Dynamics of Priority-Queue Networks

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Abstract. In this Work-in-Progress paper, we study the dynamics of priority-queue networks by generalizing the interacting priority queue model of Oliveira and Vazquez [Physica A **388**, 187 (2009)]. We show that the original AND-type protocol for interacting tasks is not scalable for the queue networks with more than two queues. We then introduce a scalable interaction protocol, an OR-type one, and examine the effects of the number of queues and the network topology on the waiting time dynamics of the priority-queue networks. We also study the effect of synchronicity in task executions to the waiting time dynamics in the priority-queue networks.

Keywords: Priority queue, human dynamics, social network.

1 Motivations

Priority queue models of human dynamics have received much attention among statistical physics community lately [1,2,3,4,5,6,7,8]. Since the introduction of the Barabasi model [1], the priority queue model aims mainly to account for the heavy-tailed waiting time distributions observed for a number of human dynamics data [1,5,8]. The emergence of distinct "univerality classes" by the power-law exponent α of the waiting time distribution $P(\tau) \sim \tau^{-\alpha}$ has been the cornerstone in the development of the subject. Yet, it has remained to be seen how robust the dynamic properties of the priority-queue model are under generalizations towards more realistic modeling of human dynamics.

In this respect, the recent work by Oliveira and Vazquez [9] on the *interacting* priority queues represents a crucial first step forward. They introduced tasks that require simultaneous actions of two individuals, that is, *interacting* tasks, and constructed a minimal interacting priority model consisting of two queues. An interacting task, or I-task, is executed only when it is the highest priority task for both the individuals. Thus the interaction in the original OV model is a kind of AND-type protocol for I-tasks. With the model, they found that the power-law exponent α of the waiting time distribution for I-tasks takes values $2 \geq \alpha > 1$, different from the Barabasi single-queue model, while it approaches to the single-queue value $\alpha = 1$ in the limit of infinite queue size. Thus the OV model showed that the interaction between queues is a relevant factor for the waiting time dynamics of the priority queue models.

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In this Work-in-Progress paper, we generalize the OV model for more than two queues and other interaction protocols to present a priority-queue *network*, and study its waiting time dynamics.

2 Preliminary Results

We build the priority queue network composed of I-tasks with pairwise interactions. Upon an arbitrary network configuration, we put a priority queue on each node, with the queue-length $\ell_i = k_i + 1$; k_i I-tasks and an additional non-interacting task, where k_i is the degree of the node i. Each of the I-tasks represents the pairwise interaction between i and its neighbor, say j, denoted by I_{ij}. Here we summarize some of the preliminary results.

2.1 OV Model with $N > 2$

We first consider the generalizibility of the OV model for more than two queues. With the AND-type interaction protocol for I-tasks, we found the OV model generically not amenable for increasing network size with $N > 2$. This suggests that the OV model in its original form alone may not be appropriate for the realistic modeling of human network behaviors. Thus we turn to a different interaction protocol, an OR-type one, which leads to nontrivial active dynamics for $N > 2$.

2.2 Priority-Queue Network with OR-Type I-Tasks

We build a priority-queue network with OR-type I-tasks as follows: i) Each step, a node is randomly chosen (say, i), and its highest priority task is identified. ii) If the highest-priority task is an I-task, say I_{ij}, then it and its conjugate task I_{ji} of the node j are excuted simultaneously. Otherwise, the non-interacting task O_i is executed. iii) The priorities of all executed tasks are newly assigned. In the case of 2 queues, the power-law exponent α of the I-tasks and O-task is founded to be $\alpha_I = 3$ and $\alpha_O = 2$, respectively. We then apply the model to fully-connected networks (complete graphs) with $N > 2$, and found that the exponent α_I depends on the network size as α_I decreases as N increases, while α_O is rather stable against N. We then apply the model on star networks (graphs with the single hub node and all other "leaf" nodes connected only to the hub). For the star network, we obtained different behaviors such as different α_O for the hub- and leaf-nodes. These indicate a significant effect of network topological position of the queue node on its waiting time dynamics.

2.3 Effect of Synchronicity in Task Executions

We modify our priority-queue network model for parallel task executions, to study the effect of synchronicity of task processing in queue networks. To this end, we modify the dynamic rules in the previous section as follows: i) Each step, each node chooses its highest priority task. There might be *priority conflicts*, such that a node i chooses I_{ij}, while the partner node j chooses I_{jk} as her highest-priority task, incompatible with each other. To resolve it, ii) we sort all chosen tasks in i) by the priority values, and execute them in order of priority, while each node can execute at most one task each step. That is, if the priority of I_{ij} is higher than that of I_{jk}, than the node j executes the task I_{ji} upon request from i before I_{jk}, which subsequently cannot be executed at this step. iii) After completing ii), all the executed tasks assigned new random priorities. We apply the modified model to fully-connected networks and star networks, finding yet distinct behaviors for both cases. In comparison with the "sequential" task execution cases in the previous section, these indicate that the synchronicity effect, by way of accompanying priority conflicts, is a relevant factor for the waiting time dynamics of priority-queue networks.

3 Discussions

In this Work-in-Progress, we have presented that the priority-queue models exhibit rich waiting time dynamics and the waiting time dynamics is strongly affected by the topology of the network as well as the queue disciplines. It remains to be seen how our results, together with that of OV model, can improve our understanding of human dynamics from the perspective of priority-queue networks.

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