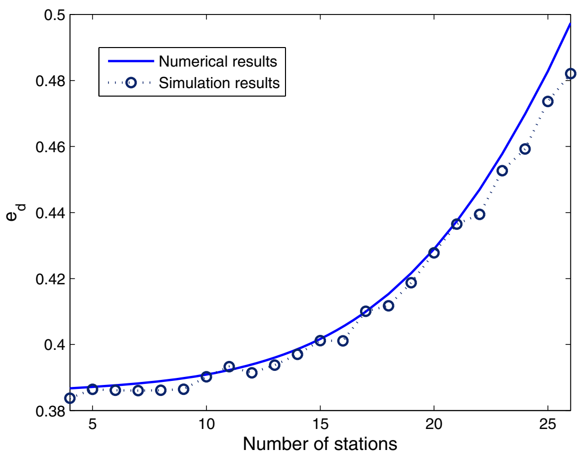


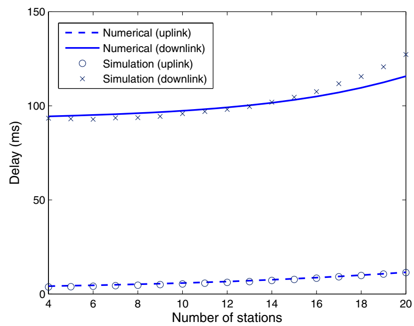
Fig. 4. Numerical and simulation results for e_d

3.2 Performance Measures

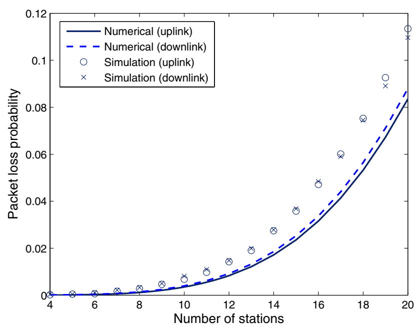
In this subsection, we obtain several performance measures such as throughput, delay, loss probability and energy consumption.

Throughput. The normalized system throughput S, defined as the fraction of time the channel is used to transmit downlink data packet successfully, is given as follows.

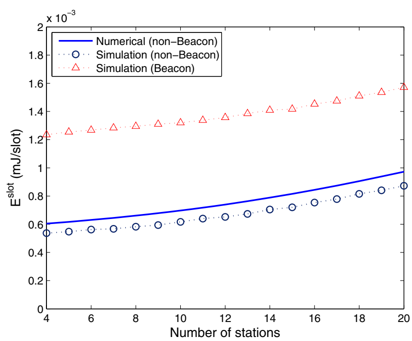
$$S = n \cdot \pi_{Rx_{\mathrm{d}}} \cdot P_s \tag{7}$$



(a) Expected delay for uplink and downlink



(b) Packet loss probability for uplink and downlink



(c) Energy consumption for a device

Fig. 5. Numerical and simulation Results : Performance measures

Delay. The expected delay $\mathrm{E}[D_{\mathrm{d}}]$ from the moment of downlink data packet arrival at the PAN coordinator to service completion point is approximately calculated by :

$$\mathrm{E}[D_{\mathrm{d}}] \approx \mathrm{E}[D_{\mathrm{r}}] + A + \mathrm{E}[D_{\mathrm{d}}*] - \frac{\int_0^{\mathrm{E}[D_{\mathrm{r}}]} x \cdot \lambda_{\mathrm{d}} e^{-\lambda_d x}\, dx}{1 - e^{-\lambda_{\mathrm{d}} \mathrm{E}[D_{\mathrm{r}}]}} \tag{8}$$

where $\mathrm{E}[D_{\mathrm{d}}*]$ is the expected duration from the beginning of backoff procedure for downlink data packet transmission to the moment of service completion of the downlink data packet (given by (24) in Appendix 6). The last term in the right-hand side represents the average duration from the completion of previous downlink procedure to a arrival of next downlink data packet.

Packet Loss Probability. The probabilities of losing downlink request packet and downlink data packet are same and let it denoted by P_{loss}. Then we have

$$\begin{aligned} P_{\mathrm{loss}} =& \sum_{v=0}^{M} \sum_{w=0}^{v} {}_{v}\mathrm{C}_{w} \alpha^{w} \{(1-\alpha)\beta\}^{v-w} (1-\alpha)(1-\beta)(1-P_s) P_{\mathrm{loss}} \\ &+ \sum_{w=0}^{M} {}_{M}\mathrm{C}_{w} \alpha^{w} \{(1-\alpha)\beta\}^{M-w} \{\alpha + (1-\alpha)\beta\} \end{aligned} \tag{9}$$

The general term in the first summation of (9) is the probability that the packet suffers loss after collision at the v^{th} backoff stage in the first backoff procedure. Note that after collision the procedure starts from the 0^{th} backoff stage again. The second term of (9) is the probability that the packet in the first backoff procedure suffers loss because channel is busy at the first CCA or the second CCA at the M^{th} backoff stage.

Energy Consumption. Since power is quite critical in a sensor network, energy consumption is the most important performance measure. To obtain the total lifetime of a battery, we need a concept of average energy consumption. Park et al. [7] and Pollin et al. [6] define the normalized energy consumption as the average energy consumption to transmit one slot amount of payload. Their definition has good explanation in saturation mode. However, in non-saturation mode, their definition mismatches with our intuition, as they [6] mentioned that the energy consumption increases as the arrival rate decreases, or equivalently idle period increases. See Fig. 9 in [6]. So,we calculate the average energy consumption E^{slot} per one slot(mJ/slot). Let E_{idle}, E_{Tx} and E_{Rx} be the energy consumption for idle slot, transmission slot and reception(or CCA) slot, respectively. Since energy consumption for reception slot and CCA slot are equal, we do not distinguish the valus. Let a^{idle}, a^{Tx} and a^{Rx} be the probabilities of slot being idle, being transmission, being reception(or CCA). Then,

$$\begin{aligned} a^{\mathrm{idle}} &= \sum_{k=1}^{I} \pi_{idle[k]} + \sum_{i=0}^{M} \sum_{j=1}^{W_i - 1} \pi_{(i,j)_{\mathrm{r}}} \\ a^{Tx} &= \pi_{Tx_{\mathrm{r}}[k]} + \left(\sum_{j=0}^{A} \pi_{(-1,k)_{\mathrm{d}}} \right) P_s \\ a^{Rx} &= 1 - a^{\mathrm{idle}} - a^{Tx} \end{aligned}$$

Note that a device consumes E_{Rx} per one slot when it waits the downlink data from PAN coordinator.

The average energy consumption E^{slot} per one slot is obtained as follows.

$$E^{\mathrm{slot}} = a^{\mathrm{idle}} E_{idle} + a^{Tx} E_{Tx} + a^{Rx} E_{Rx} \tag{10}$$

4 Analysis for a Device with Both Uplink and Downlink Traffic

We assume that uplink traffic has higher priority than downlink traffic in a sense that, if uplink data packet and downlink request packet attempt to transmit at the same slot, then uplink data packet is allowed to transmit and downlink request packet acts as if it suffers a collision. A device generates uplink data packets according to Poisson process with rate λ_{u} and the PAN coordinator generates downlink data packets destined to the tagged device according to Poisson process with rate λ_{d}, independently each other. We assume that a device and the PAN coordinator can accommodate only one uplink packet and only one downlink packet for the tagged device, respectively. A device generates a downlink request packet after fixed number I of slots from the completion of downlink procedure.

4.1 Mathematical Models

We combine the Markov chain model (called uplink model here) proposed in [3] for uplink traffic with different values α, β and P_{s} from ones in [3] and the Markov chain model (called downlink model here) proposed in Section 3 with different values α, β and P_{s} from ones in section 3. All traffics share a common channel and so they compete with each other to catch the common channel. Therefore, the probabilities α, β and P_{s} in each model (e.g. uplink model) relate not only with steady-state probability in its own model (e.g. uplink model) but also with steady-state probability in the other model (e.g. downlink model). For classification of index, we will use the subscript 'u', 'r' and 'd' for uplink data, downlink request, and downlink data, respectively. Note that, in the downlink model, the probability β_{r} for the downlink request packet are different from the probability β_{d} for downlink data packet because uplink traffic has higher priority than downlink traffic as mentioned in the beginning of this section.

The probability of the channel being idle at the first CCA for the tagged device is equal to the probability that any other transmission does not occur in the first CCA slot. Therefore α_{u}, α_{r} and α_{d} are given by

$$\alpha_{\mathrm{u}} = 1 - (1 - \pi_{\mathrm{u}})^{n-1}(1 - \pi_{\mathrm{d}})^{n} \tag{11}$$

and

$$\alpha_{\mathrm{r}}(= \alpha_{\mathrm{d}}) = 1 - (1 - \pi_{\mathrm{u}})^{n}(1 - \pi_{\mathrm{d}})^{n-1} \ , \tag{12}$$

where

$$\pi_{\mathrm{u}} = \pi_{Tx_{\mathrm{u}}} + \sum_{j=0}^{A} \pi_{(-1,k)_{\mathrm{u}}} \quad \text{and}$$

$$\pi_{\mathrm{d}} = \pi_{Rx_{\mathrm{d}}} + \sum_{k=1}^{R} \pi_{Tx_{\mathrm{r}}[k]} + \sum_{j=0}^{A} (\pi_{(-1,k)_{\mathrm{d}}} + \pi_{(-1,k)_{\mathrm{r}}}).$$

Since β_{u} is the probability that the channel is busy when the tagged device senses its second CCA for uplink data transmission given that the channel was idle slot at the first CCA, β_{u} is given by :

$$\beta_{\mathrm{u}} = 1 - \frac{\left(1-\pi_{\mathrm{u}}-\sum_{i=0}^{M} \pi_{(i,-1)_{\mathrm{u}}}\right)^{n-1}\left\{1-\pi_{\mathrm{d}}-\sum_{i=0}^{M}\left(\pi_{(i,-1)_{\mathrm{r}}}+\pi_{(i,-1)_{\mathrm{d}}}\right)\right\}^{n}}{1-\alpha_{\mathrm{u}}} \tag{13}$$

β_{r} is the probability that the tagged device fails in the second CCA for transmission of downlink request packet. The failure in the second CCA for transmission of downlink request packet comes from the following two cases. The first case is that the channel is busy at the second CCA for downlink request packet. The second case is that the the uplink data packet in the same device attempts transmission while the channel is idle at the second CCA for downlink request packet. Therefore, β_{r} is given by :

$$\beta_{\mathrm{r}} = 1 - \frac{\left\{1-\pi_{\mathrm{u}}-\sum_{i=0}^{M}\left(\pi_{(i,0)_{\mathrm{u}}}+\pi_{(i,-1)_{\mathrm{u}}}\right)\right\}\left[\left(1-\pi_{\mathrm{u}}-\sum_{i=0}^{M} \pi_{(i,-1)_{\mathrm{u}}}\right)\left\{1-\pi_{\mathrm{d}}-\sum_{i=0}^{M}\left(\pi_{(i,-1)_{\mathrm{r}}}+\pi_{(i,-1)_{\mathrm{d}}}\right)\right\}\right]^{n-1}}{1-\alpha_{\mathrm{r}}} \tag{14}$$

β_{d} is the probability that the channel is busy when the PAN coordinator senses it second CCA for downlink data transmission for tagged device. Therefore, β_{d} is given by :

$$\beta_{\mathrm{d}} = 1 - \frac{\left(1-\pi_{\mathrm{u}}-\sum_{i=0}^{M} \pi_{(i,-1)_{\mathrm{u}}}\right)^{n}\left\{1-\pi_{\mathrm{d}}-\sum_{i=0}^{M}\left(\pi_{(i,-1)_{\mathrm{r}}}+\pi_{(i,-1)_{\mathrm{d}}}\right)\right\}^{n-1}}{1-\alpha_{\mathrm{d}}} \tag{15}$$

The successful transmission probability, i.e. $P_{\mathrm{s}}^{\mathrm{u}}$, $P_{\mathrm{s}}^{\mathrm{r}}$ and $P_{\mathrm{s}}^{\mathrm{d}}$, are all same and given by :

$$\begin{aligned} P_{\mathrm{s}}^{\mathrm{u}}(= P_{\mathrm{s}}^{\mathrm{r}} = P_{\mathrm{s}}^{\mathrm{d}}) &= P\{\text{successful Tx} \mid \text{both the first CCA and the second CCA are succeed}\} \\ &= \frac{\left\{1-\pi_{\mathrm{u}}-\sum_{i=0}^{M}\left(\pi_{(i,0)_{\mathrm{u}}}+\pi_{(i,-1)_{\mathrm{u}}}\right)\right\}^{n-1}\left\{1-\pi_{\mathrm{d}}-\sum_{i=0}^{M}\left(\pi_{(i,0)_{\mathrm{r}}}+\pi_{(i,-1)_{\mathrm{r}}}+\pi_{(i,0)_{\mathrm{d}}}+\pi_{(i,-1)_{\mathrm{d}}}\right)\right\}^{n-1}}{\left(1-\pi_{\mathrm{u}}-\sum_{i=0}^{M} \pi_{(i,-1)_{\mathrm{u}}}\right)^{n-1}\left\{1-\pi_{\mathrm{d}}-\sum_{i=0}^{M}\left(\pi_{(i,-1)_{\mathrm{r}}}+\pi_{(i,-1)_{\mathrm{d}}}\right)\right\}^{n-1}} \end{aligned} \tag{16}$$

4.2 Performance Measures

In this subsection, we obtain several performance measures such as throughput, delay, loss probability and energy consumption.

Throughput. The normalized system throughput S, defined as the fraction of time the channel is used to transmit successfully for uplink data packet and downlink data packet, is given as follows.

$$S = n \cdot (\pi_{Tx_{\mathrm{u}}} \cdot P_{\mathrm{s}}^{\mathrm{u}} + \pi_{Rx_{\mathrm{d}}} \cdot P_{\mathrm{s}}^{\mathrm{d}}) \tag{17}$$

Delay. The average delay $\mathrm{E}[D_{\mathrm{u}}]$ from the moment of uplink data packet arrival at device to the moment of service completion point, can be obtained as the same form as the equation (6) in [3] by just replacing parameters α, β and P_{s} by α_{u}, β_{u} and $P_{\mathrm{s}}^{\mathrm{u}}$, respectively. Similarly, $\mathrm{E}[D_{\mathrm{r}}]$ is obtained as the same form as the equation (5) by just replacing parameters α, β and P_{s} by α_{r}, β_{r} and $P_{\mathrm{s}}^{\mathrm{r}}$ in the equations for calculation of P_{loss}, $\mathrm{E}[D_{\mathrm{r}}^{\mathrm{L}}]$ and $\mathrm{E}[D_{\mathrm{r}}^{\mathrm{S}}]$, respectively. Then, $\mathrm{E}[D_{\mathrm{d}}]$ is calculated by the same form as the as the equation (8).

Packet Loss Probability. The uplink packet loss probability $P_{\mathrm{loss}}^{\mathrm{u}}$ can be obtained as the same form with the equation (7) in [3] by just replacing parameters α, β and P_{s} by α_{u}, β_{u} and $P_{\mathrm{s}}^{\mathrm{u}}$, respectively. Let $P_{\mathrm{loss}}^{\mathrm{r}}$ be the probability of losing downlink request packet and $P_{\mathrm{loss}}^{\mathrm{d}}$ be the probability of losing downlink data packet. Then $P_{\mathrm{loss}}^{\mathrm{r}}$ can be obtained as the same form with the equation (9) in 3.2 by just replacing parameters α, β and P_{s} by α_{r}, β_{r} and $P_{\mathrm{s}}^{\mathrm{r}}$, respectively. Moreover, $P_{\mathrm{loss}}^{\mathrm{d}}$ can be obtained as the same form with the equation (9) in 3.2 by just replacing parameters α, β and P_{s} by α_{d}, β_{d} and $P_{\mathrm{s}}^{\mathrm{d}}$, respectively.

Energy Consumption. We calculate the average energy consumption E^{slot} per one slot(mJ/slot). Let $a_{\mathrm{u}}^{\mathrm{idle}}$, a_{u}^{Tx} and a_{u}^{Rx} be the probabilities of slot being idle, being transmission, being reception(or CCA) for uplink traffic, respectively. Then,

$$a_{\mathrm{u}}^{\mathrm{idle}} = 1 - \sum_{i=0}^{M} (\pi_{(i,0)_{\mathrm{u}}} + \pi_{(i,-1)_{\mathrm{u}}}) - \pi_{Tx_{\mathrm{u}}} - \sum_{j=0}^{A} \pi_{(-1,j)_{\mathrm{u}}}$$

$$a_{\mathrm{u}}^{Tx} = \pi_{Tx_{\mathrm{u}}}$$

Similarly, let $a_{\mathrm{d}}^{\mathrm{idle}}$, a_{d}^{Tx} and a_{d}^{Rx} be the probabilities of slot being idle, being transmission, being reception(or CCA) for downlink traffic. Then

$$a_{\mathrm{d}}^{\mathrm{idle}} = \sum_{k=1}^{I} \pi_{idle[k]} + \sum_{i=0}^{M} \sum_{j=1}^{W_i - 1} \pi_{(i,j)_{\mathrm{r}}}$$

$$a_{\mathrm{d}}^{Tx} = \pi_{Tx_{\mathrm{r}}[k]} + \left(\sum_{j=0}^{A} \pi_{(-1,k)_{\mathrm{d}}} \right) P_s$$

Thus E^{slot} is calculated as follows.

$$E^{\mathrm{slot}} = a_{\mathrm{u}}^{\mathrm{idle}} a_{\mathrm{d}}^{\mathrm{idle}} E_{idle} + (a_{\mathrm{u}}^{Tx} + a_{\mathrm{d}}^{Tx}) E_{Tx} + \{1 - a_{\mathrm{u}}^{\mathrm{idle}} a_{\mathrm{d}}^{\mathrm{idle}} - (a_{\mathrm{u}}^{Tx} + a_{\mathrm{d}}^{Tx})\} E_{Rx} \tag{18}$$

5 Numerical Results and Simulation Results for Both Uplink and Downlink Traffic

In this section, numerical results for performance measures of the network with both uplink and downlink traffic are presented. For our numerical results, I is set to 500 backoff slots. The average length of a uplink data packet, $\frac{1}{1-P_{\mathrm{Tx,u}}}$, is set to 4 and the average length of a downlink data packet, $\frac{1}{1-P_{\mathrm{Rx,d}}}$, is set to 4. Note that $\sigma = 0.32$ms in case of 250 Mbps, 2.4 GHz. N and M are 2 and 4, respectively. W_0 is set to $2^3 = 8$ in our experiment. The energy consumptions at T_x, R_x, and CCA states are 0.0100224mJ, 0.0113472mJ and 0.0113472mJ, respectively, [7]. A device consumes 0.000056736mJ during idle state.

Fig. 5(a) depicts the expected delay $\mathrm{E}[D_{\mathrm{u}}]$ for uplink traffic and the expected delay $\mathrm{E}[D_{\mathrm{d}}]$ for downlink traffic. As the number of devices increases, $\mathrm{E}[D_{\mathrm{u}}]$ and $\mathrm{E}[D_{\mathrm{d}}]$ increase due to the exponential backoff by competitions of each other. Fig. 5(b) depicts the packet loss probability P^u_{loss} and P^d_{loss} for uplink traffic and downlink traffic, respectively. Also P^u_{loss} and P^d_{loss} increase as the number of devices increases. Fig. 5(c) depicts the average energy consumption E^{slot} per one backoff slot. Fig. 5 shows that the numerical results and simulation results for performance measures differ slightly. This may be caused by the analytical model where two approximations (6) and (8) are used. In Fig. 5(c), we also compare the energy consumption between the non-beacon mode and the beacon mode through simulation. For this comparison, we set the superframe duration by 96 backoff slots. From the simulation results, we find out that the non-beacon mode reduces the energy consumption about 50% compared with beacon mode, which the downlink delay is within $100msec$ (See Fig. 5(a)). Finally, our results are used for determining the optimal number of devices which can be accommodated in the system while supporting the required QoS on the expected packet delay and the packet loss probability. For instance, with the requirements of $\mathrm{E}[D_{\mathrm{u}}]\leq 20ms$, $\mathrm{E}[D_{\mathrm{d}}]\leq 100ms$, $P^u_{\mathrm{loss}} \leq 2\%$ and $P^d_{\mathrm{loss}} \leq 2\%$, the optimal number of devices in the network is from Fig. 5(a) and Fig. 5(b). With this case, we obtain from Fig. 5(c) that the average energy consumption E^{slot} per one backoff slot is 7.1×10^{-4}mJ/slot.

6 Appendix : Delay for Downlink

In this section, we obtain the expected durations $\mathrm{E}[D^{\mathrm{L}}_{\mathrm{r}}]$, $\mathrm{E}[D^{\mathrm{S}}_{\mathrm{r}}]$ and $\mathrm{E}[D_{\mathrm{d}}*]$ (See Fig. 2). $\mathrm{E}[D^{\mathrm{L}}_{\mathrm{r}}]$ is the expected time duration from the moment of generation of downlink request packet to the moment of discarding the packet, and $\mathrm{E}[D^{\mathrm{S}}_{\mathrm{r}}]$ is the expected time duration from the moment of generation of downlink request packet to the moment of beginning of downlink request packet transmission. To obtain $\mathrm{E}[D^{\mathrm{L}}_{\mathrm{r}}]$ and $\mathrm{E}[D^{\mathrm{S}}_{\mathrm{r}}]$, let P^{c} be the probability that a packet suffers collision in a backoff procedure. Then,

$$P^{\mathrm{c}} = \sum_{v=0}^{M}\sum_{r=0}^{v} {}_v\mathrm{C}_r \alpha^r \{(1-\alpha)\beta\}^{v-r}(1-\alpha)(1-\beta)(1-P_{\mathrm{s}}) \ . \qquad (19)$$

Let $\mathrm{E}[D^{\mathrm{T}}_{\mathrm{backoff}}]$ and $\mathrm{E}[D^{\mathrm{L}}_{\mathrm{backoff}}]$ be the expected number of backoff slots that a packet experience until the moment of transmission attempt in a backoff procedure and the expected number of backoff slots that a packet experience until the moment of discarding in a backoff procedure, respectively. Then,

$$\mathrm{E}[D^{\mathrm{T}}_{\mathrm{backoff}}] = \frac{\sum_{v=0}^{M}\sum_{r=0}^{v}\ {}_v\mathrm{C}_r\alpha^r\{(1-\alpha)\beta\}^{v-r}(\sum_{i=0}^{v}\frac{W_i-1}{2}+2v-r+2)}{\sum_{v=0}^{M}\sum_{r=0}^{v}\ {}_v\mathrm{C}_r\alpha^r\{(1-\alpha)\beta\}^{v-r}} \tag{20}$$

$$\begin{aligned}\mathrm{E}[D^{\mathrm{L}}_{\mathrm{backoff}}] =& \frac{\sum_{r=0}^{M}\ {}_M\mathrm{C}_r\alpha^r\{(1-\alpha)\beta\}^{M-r}}{\sum_{r=0}^{M+1}\ {}_{M+1}\mathrm{C}_r\alpha^r\{(1-\alpha)\beta\}^{M+1-r}} \\ &\times\{\alpha(\sum_{i=0}^{M}\frac{W_i-1}{2}+2M-r+1+(1-\alpha)\beta(\sum_{i=0}^{M}\frac{W_i-1}{2}+2M-r+2)\}\end{aligned} \tag{21}$$

Note that a downlink packet is discarded when the CCA fails at the Mth backoff stage. So, the expected duration $\mathrm{E}[D^{\mathrm{L}}_{\mathrm{r}}]$ is given by :

$$\mathrm{E}[D^{\mathrm{L}}_{\mathrm{r}}] = \sum_{k=0}^{\infty}(P^c)^k(1-P^c)\left\{k\left(D^{\mathrm{T}}_{\mathrm{backoff}}+R+A\right)+D^{\mathrm{L}}_{\mathrm{backoff}}\right\}\sigma\ . \tag{22}$$

The general term in (22) is the expected duration for the case that a packet is discarded after the kth collision. Similarly, the expected duration $\mathrm{E}[D^{\mathrm{S}}_{\mathrm{r}}]$ is given by :

$$\mathrm{E}[D^{\mathrm{S}}_{\mathrm{r}}] = \sum_{k=0}^{\infty}(P^c)^k(1-P^c)\left\{k\left(D^{\mathrm{T}}_{\mathrm{backoff}}+R+A\right)+D^{\mathrm{T}}_{\mathrm{backoff}}\right\}\sigma\ . \tag{23}$$

The general term in (23) is the expected duration for the case that a packet is successfully transmitted after the kth collision.

The expected delay $\mathrm{E}[D_{\mathrm{d}}*]$, from the beginning of backoff procedure for downlink data packet transmission to the moment of service completion of the downlink data packet, is given as follows.

$$\begin{aligned}\mathrm{E}[D_{\mathrm{d}}*] =& \sum_{v=0}^{M}\sum_{r=0}^{v}\ {}_v\mathrm{C}_r\alpha^r\{(1-\alpha)\beta\}^{v-r}(1-\alpha)(1-\beta)P_{\mathrm{s}}\left(\sum_{i=0}^{v}\frac{W_i-1}{2}+2v-r+2+\frac{1}{1-P_{\mathrm{Rx,d}}}+A\right)\sigma \\ &+\sum_{v=0}^{M}\sum_{r=0}^{v}\ {}_v\mathrm{C}_r\alpha^r\{(1-\alpha)\beta\}^{v-r}(1-\alpha)(1-\beta)(1-P_{\mathrm{s}}) \\ &\quad\times\left\{\left(\sum_{i=0}^{v}\frac{W_i-1}{2}+2v-r+2+\frac{1}{1-P_{\mathrm{Rx,d}}}+A\right)\sigma+\mathrm{E}[D_{\mathrm{d}}*]\right\} \\ &+\sum_{r=0}^{M}\ {}_M\mathrm{C}_r\alpha^r\{(1-\alpha)\beta\}^{M-r} \\ &\quad\times\left\{\alpha\left(\sum_{i=0}^{M}\frac{W_i-1}{2}+2M-r+1\right)+(1-\alpha)\beta\left(\sum_{i=0}^{M}\frac{W_i-1}{2}+2M-r+2\right)\right\}\sigma\end{aligned} \tag{24}$$

The first summation and second summation of the equation (24) describe the cases of successfully transmission and collision in the first transmission attempt, respectively. The last summation of the equation (24) describes the case that a packet is discarded at the first backoff procdure because the channel is continuously sensed due to busy condition in CCA.

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