Performance Analysis of Power Saving Class of Type 1 with Both Downlink and Uplink Traffics in IEEE 802.16e

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Abstract. We investigate power consumption of a mobile station with the power saving class of type 1 in the IEEE 802.16e. We deal with stochastic behavior of mobile station during not only sleep mode period but also awake mode period with both downlink and uplink traffics. Our methods for investigating the power saving class of type 1 are to construct the embedded Markov chain and the semi-Markov chain generated by the embedded Markov chain. To see the effect of the sleep mode, we obtain the average power consumption of a mobile station and the mean queueing delay of a message. Numerical results show that the larger size of the sleep window makes the power consumption of a mobile station smaller and the queueing delay of a downlink message longer.

Keywords: power saving class, IEEE 802.16e, sleep mode, embedded Markov chain, semi-Markov chain.

1 Introduction

The IEEE 802.16 standard [1] has been designed to provide communication paths between subscriber stations and the base station as an emerging broadband wireless access system. An amendment to the standard, the IEEE 802.16e, has been concluded in 2006 [2] with mobility so that subscriber stations can move during services. This amendment[2] supports handover procedure and power saving of the mobile station(MS).

Due to the mobility of stations, power saving is one of the significant issues for the battery-powered MSs. The IEEE 802.16e defines sleep mode operations called power saving classes of type 1, 2 and 3. Power saving class of type 1 is recommended for connections of Best Effort (BE) and non-real time variable rate (NRT-VR) types, power saving class of type 2 for connections of unsolicited grant service (UGS) and real time variable rate (RT-VR) types, and power saving class of type 3 for multicast connection and managements operations.

As for the sleep mode operation of the power saving class of type 1 in the IEEE 802.16e, a few studies have been done to evaluate its performance. Xiao[3] proposed a simple model for power saving class of type 1 in the IEEE 802.16e

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for downlink traffic by focusing on sleep mode period. Xiao[4] and Zhang and Fujise[5] extended Xiao's model[3] to cover uplink and downlink traffics. Nejatian et al.[6][7] and Zhang[8] developed analytical models for non-Poisson traffic arrivals. Nejatian et al.[6][7] assumed that the interarrival times of packets follow Erlang distribution, whereas Zhang[8] assumed that the interarrival time of downlink packet follows hyper-Erlang distribution. They[3][4][5][6][7][8] obtained the average power consumption and the average delay during the sleep mode period. They[3][4][5][6][7][8] focused only on the sleep mode period, so they didn't model any stochastic behaviors of packets during the awake mode period.

Seo et al.[9], Park et al.[10], Han et al.[11] and Kong et al[12] proposed analytical models considering both awake mode period and sleep mode period. In Seo et al.[9] and Park et al.[10], the mobile station was modeled as an M/G/1/K queue. They[9][10] obtained the power consumption, mean downlink packet delay and packet blocking probability. Han et al.[11] proposed an analytical model using semi-Markov chain. Kong[12] mathematically analyzed power saving classes of both type 1 and type 2. Their models[9][10][11][12] focused only on the downlink traffic, i.e., they didn't model any uplink traffic.

We investigate analytical performance on sleep mode operation of power saving class-type 1 in the IEEE 802.16e with both downlink and uplink traffics. The main contribution of this paper is that we deal with stochastic behavior of mobile stations during not only sleep mode period but also awake mode period with both downlink and uplink traffics. In the literature mentioned above, they analyzed either only downlink traffic [9][10][11][12], or only sleep mode without awake mode[3][4][5][6][7][8]. Additionally, we find the mean queueing delay to see how sleep mode affects on queueing delay. Our model also takes account of practical setup times such as switching time from awake mode to sleep mode, switching time from sleep mode to awake mode and close-down time.

We organize the rest of this paper as follows. In Section 2, we describe sleep mode operation of power saving class of type 1 in the IEEE 802.16e. In Section 3, we present a mathematical model for the power saving class of type 1. Embedded Markov chains and semi-Markov chains generated by the embedded Markov chain of MS are constructed. We obtain the average power consumption. In section 4, we give numerical results and evaluate the performance of the power saving class of type 1.

2 Sleep Mode Operation of Power Saving Class of Type 1 in IEEE 802.16e

A mobile station(MS) has two modes: awake mode and sleep mode. While the MS is in the awake mode, it can send or receive data according to the BS's bandwidth scheduling. The MS sends a sleep mode request message (MOB_SLP-REQ) to the BS when there is no arrival message destined to the MS during the following close-down time (called *idle frame threshold*[11]) after both the downlink buffer at the BS and the uplink buffer at the MS become empty. The MS receives a sleep mode response message (MOB_SLP-RSP) from the BS, which contains the sizes of the initial sleep window, the final sleep window and the listening window, and the MS goes into the sleep mode. A sleep mode consists of sleep windows and listening windows, which are switched alternatively until the MS is notified of the buffered data at the BS, or an uplink data message arrives at the MS. During the sleep windows the MS may power down physical operation components. If no messages arrive during a sleep window and a following listening window, the size of the next sleep window is doubled, but not greater than the final sleep window. A traffic indication message (MOB_TRF-IND) shall be broadcasted by the BS during the listening window to alert the MS of the appearance of downlink traffic demand. When the MS receives a positive MOB_TRF-IND, the MS terminates the sleep mode and receives the pending data after the switching time from sleep mode to awake mode. During the switching time from sleep mode to awake mode, the BS requests the MS to send a bandwidth request header and confirms that the MS is in awake mode. The sleep mode is also terminated immediately if an uplink message arrives at the MS. Fig. 1 illustrates power saving class of type 1.

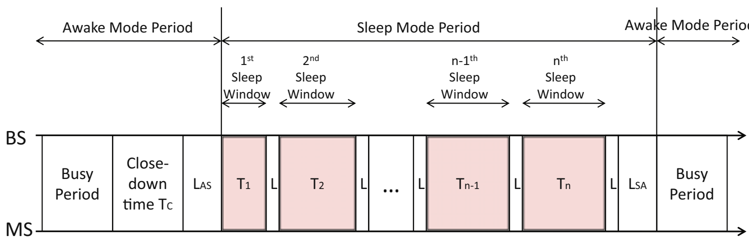


Fig. 1. Power saving class of type 1

3 Analytical Model

3.1 Assumtions

We assume that the downlink message arrivals at the BS toward the tagged MS and the uplink message arrivals at the MS toward the BS follow independent Poisson processes of rate λ_d and λ_u, respectively. The number of packets in a downlink message has a geometric distribution with parameter ρ_d and the number of packets in a uplink message has a geometric distribution with parameter ρ_u. Let T_j denote the length of the j-th sleep window. Let L denote the fixed length of a listening window. The MS stays in the awake mode period for an additional close-down time T_C after the buffers become empty Let L_{AS} denote the switching time from awake mode to sleep mode. It is known that L_{AS} needs at least 4 frames. During the switching time, the MS sends MOB_SLP-REQ and receive MOB_SLP-RSP. Let L_{SA} denote the switching time from sleep mode to awake mode. It is known that L_{SA} needs at least 3 frames.

Even though uplink arrivals and downlink arrivals are independent and are served by uplink subframe and downlink subframe, separately, they share a common sleep mode period. Thus, two traffics interact each other. We assume that capacity of each queue (in the BS for downlink message and in the MS for uplink message, respectively) is infinite.

3.2 Analysis of Sleep Mode Period

We calculate the necessary probabilities for finding the steady-state probabilities of the embedded Markov chain. We find the probability that the MS enters sleep mode period from awake mode period after both queues become empty. If there are neither downlink arrivals nor uplink arrival during a close-down time T_C, the MS sends a MOB_SLP-REQ to the BS. After getting approval(MOB_SLP-RSP) at the last frame of switching time L_{AS} from awake mode to sleep mode, the MS goes into sleep mode period and powers down its physical operation components. If any message arrives during the close-down time or the switching time L_{AS} from awake mode to sleep mode, the MS keeps on the awake mode period and sends/receives the message. However, the BS cannot cancel the MS's initiation of the sleep mode period due to downlink arrivals during the last frame of L_{AS}. Let q_1 be the probability that there are neither downlink nor uplink arrivals during $T_C + L_{AS} - 1$ frames. Let q_2 be the probability that there are no uplink arrivals during the last frame of L_{AS}.

$$\begin{aligned} q_1 &= e^{-(\lambda_d+\lambda_u)(T_C+L_{AS}-1)} \\ q_2 &= e^{-\lambda_u}. \end{aligned} \tag{1}$$

After both queues become empty, the MS enters the sleep mode period with probability $q_1 q_2$.

The instants in terminating sleep mode period are different for uplink traffic and downlink traffic. In case that downlink traffic occurs during sleep mode period, sleep mode period is ended in the next listening window. In contrast, if there are uplink arrivals during sleep mode period, the sleep mode period is ended immediately.

We consider two cases separately, and find the probability that there are i downlink messages in the BS and j uplink messages in the MS at the beginning of awake mode period, i.e. the end of sleep mode period.

- Case 1 - termination by uplink messages: If an uplink message arrives at the MS during a sleep mode period, the MS terminates the sleep mode period at the arrival frame and sends the message after the switching time L_{SA} from sleep mode to awake mode. Note that the arrivals of downlink messages during the same sleep window and the listening window do not affect the termination of the sleep mode period. Let α_n^u denote the probability that a sleep mode period is terminated by uplink arrivals at the MS during the n-th sleep window or the n-th listening window.

$$\alpha_1^u = 1 - e^{-\lambda_u(T_1+L)}$$
$$\alpha_n^u = e^{-\lambda_d(1-L+\sum_{k=1}^{n-1}(T_k+L))} e^{-\lambda_u \sum_{k=1}^{n-1}(T_k+L)}(1-e^{-\lambda_u(T_n+L)}) \quad n = 2, 3, \cdots . \tag{2}$$

Let $h_{i,j}^u$ be the probability that a sleep mode period is terminated by uplink arrivals and there are i downlink messages in the BS and j uplink messages in the MS at the beginning of the awake mode period. The probability $h_{i,j}^u$ is obtained by

$$\begin{aligned} h_{i,j}^u =& \alpha_1^u \sum_{k=1}^{T_1+L} \frac{e^{-\lambda_u(k-1)}(1-e^{-\lambda_u})}{1-e^{-\lambda_u(T_1+L)}} \frac{(\lambda_d(1+k+L_{SA})^i e^{-\lambda_d(1+k+L_{SA})})}{i!} \\ & \frac{e^{-\lambda_u(1+L_{SA})}((\lambda_d(1+L_{SA}))^j - (\lambda_d L_{SA})^j)}{j!} \\ & + \sum_{n=2}^{\infty} \alpha_n^u \sum_{k=1}^{T_n+L} \frac{e^{-\lambda_u(k-1)}(1-e^{-\lambda_u})}{1-e^{-\lambda_u(T_1+L)}} \frac{(\lambda_d(L+k+L_{SA})^i e^{-\lambda_d(L+k+L_{SA})})}{i!} \\ & \frac{e^{-\lambda_u(1+L_{SA})}((\lambda_d(1+L_{SA}))^j - (\lambda_d L_{SA})^j)}{j!} \qquad for\ i \geq 0\ and\ j \geq 1 \end{aligned} \tag{3}$$

– Case 2 - termination by downlink messages: If a downlink message arrives in the BS during a sleep mode period, a positive traffic indication message (MOB_TRF-IND) is sent at the beginning of the following listening window and the sleep mode period is terminated. Note that the case that any uplink messages arrive in the MS until the listening window in which the positive traffic indication message is sent belongs to *Case* 1. Let α_n^d denote the probability that a sleep mode period is terminated by downlink arrivals in the BS during the $(n-1)$th listening window or the n-th sleep window.

$$\alpha_1^d = (1 - e^{-\lambda_d(1+T_1)})e^{-\lambda_u(T_1+L)}$$
$$\alpha_n^d = e^{-(\lambda_d+\lambda_u)\sum_{k=1}^{n-1}(T_k+L)-\lambda_d(1-L)-\lambda_u(T_n+L)}(1-e^{-\lambda_d(L+T_n)}), \quad n = 2, 3, \cdots . \tag{4}$$

Let $a_{i,0}$ denote the probability that a sleep mode period is terminated by downlink messages and there are i downlink messages in the BS at the end of the last sleep window. By the assumption that the sleep mode period is terminated by downlink messages, any uplink messages cannot arrive in the MS until the end of the last sleep window. $a_{i,0}$ is obtained by

$$\begin{aligned} a_{i,0} =& \alpha_1^d \frac{1}{1-e^{-\lambda_d(T_1+1)}} \frac{(\lambda_d(T_1+1))^i e^{-\lambda_d(T_1+1)}}{i!} \\ & + \sum_{n=2}^{\infty} \alpha_n^d \frac{1}{1-e^{-\lambda_d(T_n+L)}} \frac{(\lambda_d(T_n+L))^i e^{-\lambda_d(T_n+L)}}{i!} \end{aligned} \tag{5}$$

Let $\tilde{a}_{i,j}$ denote the probability that i downlink message arrive during the last listening window(the listening window during which a positive traffic indication message is sent) and the switching time L_{SA} from sleep mode to awake mode and j uplink messages arrive during the switching time L_{SA}.

$$\tilde{a}_{i,j} = \frac{(\lambda_d(L + L_{SA}))^i e^{\lambda_d(L+L_{SA})}}{i!} \frac{(\lambda_u L_{SA})^j e^{\lambda_u L_{SA}}}{j!} \tag{6}$$

Let $h^d_{i,j}$ be the probability that a sleep mode period is terminated by downlink arrivals and there are i downlink messages in the BS and j uplink messages in the MS at the beginning of the awake mode period. Since we consider no uplink arrivals during the last listening window, $h^u_{i,j}$ is obtained as follows

$$h^d_{i,j} = \sum_{k=1}^{i} a_{k,0}\tilde{a}_{i-k,j} \quad for\ i \geq 1. \tag{7}$$

Thus the probability $h_{i,j}$ that there are i downlink messages in the BS and j uplink messages in the MS at the beginning of awake mode period is given by

$$h_{i,j} = h^u_{i,j} + h^d_{i,j}. \tag{8}$$

3.3 Embedded Markov Chain

We consider an embedded Markov chain representing the numbers of downlink and uplink messages, respectively, immediately after the following embedded points:

– time epoch where an uplink or downlink message completes its transmission,
– the beginning of the transmission of a uplink message at the MS after the uplink queue becomes empty,
– the beginning of the transmission of a downlink message at the BS after the downlink queue becomes empty.

Let X_n be the number of downlink messages and Y_n the number of uplink messages immediately after the n-th embedded point. Then, $\{(X_n, Y_n)\}$ is a discrete time embedded Markov chain with state space $\{(0,0), (i,0), (0,j), (i,j)|i \geq 1, j \geq 1\}$. Let $\{\pi_{i,j}\}$ be the steady-state probability of the embedded Markov chain. Before we find one-step transition probability, we calculate the distribution of the residence time of state (i,j) for the semi-Markov chain generated by the embedded Markov chain.

Distribution of Residence Times. Let $b_{i,j}(n)$ be the probability that the residence time at state (i,j) is n frames.

– $b_{0,0}(n)$: We obtain the distribution of the residence time at state $(0,0)$ as follows.

$$b_{0,0}(n) = \begin{cases} e^{-(\lambda_d+\lambda_u)(n-1)}(1-e^{-(\lambda_d+\lambda_u)}) \\ \qquad for\ 1 \le n \le T_C + L_{AS} - 1 \\ e^{-\lambda_d(T_C+L_{AS}-1)}e^{-\lambda_u(T_C+L_{AS}-1-L_{SA})}(1-e^{-\lambda_u}) \\ \qquad for\ n = T_C + L_{AS} \\ 0 \qquad for\ T_C + L_{AS} + 1 \le n \le T_C + L_{AS} + L_{SA} + 1 \\ e^{-\lambda_d(T_C+L_{AS}-1)}e^{-\lambda_u(n-1-L_{SA})}(1-e^{-\lambda_u}) \\ \qquad for\ T_C + L_{AS} + L_{SA} \le n \le T_C + L_{AS} + T_1 + L - 1 + L_{SA} \\ e^{-\lambda_d(T_C+L_{AS}-1)-\lambda_u(T_C+L_{AS}+T_1+L-1)}(1-e^{-\lambda_d(T_1+1)-\lambda_u}) \\ \qquad for\ n = T_C + L_{AS} + T_1 + L + L_{SA} \\ e^{-\lambda_d(T_C+L_{AS}+\sum_{k=1}^{N(n)} T_k+(N(n)-1)L)-\lambda_u(n-1-L_{SA})}(1-e^{-\lambda_u}) \\ \qquad for\ T_C + L_{AS} + \sum_{k=1}^{N(n)} T_k + (N(n)-1)L - 1 + L_{SA} \le n \\ \qquad\quad \le T_C + L_{AS} + \sum_{k=1}^{N(n)+1} T_k + N(n)L - 1 + L_{SA} \\ e^{-\lambda_d(T_C+L_{AS}+\sum_{k=1}^{N(n)-1} T_k+(N(n)-2)L)-\lambda_u(n-1-L_{SA})}(1-e^{-\lambda_d(T_{N(n)}+L)-\lambda_u}) \\ \qquad for\ n = T_C + L_{AS} + \sum_{k=1}^{T_{N(n)}} T_k + N(n)L + L_{SA} \end{cases} \tag{9}$$

where $N(n) = max\{m | T_C + L_{AS} + \sum_{k=1}^{m}(T_k + L) + L_{SA} \le n\}$, the number of the sleep window during a sleep mode period.

– $b_{0,j}(n),\ j \ge 1$: Let $b^u_{0,j}(n)$ be the probability that the residence time at state $(0, j)$ is n frames and the next embedded point is the completion of a transmission of an uplink message. Let $b^d_{0,j}(n)$ be the probability that the residence time at state $(0, j)$ is n frames and the next embedded point is the arrival epoch of a downlink message during an uplink transmission. $b^u_{0,j}(n)$ and $b^d_{0,j}(n)$ are obtained by

$$b^u_{0,j}(n) = e^{-\lambda_d(n-1)}(1-\rho_u)^{n-1}\rho_u \tag{10}$$

$$b^d_{0,j}(n) = e^{-\lambda_d(n-1)}(1-e^{-\lambda_d})(1-\rho_u)^n \tag{11}$$

Additionally, the distribution of the residence time from state $(0, j)$ is obtained by

$$b_{0,j}(n) = b^u_{0,j}(n) + b^d_{0,j}(n) \quad for\ j \ge 1. \tag{12}$$

– $b_{i,0}(n),\ i \ge 1$: Let $b^d_{i,0}(n)$ be the probability that the residence time at state $(i, 0)$ is n frames and the next embedded point is the completion of a transmission of a downlink message. Let $b^u_{i,0}(n)$ be the probability that the residence time at state $(i, 0)$ is n frames and the next embedded point is the arrival epoch of an uplink message during a downlink transmission. $b^d_{i,0}(n)$ and $b^u_{i,0}(n)$ are obtained by

$$b^d_{i,0}(n) = e^{-\lambda_u(n-1)}(1-\rho_d)^{n-1}\rho_d \tag{13}$$

$$b^u_{i,0}(n) = e^{-\lambda_u(n-1)}(1-e^{-\lambda_u})(1-\rho_d)^n. \tag{14}$$

Additionally, the distribution of the residence time at state $(i,0),\ i \geq 1$ is obtained by

$$b_{i,0}(n) = b^d_{i,0}(n) + b^u_{i,0}(n) \qquad for\ i \geq 1 \tag{15}$$

– $b_{i,j}(n),\ i,j \geq 1$: The residence time at state (i,j) is the minimum of transmission time of a uplink message and a downlink message from the beginning of the embedded point. Since we assume that the number of packets in downlink and uplink messages have geometric distributions with parameter ρ_d and ρ_u, respectively, the minimum of two independent geometric distributions is also a geometric distribution with parameter $\rho = \rho_d + \rho_u - \rho_d\rho_u$. The distribution of the residence time at state (i,j) is obtained by

$$b_{i,j}(n) = (1-\rho)^{n-1}\rho \qquad for\ i \geq 1, j \geq 1 \tag{16}$$

One-step Transition Probabilities. To find the one-step transition probabilities $p_{(i,j),(k,l)}$ from state (i,j) to state (k,l), we consider all possible transitions at the following each state:

– $p_{(0,0),(k,l)}$: If a message arrives in the BS or the MS during a close-down time or a switching time except the last frame ($T_C + L_{AS} - 1$ frames), the MS stays in awake mode period and receives or sends the message. In this case, the conditional probability $g_{k,l}$, given that messages arrive during a close-down time or the switching time except the last frame, that there are k downlink messages and l uplink messages at the beginning of the frame where the transmission starts again after the queues become empty is given by

$$g_{k,l} = \frac{1}{1-e^{-(\lambda_d+\lambda_u)}} \frac{\lambda_d^k e^{-\lambda_d}}{k!} \frac{\lambda_u^l e^{-\lambda_u}}{l!}. \tag{17}$$

If an uplink message arrives in the MS during the last frame of switching time, the MS doesn't enter the sleep mode period, and it sends the message to the BS. In this case, the conditional probability $g'_{k,l}$, given that uplink messages arrive during the last frame of switching time, that there are k downlink messages and l uplink messages at the beginning of the frame where the transmission starts again after the queues become empty is given by

$$g'_{k,l} = \frac{1}{1-e^{-\lambda_u}} \frac{\lambda_d^k e^{-\lambda_d}}{k!} \frac{\lambda_u^l e^{-\lambda_u}}{l!}. \tag{18}$$

The probability $s_{k,l}$ that there are k downlink messages and l uplink messages in each queues at the beginning of the frame where the transmission start again after the queues become empty is obtained by

$$s_{k,l} = (1-q_1)g_{k,l} + q_1(1-q_2)g'_{k,l} + q_1q_2h_{k,l}. \tag{19}$$

In other word, $p_{(0,0),(k,l)}$, the one-step transition probability from $(0,0)$ to (k,l) is $s_{k,l}$.

– $p_{(0,j),(k,l)}$: The probability that the next embedded point is the completion of a transmission of an uplink message and k downlink messages and l uplink messages arrive in a state $(0,j)$ is given by

$$v^u_{k,l} = \sum_{n=1}^{\infty} b^u_{0,j}(n) \frac{\lambda_d^k e^{-\lambda_d}}{k!} \frac{(n\lambda_u)^l e^{-n\lambda_u}}{l!}. \tag{20}$$

The probability that the next next embedded point is the arrival epoch of a downlink message during an uplink transmission and k downlink messages and l uplink messages arrive in a state $(0,j)$ is given by

$$\begin{aligned} v^d_{k,l} &= \sum_{n=1}^{\infty} b^d_{0,j}(n) \frac{1}{1-e^{-\lambda_d}} \frac{\lambda_d^k e^{-\lambda_d}}{k!} \frac{(n\lambda_u)^l e^{-n\lambda_u}}{l!} \quad for\ k \neq 0 \\ v^d_{0,l} &= 0. \end{aligned} \tag{21}$$

The one step transition probability $p_{(0,j),(k,l)},\ j \geq 1$ from state $(0,j)$ to state (k,l) is obtained by

$$p_{(0,j),(k,l)} = \begin{cases} v^u_{k,l-j+1} + v^d_{k,l-j}, & l \geq j \\ v^u_{k,0}, & l = j-1 \end{cases} \tag{22}$$

– $p_{(i,0),(k,l)}$: The probability that the next embedded point is the arrival epoch of an uplink message during a downlink tranmission and k downlink messages and l uplink messages arrive in a state $(i,0)$ is given by

$$\begin{aligned} w^u_{k,l} &= \sum_{n=1}^{\infty} b^u_{i,0}(n) \frac{1}{1-e^{-\lambda_u}} \frac{(n\lambda_d)^k e^{-n\lambda_d}}{k!} \frac{\lambda_u^l e^{-\lambda_u}}{l!} \quad for\ l \neq 0 \\ w^u_{k,0} &= 0. \end{aligned}$$

The probability that the next embedded point is the completion of a transmission of a downlink message and k downlink messages and l uplink messages arrive in a state $(i,0)$ is given by

$$w^d_{k,l} = \sum_{n=1}^{\infty} b^d_{i,0}(n) \frac{\lambda_d^k e^{-\lambda_d}}{k!} \frac{(n\lambda_u)^l e^{-n\lambda_u}}{l!} \tag{23}$$

The one step transition probability $p_{(i,0),(k,l)},\ i \geq 1$ from state $(i,0)$ to state (k,l) is obtained by

$$p_{(i,0),(k,l)} = \begin{cases} w^u_{k-i,l} + w^d_{k-i+1,l}, & k \geq i \\ w^d_{0,l}, & k = i-1 \end{cases} \tag{24}$$

– $p_{(i,j),(k,l)},\ i,j \geq 1$: The probability that k downlink messages and l uplink messages arrive in a state (i,j) is given by

$$r_{k,l} = \sum_{n=1}^{\infty} (1-\rho)^{n-1} \rho \frac{(n\lambda_d)^k e^{-n\lambda_d}}{k!} \frac{(n\lambda_u)^l e^{-n\lambda_u}}{l!}. \tag{25}$$

A transition can occur by the transmission of a downlink or uplink message. The probability that a transition occurs by the transmission of a downlink message is given by

$$p_d = \frac{\rho_d - \rho_d\rho_u}{\rho_d + \rho_u - \rho_d\rho_u} \tag{26}$$

The probability that a transition occurs by the transmission of an uplink message is given by

$$p_u = \frac{\rho_u - \rho_d\rho_u}{\rho_d + \rho_u - \rho_d\rho_u} \tag{27}$$

The probability that a transition occurs by the transmission of both downlink and uplink messages is given by

$$p_{du} = \frac{\rho_d\rho_u}{\rho_d + \rho_u - \rho_d\rho_u} \tag{28}$$

The one step transition probability $p_{(i,j),(k,l)},\ i,j \geq 1$ from state (i,j) to state (k,l) is obtained by

$$p_{(i,j),(k,l)} = \begin{cases} r_{k-i+1,l-j}p_d + r_{k-i,l-j+1}p_u + r_{k-i+1,l-j+1}, & k \geq i \text{ and } l \geq j \\ r_{0,l-j}p_d + r_{0,l-j+1}p_{du}, & k = i-1 \text{ and } l \geq j \\ r_{k-i,0}p_u + r_{k-i+1,0}p_{du}, & k \geq i \text{ and } j = l-1 \\ r_{0,0}p_{du}, & k = i-1 \text{ and } j = l-1 \end{cases} \tag{29}$$

Balance Equations By (19)-(29), the steady-state probabilities $\{\pi_{i,j}\}_{i,j=0}^{\infty}$ of the embedded Markov chain satisfy the following balance equations

$$\pi_{k,l} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \pi_{i,j}p_{(i,j),(k,l)} \tag{30}$$

and the normalization condition $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \pi_{i,j} = 1$. In calculation, we restrict the state space by large i and j.

3.4 Power Consumption of Mobile Station

Now we consider a semi-Markov chain generated by the embedded Markov chain to find the mean residence time of the semi-Markov chain at state (i,j) and to derive sleep mode ratio which is defined as the proportion of the sleep mode period. The mean residence time $\eta_{i,j}$ of the semi-Markov chain at state (i,j) is given by

$$\eta_{i,j} = \sum_{n=1}^{\infty} nb_{i,j}(n) \tag{31}$$

We obtain the expected length of a sleep mode period as follows.

$$R_{sleep} = \sum_{n=1}^{\infty} \alpha_n^u [\sum_{j=1}^{n-1}(T_j+L) + \sum_{k=1}^{T_n+L} k\frac{e^{-\lambda_u(k-1)}(1-e^{-\lambda_u})}{1-e^{-\lambda_u(T_n+L)}}] + \sum_{n=1}^{\infty} \alpha_n^d \sum_{j=1}^{n}(T_j+L) + L_{SA} \tag{32}$$

Sleep mode ratio is given by

$$Sleep\ Mode\ Ratio = \frac{\pi_{0,0} q_1 q_2 R_{sleep}}{\sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \pi_{a,b} \eta_{a,b}} \tag{33}$$

The MS is assumed to consume its energy by E_{awake} (mJ) per one frame in the awake mode period and $E_{powersaving}$ per one frame in the sleep window. The energy of the MS is consumed additionally by E_{change} when the MS switches from a sleep window to a listening window. The average energy consumption during a sleep mode period is obtained by

$$\begin{aligned} E_{sleep} = & \sum_{n=1}^{\infty} \alpha_n^u \Big[\sum_{j=1}^{n-1} (T_j E_{powersaving} + L E_{awake}) + \sum_{k=1}^{T_n} k E_{powersaving} \frac{e^{-\lambda_u (k-1)}(1 - e^{-\lambda_u})}{1 - e^{-\lambda_u (T_n + L)}} \\ & + \sum_{k=1}^{L} (T_n E_{powersaving} + k E_{awake}) \frac{e^{-\lambda_u (T_n + k - 1)}(1 - e^{-\lambda_u})}{1 - e^{-\lambda_u (T_n + L)}} + n E_{change} \Big] \\ & + \sum_{n=1}^{\infty} \alpha_n^d \sum_{j=1}^{n} (T_j E_{powersaving} + L E_{awake} + E_{change}) + L_{SA} E_{awake}. \end{aligned} \tag{34}$$

The average energy consumption per one frame during a sleep mode period is given by

$$E_S = \frac{E_{sleep}}{R_{sleep}} \tag{35}$$

Thus the average power consumption with both uplink and downlink traffics can be derived as follows:

$$Power\ Consumption = (Sleep\,mode\ Ratio) E_S + (1 - (Sleep\,mode\ Ratio)) E_{awake} \tag{36}$$

3.5 Queueing Delay

The steady-state probability $\pi_{i,j}^*$ for the semi-Markov chain generated by the embedded Markov chain in subsection 3.3 is given by

$$\pi_{i,j}^* = \frac{\pi_{i,j} \eta_{i,j}}{\sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \pi_{a,b} \eta_{a,b}}. \tag{37}$$

Let $X(t)$ be the number of downlink messages and $Y(t)$ the number of uplink messages at time t. To find the queueing delay for both uplink and downlink traffics, the steady-state probability $p_{k,l}$ for a stochastic process $(X(t), Y(t))$ is given as follows[13]

$$\begin{aligned} p_{k,l} &= \sum_{i=0}^{k} \sum_{j=0}^{l} \frac{\pi_{i,j}^*}{\eta_{i,j}} \sum_{n=1}^{\infty} P(k-i\ downlink\ arrivals, l-j\ uplink\ arrivals\ in\ [0,n]) b_{i,j}(n) \\ &= \sum_{i=0}^{k} \sum_{j=0}^{l} \frac{\pi_{i,j}^*}{\eta_{i,j}} \sum_{n=1}^{\infty} \frac{(\lambda_d n)^{k-i} e^{-\lambda_d n}}{(k-i)!} \frac{(\lambda_u n)^{l-j} e^{-\lambda_u n}}{(l-j)!} b_{i,j}(n) \end{aligned} \tag{38}$$

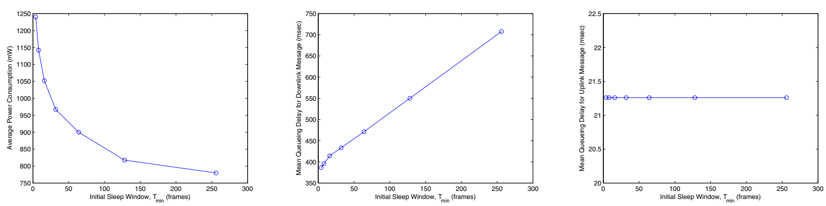
Thus, the downlink mean queueing delay and uplink mean queueing delay are respectively obtained by

$$\begin{aligned} mean\ queueing\ delay\ for\ downlink\ message &= \frac{1}{\lambda_d}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} i p_{i,j} - \frac{1}{\rho_d} \\ mean\ queueing\ delay\ for\ uplink\ message &= \frac{1}{\lambda_u}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} j p_{i,j} - \frac{1}{\rho_u}. \end{aligned} \tag{39}$$

4 Numerical Results

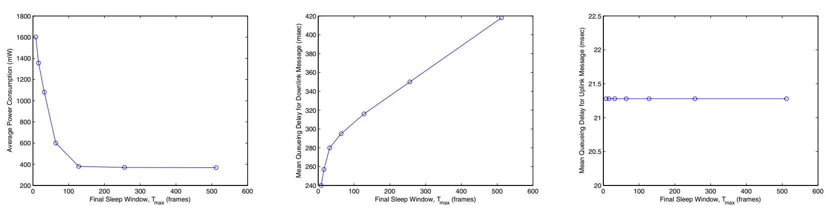
For numerical results, we use the following parameters:

- Listening Window L : 2 frames
- Close-down time T_C : 5 frames
- L_{AS} : 4 frames
- L_{SA} : 3 frames
- E_{awake} : 10 mJ
- $E_{powersaving}$: 1 mJ



(a) Power consumption (b) Mean queueing delay for a downlink message (c) Mean queueing delay for an uplink message

Fig. 2. Effect of the Initial Sleep Window



(a) Power consumption (b) Mean queueing delay for a downlink message (c) Mean queueing delay for an uplink message

Fig. 3. Effect of the Final Sleep Window

– E_{change} : 10 mJ
– λ_d : 0.01 /frame
– λ_u : 0.01 /frame

The length of a message is assumed to be geometrically distributed with mean 10 frames.

Fig. 2 depicts (a) the average power consumption, (b) the mean queueing delay for a downlink message and (c) the mean queueing delay for an uplink message as the initial sleep window increases from 4 frames to 256 frames. In Fig. 2, we assume T_{max} = 256 frames. We see that the average power consumption decreases and the mean queueing delay for a downlink message increases, as the initial sleep window increases. However, the mean queueing delay for an uplink message does not depend on the initial sleep window size.

Fig. 3 depicts (a) the average power consumption, (b) the mean queueing delay for a downlink message and (c) the mean queueing delay for an uplink message as the final sleep window increases from 8 frames to 512 frames. In Fig. 3, we assume T_{min} = 8 frames. We also see that the average power consumption decreases and the mean queueing delay for a downlink message increases, as the final sleep window increases. The final sleep window does not influence the mean queueing delay for an uplink message.

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References

1. IEEE 802.16-2004, Part 16: Air Interface for Fixed Broadband Wireless Access Systems, Standard for Local and Metropolitan Area Networks (2004)
2. IEEE 802.16e-2005, Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems - Amendment for Physical and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands (2006)
3. Xiao, Y.: Energy saving mechanism in the IEEE 802.16e wireless MAN. IEEE Communications Letters 9(7), 595–597 (2005)
4. Xiao, Y.: Performance Analysis of an Energy Saving Mechanism in the IEEE 802. 16e Wireless MAN. In: Consumer Communications and Networking Conference (CCNC) 2006, vol. 1, pp. 8–10 (2006)
5. Zhang, Y., Fujise, M.: Energy management in the IEEE 802.16e MAC. IEEE Communications Letters 10(4), 311–313 (2006)
6. Nejatian, N.M.P., Nayebi, M.M., Ashtiani, F.: Effect of different traffic patterns on power consumption of sleep mode in the IEEE 802.16e MAC. In: IEEE ICT-MICC 2007, pp. 649–653 (2007)

7. Nejatian, N.M.P., Nayebi, M.M.: Evaluating the Effect of non-Poisson Traffic Patterns on Power Consumption of Sleep Mode in the IEEE 802.16e MAC. In: IFIP WOCN 2007, pp. 1–5 (2007)
8. Zhang, Y.: Performance Modeling of Energy Management Mechanism in IEEE 802.16e Mobile WiMAX. In: IEEE WCNC 2007, pp. 3205–3209 (2007)
9. Seo, J., Lee, S., Park, N., Lee, H., Cho, C.: Performance analysis of sleep mode operation in IEEE 802.16e. In: IEEE VTC 2004-Fall, vol. 2, 26-29, pp. 1169–1173 (2004)
10. Park, Y., Hwang, G.U.: Performance Modelling and Analysis of the Sleep-Mode in IEEE802.16e WMAN. In: IEEE VTC 2007-Spring, pp. 2801–2806 (2007)
11. Han, K., Choi, S.: Performance Analysis of sleep mode Operation in IEEE802.16e Mobile Broadband Wireless Access Systems. In: IEEE VTC 2006-Spring, vol. 3 (2006)
12. Kong, L., Tsang, D.H.K.: Performance Study of Power Saving Classes of Type I and II in IEEE 802.16e. In: IEEE Conference on Local Computer Networks, pp. 20–27 (2006)
13. Takagi, H.: Queueing Analysis. Vacation and Priority Systems, vol. 1. North-Holland, Amsterdam (1991)
14. Gross, D., Harris, C.: Queueing Theory. Wiley, Chichester (1998)