Rank-Size Distribution of Notes in Harmonic Music: Hierarchic Shuffling of Distributions

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Abstract. We trace the rank size distribution of notes in harmonic music, which on previous works we suggested was much better represented by the Two-parameter, first class Beta distribution than the customary power law, to the ranked mixing of distributions dictated by the harmonic and instrumental nature of the piece. The same representation is shown to arise in other fields by the same type of ranked shuffling of distributions. We include the codon content of intergenic DNA sequences and the ranked distribution of sizes of trees in a determined area as examples. We show that the fittings proposed increase their accuracy with the number of distributions that are mixed and ranked.

Keywords: Ranking distributions, Power law distribution, Zipf law in Music.

1 Introduction

Since the publication of Zipf's law, Power Law and Rank-size distributions have been often tried in fields other than languages, Music among others. There have been several recent papers proposing different explanations and/or modifications to the pure Power Law [1,2,3,4]. In a previous work [5], we have shown that pure Power Laws are not a good representation of the rank size distribution of notes of a musical piece throughout its domain, and that the two-parameter, first class Beta Distribution (TPBF) [6]:

$$S(r) = \frac{N(R+1-r)^{\beta}}{r^{\alpha}}, \tag{1}$$

where $S(r)$ is the size against which the distribution is ranked, N a normalisation factor, R the number of total ranks and r the rank variable, can account for a better fit on both tails of the distribution. Refer to Figure 1 for some sample fittings, and to Figure 2 for a comparison of several types of representations.

When analysing the results of over 1500 fitted musical pieces in parameter space, a clear trend over major and minor modes was noted. To get a clearer picture of what made the parameters different for major and minor modes, the rank-size distribution of notes was broken into a distribution of octaves ranked

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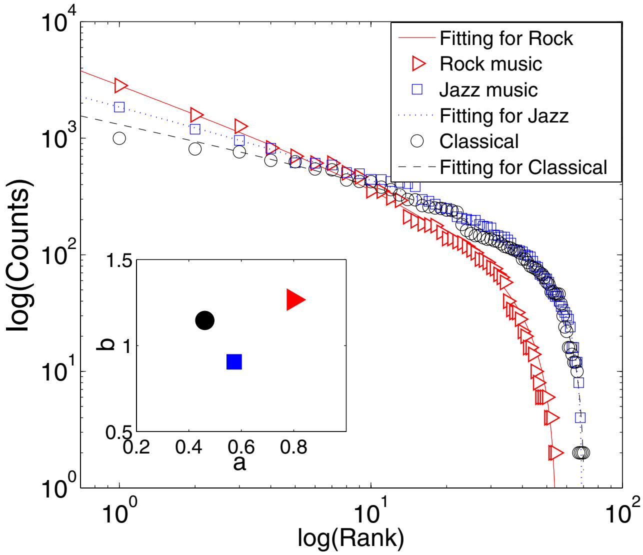

Fig. 1. A log-log plot of the frequency of notes against the rank. The classical piece is Beethoven´s Quartet No.14 Op. 131 ($\alpha = 0.461$, $\beta = 1.147$, $r^2 = 0.9968$), the jazz piece is "A good one" by Benny Goodman ($\alpha = 0.573$, $\beta = 0.905$, $r^2 = 0.9993$) and for rock music, the song "Sweet child of mine" by Guns&Roses ($\alpha = 0.798$, $\beta = 1.267$, $r^2 = 0.9992$).

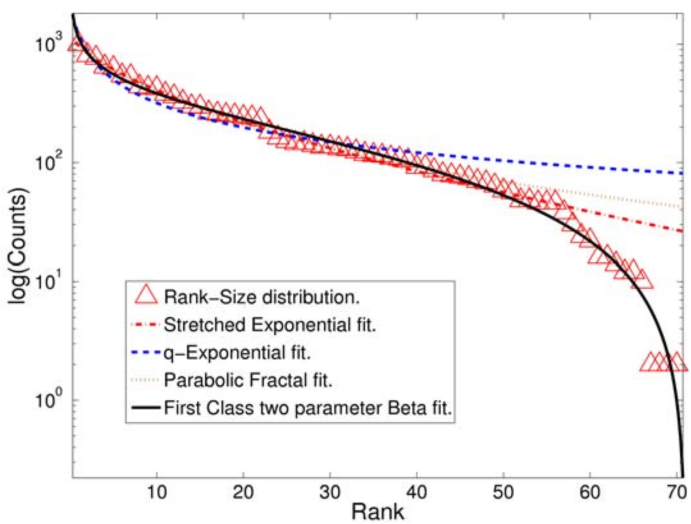

Fig. 2. Semilog plot of the rank-size distribution of Brahms' Symphony No.4, 1st movement with superimposed fittings. the fittings to be compared are a stretched exponential, a q-exponential, a parabolic fractal distribution, and the first class two parameter Beta distribution. Note the correct qualitative behaviour of the TPBF in the rightmost region of the distribution.

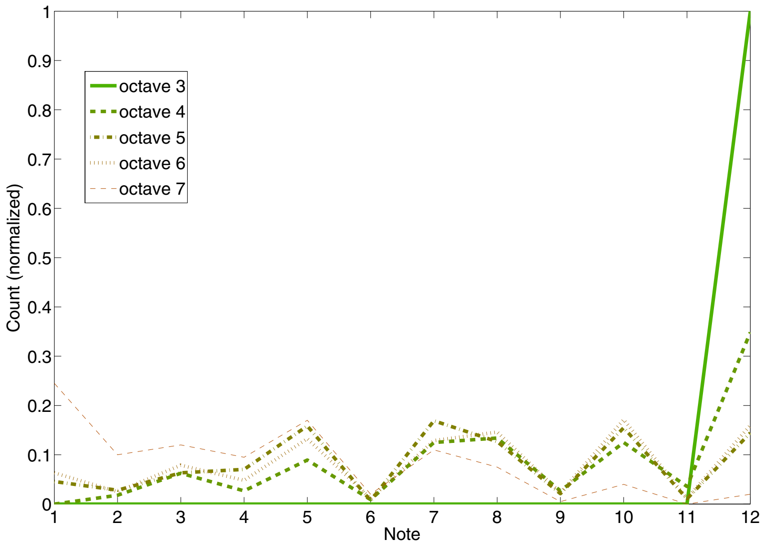

Fig. 3. Histogram per octave of the number of times that a certain note is used. The histogram is normalized to the frequency of each note in a certain octave. The number 1 note is C# and number 12 is C. Notice that F#, A and B are rarely used, as can be expected from harmonic arguments. The piece is J.S. Bach's prelude BWV 881 in F minor.

by the number of notes used within, and a distribution of the twelve notes of the scale, per octave, ranked by frequency of appearance. The second group of the above mentioned revealed that the use of a particular note was qualitatively the same in each octave, thus suggesting that the use of a particular note can be simulated stochastically according to its importance within the particular scale used, i.e. favouring, for example, the tonic, the dominant, etc... over notes that are not in the scale of the main key or those of the most common modulations. See Fig. 3. This subject is treated in [7] from the perspective of perception psychology.

2 TPBF and Ranked Shuffling of Distributions

Until now all work has been descriptive, showing the phenomenology and making emphasis on the ubiquity of the TPBF [8,9,10], which suggests an approach of great generality. In the following we show that it is suspiciously often the case that, under certain restrictions yet to be fully characterised, the mixing of distributions and their posterior ranking produces functional forms well represented by the TPBF, much in the same spirit as convolution leads to a Normal distribution, given certain restrictions and limits. The ubiquity of the findings is the prime motivation to believe that the mentioned restrictions shouldn't be too strong as to make the systems of interest abound as they seem to.

The separation of the music rank-size histogram into distributions of the individual octaves, made it clear that the place a note adopted after being ranked was dependent on its function within the tonal scale on which the piece was set, and on the octave on which it was played. The final form of the distribution was thus shown to be a collection of regions that could be characterised by the different combinations of those two parameters that determine the "importance" of a note within the piece. For example, the leftmost region of the rank-frequency distribution, that which contains the notes with higher frequency, is characterised by containing notes placed typically in the middle octaves (the histogram of the octaves is usually unimodal and non monotonous) and belonging to the tonal scale, the intermediate region consisted predominantly on notes either played on border octaves and belonging to the scale or on middle octaves and not on the scale, etc... That music can be decomposed in that way also explains the major-minor modes trend in parameter space, since the 12 note histogram of each is different, given the inclusion of two extra notes in the minor mode scale. See Figs. 4. and 5.

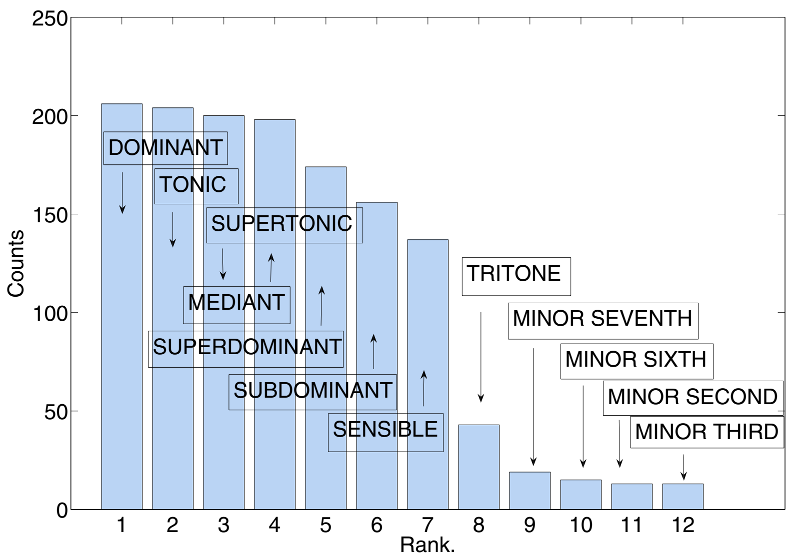

Fig. 4. Typical twelve-note rank-size distribution of a composition written in major mode. The sharp cut after the seventh rank marks the line between the notes on the scale and those used only as passing or grace notes.

It is this play between hierarchies what is consistently present in systems that show ranked distributions well fitted by the TPBF. Evidently, any distribution can be separated arbitrarily into smaller distributions in such way that the ranked shuffling of the constituents is the original distribution, so it is possible

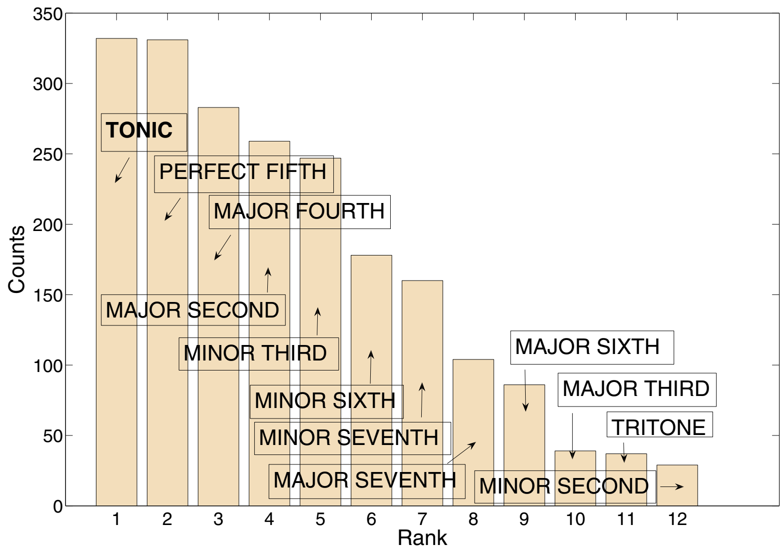

Fig. 5. Typical twelve-note rank-size distribution of a composition written in minor mode. This distribution has a less dramatic fall than that for major keys because minor tones have two more notes in the scale, depending on whether it is rising or falling.

that the restrictions needed on the ranked shuffling of distributions as to produce TPBF's act on the way the individual constituents of the distribution overlap.

2.1 Examples

To illustrate the ranked shuffling of distributions we present data taken from [11]. We obtained the distribution of trees, herbs and shrubs in a particular abandoned field in Illinois, USA, ranked according to the area of coverage. The fittings for each of these are not quite as satisfactory as that for the distribution of all areas together, that is, that for the ranked shuffle of the three distributions. Refer to Fig 6.

This progressive approach to a better TPBF fit with further shuffling also happens in the previous examples on Music. See Fig 7.

To end this section we mention, without much detail, that another important example we have worked with is the ranked distribution of triplets of nucleic bases in DNA. This last distributions reveals a structure similar to that of music, in that it can be separated in regions characterised by the degeneration of the parameters that determine the likeliness of appearance of a triplet. Whereas in Music this likeliness was determined by the octave and whether the note is in the tonal scale or not, in the distribution of triplets it is characterised by whether the first, second and third base are a "strong" or "weak" base, which in a loose sense equals to say that they belong or not to a privileged set, like, in music, the tonal scale.

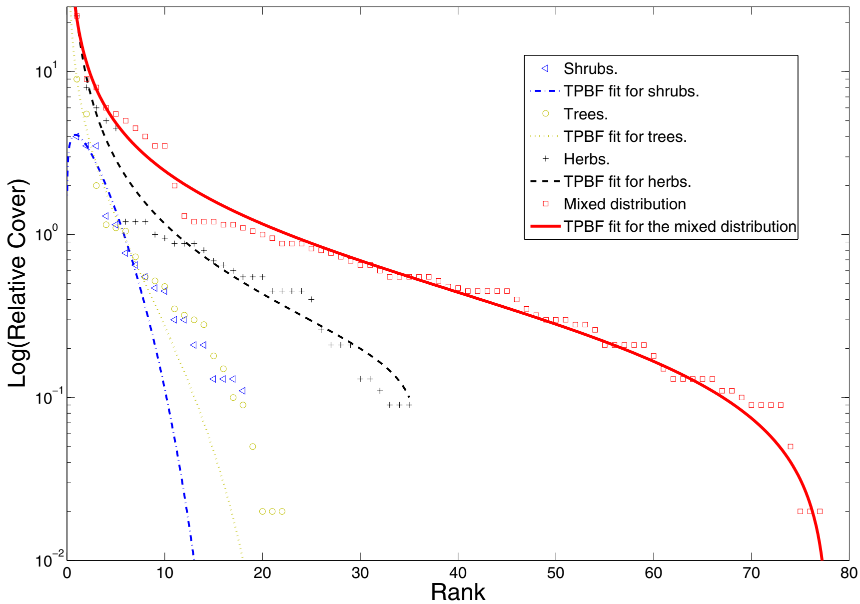

Fig. 6. Log Plot of the distribution of trees, herbs and shrubs, ranked according to relative cover (percentage of total cover) in an abandoned lot in Illinois, USA

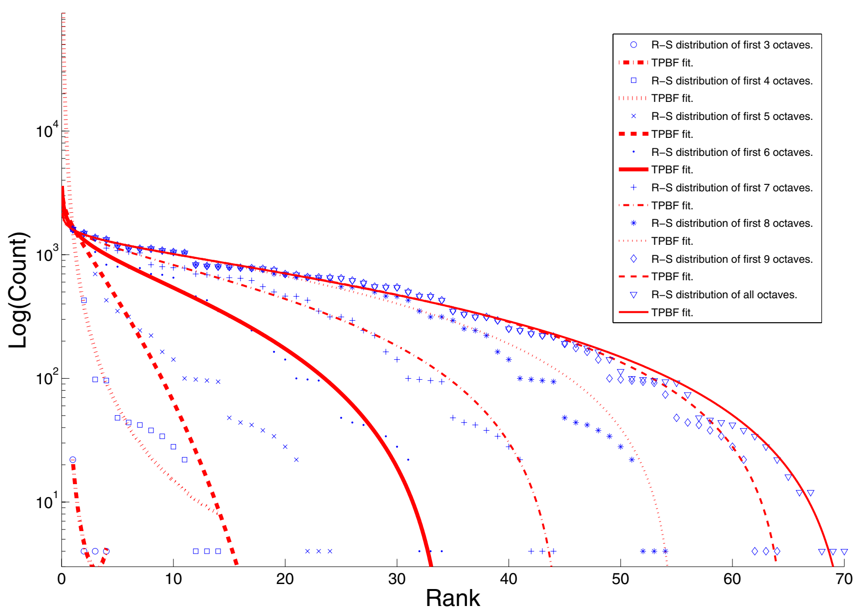

Fig. 7. Log Plot of the rank-frequency distributions and their TPBF fittings of an increasing number of octaves in Isaac Albeniz's *Sevilla* from *Suite Iberia*

3 Conclusions

The ranked distributions of notes in harmonic musical compositions are consistently well represented by the TPBF, and the parameters arising from the fittings have tendencies that become clear once one separates the whole into distributions of individual octaves, where the importance of the main key of the composition is revealed. Such separation leads to believe that the presence of the TPBF may arise from the ranked shuffling of distributions. Evidence from other fields, and from the general observation that goodness-of-fit parameters increase progressively on the number of distributions that are shuffled and ranked, strengthen this last idea. The necessary restrictions on the ranked shuffling to produce the TPBF can't be too strong since they need to account for the ubiquity of the findings.

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