Music, New Aesthetic and Complexity

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Abstract. This paper illustrates an algorithm to generate a complex acoustic stimulus whose statistical properties are as close as possible to the non-stationary dynamics revealed by the current analysis of the electro-encephalogram activity of the human brain. Thus, the composition is driven by crucial events, namely renewal non-Poisson events with an inter-time distribution density $\psi(\tau)$, which is an inverse power law with index μ, fitting the condition $1 \leq \mu \leq 2$. We find that the music composition is more attractive when we fill the time region between two consecutive crucial events so as to enhance the leading role of μ. In all cases the spectra markedly depart from the ideal $1/f$ condition, thereby suggesting a shift from the $1/f$ noise perspective of the pioneer work of Voss and Clark to the Zipf's law perspective advocated by more recent work on music composition.

Keywords: music composition, brain, complexity matching, $1/f$ noise, Zipf's law.

1 Introduction

Since the pioneer work of Voss and Clarke, [1,2,3] there has been a widely accepted conviction that music generates $1/f$ noise and that the fractal structure of music and painting corresponds to the aesthetic needs of human psychology [4]; human beings live in an out of equilibrium condition that makes them sensitive through a still unknown form of resonance to the beauty of the arts, and to the environment fractal structure as well. This is so because both arts and environment live in the same out of equilibrium condition as the human brain. However, the current understanding of the real nature of $1/f$ noise is incomplete. The theory of Voss and Clarke [3] shows that $1/f$ noise emerges from a stationary correlation function, and this conflicts with the attractive conjecture that music pleases us as a consequence of its correspondence with the non-stationary nature of the brain processes [4,5,6]. This conjecture is supported, for instance, by Anderson [7] who advocates the perspective of the brain self-organizing itself via a vertical collation of the $1/f$ fluctuations in order to perceive the world and generate adaptive behavior.

The work of Gong *et al.*[8], based on the analysis of the electro encephalographic activity of the human brain in the alpha range, affords compelling evidence of collective intermittent dynamics as an emergent property of globally

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coupled phase oscillators near the critical point of a phase transition. The conclusion of these authors is that the brain is characterized by type-I intermittent behavior [9], namely, a dynamic process with regular, quiet behavior interrupted by abrupt bursts of chaotic and irregular activity. The time spent by the system in the quiet (laminar) state is very extended and the time distance τ between two consecutive bursts of activity is given by an inverse power law distribution density $\psi(\tau)$

$$\psi(\tau) = \frac{A}{\tau^{\mu}}, \quad (1)$$

where A is a non vanishing positive number. Note that, as shown in Refs.[10], the non-stationary condition is generated by

$$\mu < 2. \quad (2)$$

In fact, the mean time length of the laminar region is infinite and upon time increase, quiet regions of larger and larger size are expected to appear. This generates ergodicity breakdown [10] and, consequently, an out of equilibrium condition which, according to Gong *et al.* [8] is shared by the brain. The conclusions of Gong *et al* [8] agree with the intermittence perspective advocated by other authors [11,12] and with the brain non-stationarity of [13] as well. It is plausible that the brain non-stationarity, emphasized by the recent psychological studies [4,7], corresponds to the occurrence of unpredictable quakes and that the time distance between two consecutive quakes has the waiting time distribution of Eq. (1) supplemented by (2).

The main aim of this paper is to design an complex acoustic stimulus matching brain intermittence. According to the Complexity Matching Principle (CMP)[14], the transport of information from one complex system to another is facilitated if the two systems share the same μ. An acoustic stimulus matching the brain may have therapeutic effects. Having in mind the biblical harp that David used to sooth King Saul, we call this composition David's Harp, or, in short, Harp.

2 Double Truncation Algorithm

The authors of Ref.[15] determine τ through

$$\tau = T\left[\frac{1}{y^{\frac{1}{\mu-1}}} - 1\right], \quad (3)$$

where y is a number of the interval $[0, 1]$ generated by a random number generator. The non-linear transformation of Eq. (3) converts the number y into a number τ, whose distribution density is given by

$$\psi(\tau) = \frac{A}{(\tau + T)^{\mu}}, \quad (4)$$

with the normalization factor $A = (\mu - 1)T^{\mu-1}$. The lack of correlation between two different values of y makes the corresponding τ values uncorrelated as well, thereby generating the rejuvenation condition associated with type-I intermittence. The finite value of μ makes the resulting process depart from the ordinary Poisson condition, which corresponds with $\mu = \infty$. It is important to point out that this transformation, with no apparent psychological significance at a merely mathematical level, becomes very attractive when we assign a temporal meaning to τ. The nth event occurs at time $t_n = t_n = \tau_1 + ... + \tau_n$ for $n > 1$. If the number of events per unit of time is constant, they generate a boring random process. This is the property of the regime $\mu > 3$. The region $1 \leq \mu \leq 3$ generates events strongly departing from the Poisson statistics [14]. The condition $\mu < 2$ corresponds to a vanishing Kolmogorov complexity [16], with a number of crucial events per unit of time decreasing with time as $1/t^{2-\mu}$. This is the source of a non-stationary and rejuvenation process fitting the psychological needs of the human brain as we shall see in Section 3. Properly filling the laminar regions generates $1/f$ noise.

The algorithm of Eq. (3) yields an excessive number of short times $\tau < T$. These times have a probability larger than the long times and do not carry information on the parameter μ, which is the essential property for matching the brain [17]. The excessive number of these times makes the music composition unattractive and computationally heavy [18]. Furthermore, we note that the EEG analysis [17] reveals that the region of very long waiting times is truncated. Thus, to match the human brain, we use the following algorithm

$$\tau = \left[T_{max}^{\beta} - y\left(T_{max}^{\beta} - T_{min}^{\beta}\right)\right]^{\frac{1}{\beta}}, \quad (5)$$

with

$$\beta \equiv 1 - \mu, \quad (6)$$

for $\mu > 1$ and

$$\tau = T_{min}\left(\frac{T_{max}}{T_{min}}\right)^{y}, \quad (7)$$

for $\mu = 1$. As in the case of the algorithm of Eq.(3), y is a number selected randomly and with equal probability from the interval $[0, 1]$, inclusive. This algorithm, in line with the statistical information emerging from the EEG analysis, confines the power law behavior to a limited region ranging from T_{min} to T_{max} while setting $\psi(\tau) = 0$ for both $\tau < T_{min}$ and $\tau > T_{max}$.

Note that in the time scale $\tau > T_{max}$ this music composition is ergodic, just as the brain in the same time region. However, in the time scale $\tau < T_{max}$, with the inverse power law nature of $\psi(\tau)$ and $\mu \leq 2$, this music composition turns out to be non-ergodic, thereby matching the brain's non-stationarity.

3 1/f Noise

When $T_{max} = \infty$, the region $\mu < 2$ is characterized by a perennial non-stationary condition. In the case $\mu > 3$, the stationary condition exists but the regression to equilibrium is infinitely slow [14]. Consequently, the whole region $1 \leq \mu \leq 3$, must be considered anomalous. When $\mu < 2$, the Kolmogorov entropy vanishes. When $2 < \mu \leq 3$, the sequence of crucial events generate Lévy rather than Gauss statistics [16]. We define this as a *weak chaos condition*. The pioneer work of Voss and Clarke [1,2,3] rests on the assumption of stationarity. Their theoretical arguments for the evaluation of spectrum of the squared composition, S_{V^2}, do not apply to the Harp. However, if we interpret V^2 as the signal to study, we can in principle benefit from the work of Ref. [19]. Lowen and Teich, in fact, apply their theory to the case of renewal non-Poisson time series. They fill the laminar regions in two different ways. The former corresponds to assigning to each laminar region the value of 1 or -1, according to the fair coin tossing prescription. The latter is based on leaving the laminar regions empty by assigning to them values of vanishing duration, while giving the constant value of 1 to the times where the quakes occur. The latter prescription holds true also in the case where the signal gets the maximum value when an event occurs, and drops quickly to zero. They find

$$S(f) \propto \frac{1}{f^{\eta}}, \tag{8}$$

with

$$\eta = 3 - \mu, \tag{9}$$

in the former case, and

$$\eta = \mu - 1, \tag{10}$$

in the latter case. It is interesting to notice that $\mu = 2$, in both cases, generates the ideal $1/f$ noise condition. In the former case, the perennial out of equilibrium condition is signaled by $\eta > 1$ and in the latter by $\eta < 1$. We shall see that the weak-chaos music composition, filling the laminar region with a different prescription, departs from this theoretical prediction.

4 Weak-Chaos Music Composition

In this section we illustrate the Harp design. Actually, we study two Harps, called Harp #1 and Harp #2. Harp #1 selects the envelope decay time from the prescription of Eqs.(5) and (7), with the same μ as that determining the laminar region length. Harp #2, however, is based on a perfect correlation between the envelope decay and the time length of the laminar region, insofar as the envelope decay time is selected to be proportional to the laminar region length through a proportionality factor r. This makes Harp #1 much richer in events than Harp #2, especially in the non-ergodic case $\mu \leq 2$. It is remarkable that Harp #1 sounds much more attractive musically than Harp #2.

4.1 How to Fill the Laminar Region

The music was created in Csound with an algorithmically generated score. It consists of note events whose onsets are determined from a series of waiting times generated according to Eqs. (5) and (7). Sine waveforms, which were chosen to minimize dissonance, were multiplied by percussive exponential envelopes with short attacks and long decays. The decay times and peak amplitudes, like the waiting times, were calculated according to equation Eqs. (5) and (7). Pan positions were chosen randomly from an ordinary white probability distribution.

Pitches were chosen randomly from a chord set which progressively changed throughout the duration of the piece. A just major Ptolemaic diatonic tuning ($\frac{1}{1}, \frac{9}{8}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{15}{8}, \frac{2}{1}$) was used for the purity of its consonances. Run through a simple medium room reverberator, the end result sounds something like a large music box with an extended frequency range and more expressive dynamic range. The inverse power law distribution of the waiting times, decay times, and amplitudes results in numerous, mostly quiet notes with close to the minimum decay time, with occasional longer gaps, sustains, and volume accents occurring sporadically throughout. This produces an attractive, varied texture occupying an intermediate position between order and randomness.

4.2 Exponential Envelope

Here we describe the envelope function of the oscillations filling the laminar regions with a generic example. The envelope we used is comprised of two parts: an attack (rise) and a decay (fall). Times are expressed in milliseconds. We use the general form

$$f(x) = ka^x \tag{11}$$

for the attack, and

$$f(x) = b^{x-\Delta} \tag{12}$$

for the decay. We set $a = (1/\epsilon)^{(1/\Delta)}$. We shall use throughout $\epsilon = k = 0.001$ and $\Delta = 40$. At time $x = 0$, $f(x) = k$ and at $x = \Delta$, $f(x) = k(1/\epsilon) = 1$. For the envelope decay we set $b = (\epsilon)^{1/T_D}$, with $\epsilon = k$. Thus, at $x = \Delta$ we have $f(x) = 1$. With time increase $f(x)$ decays from the value of 1 at $x = \Delta$ to the value of ϵ at $t = T_D + \Delta$. The parameter T_D defines the decay time. The parameter T_D for Harp #1 is selected from the distribution density of Eqs. (5) and (7), with the same μ as that assigned to distribution density of the laminar region lengths, and for Harp #2 is assumed to be proportional to the waiting time τ. In the case of Harp#1, the attack or rise time is small compared to the decay or fall time which is 7000ms to 12000ms.

4.3 Spectral Analysis

Following Voss and Clarke [1,2], here we evaluate S_{V^2}. Fig. 1 refers to the case that we judge to be aesthetically attractive, with the waveform between two consecutive crucial events established independently of the laminar region length. We se that $\eta \approx 1.8$, with no dependence on the complexity parameter ranging from $\mu = 1.2$ to $\mu = 2$. In Fig. 2, we illustrate the S_{V^2} spectrum of Harp #2.

In this case, when r is small, we can compare our result to Eq. (10). We find that for $r = 1$, the slope of the spectrum, in the low-frequency regime, is indeed close to -1 and that η depends on μ. However, making the spectrum closer to the ideal $1/f$-noise condition does not make it more attractive. It is rather much less attractive than Harp #1. Interested readers can listen to excerpts of both Harp #1 and Harp #2 [18].

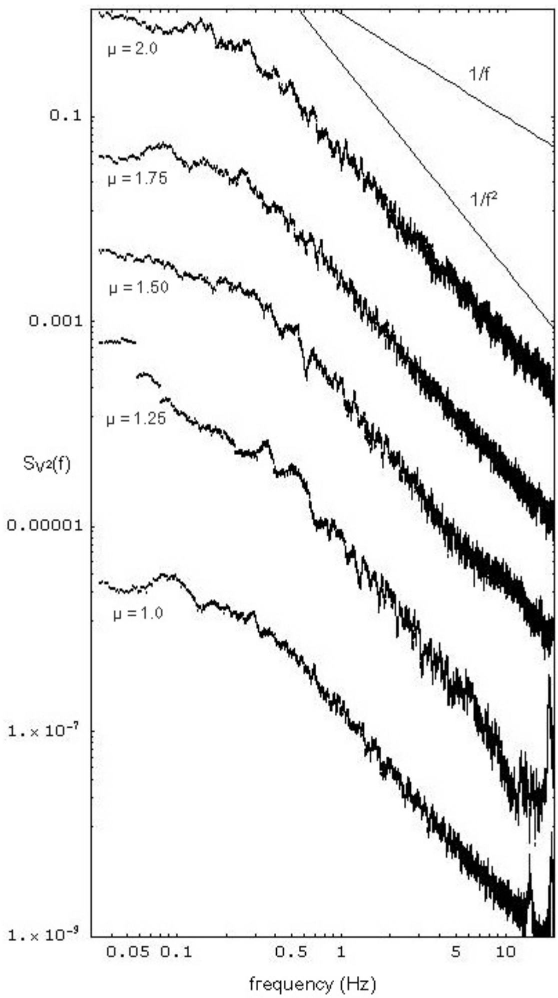

Fig. 1. $T_{max} = 2200ms$, attack $= 40ms$, decay range: $7000 - 12000ms$, amplitude range: $1200 - 30000$, frequency range: $220 - 1650Hz$. T_{min} was set for each data set such that all sets would have the same predicted number of events (approx. 3,500). In this version of the Harp, decay is not proportional to waiting time. The μ for the amplitudes equals 1.5. The μ for the decay times equals that of the waiting times.

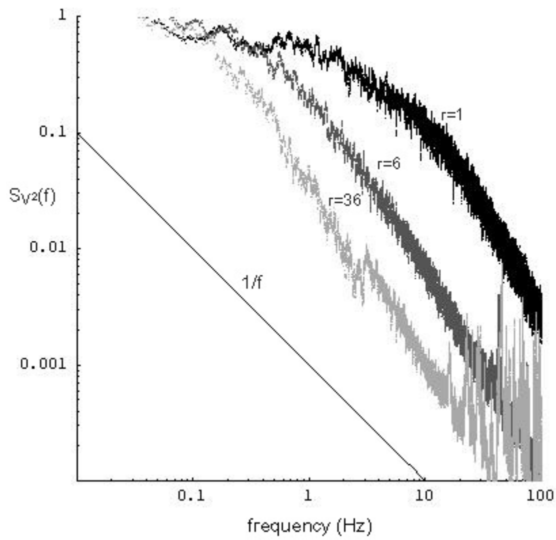

Fig. 2. The μ of the amplitudes equals 1.5. Decay is proportional to waiting time (tau). The authors do not consider the results to be aesthetically pleasing [18]. The μ of the waiting times equals 2.0. $T_{min} = 100ms$, $T_{max} = 2200ms$, attack $= 40ms$, amplitude range: 1200 to 30000, frequency range: 220-1650Hz.

5 Zipf's Law

Manaris *et al.* [20] have recently pointed out the relevance of Zipf's law for music. According to Zipf's law, after ranking the words of a text, their frequency W, expressed as a function of the rank R, turns out to obey the prescription:

$$W(R) = \frac{K}{R^{\eta}}, \tag{13}$$

with $\eta \approx 1$. We adopt the symbol W because, in accordance with the authors of Ref. [21,22], we wish to emphasize the connection between Zipf's law and Pareto's law, which refers to the wealth of individuals of a human society. However, according to Manaris *et al.* [20], W can be identified with the spectrum S and the rank R with the frequency f, thereby making Zipf's law generate the $1/f$ spectrum of Eq. (8). According to Manaris *et al.* [20], it is possible to rank many other musical properties, for instance *chromatic-tone distance*, *melodic intervals* and *note duration.* Their frequencies always obey Eq. (13).

What about the distribution density of W? We note that, according to Pareto [21,22], the probability of finding incomes larger than a given value W is proportional to $1/W^k$, thereby yielding

$$\Psi(W) = \frac{A}{W^k}, \tag{14}$$

where $\Psi(W)$ denotes the cumulative probability, which is related to distribution density $\psi(W)$ by the relation

$$\psi(W) = -\frac{d\Psi(W)}{dW}. \tag{15}$$

Thus, we get

$$\psi(W) = \frac{B}{W^{\mu}} \tag{16}$$

with $\mu = k+1$ and $B = kA$. Following the authors of Refs. [21,22], it is straightforward to show that

$$\mu = 1 + \frac{1}{\eta}. \tag{17}$$

We therefore conclude that the ideal Zipf's law condition $\eta = 1$ yields the value $\mu = 2$, which corresponds to the condition were our music composition becomes most attractive [18].

The music composition proposed in this article corresponds to identifying τ with W. Consequently, the most frequent waiting times are the short times, so that

$$W_{max} = W(1) = T_{min}. \tag{18}$$

The upper limit to the waiting times τ establishes a limit to the maximum rank of this (musical) society through

$$W(R_{max}) = T_{max}. \tag{19}$$

6 Concluding Remarks

We have designed a music composition driven by crucial events, namely renewal non-Poisson events, with the power index μ ranging from 1 to 2. According to the CMP discussed in Ref. [14], this music composition should exert a significant influence on the brain, especially when its complexity parameter μ is tuned to the EEG's μ [8]. The authors of Ref. [14] argued that there should be a correspondence between the CMP and the widely accepted fact that the brain is sensitive to $1/f$ stimuli. The main result of this paper is that the CMP is more general than the $1/f$ effect and that this influence may be good if the music is attractive. Thus, Harp #1 may have therapeutic effects via the CMP, without involving the $1/f$ channel of communication. In fact, when the lifetime of the oscillations envelope is short compared to the laminar region size and the $1/f$ prediction of Eq. (10) applies, the composition (Harp #2)is disagreeable. The S_{V^2} spectrum of the agreeable music composition is essentially independent of μ, with $\eta \approx 1.8$. The music composition designed in this paper fits Zipf's law, which according to Ref. [20] is a key aspect of music pleasantness, and the music variable fitting Zipf's law is the time duration τ of the laminar region. Of course, this is a truncated Zipf's law, namely T_{max} is finite in accordance with the recent analysis done by the authors of Ref. [23].

In conclusion, although we cannot prove that the real music composition hosts crucial events, this possibility cannot be ruled out either: the crucial event hypothesis may be compatible with the analysis made by the authors of Refs. [24] and [25]. On the other hand, we invite the readers of this paper to express their own judgment on the aesthetics of our music composition, which does host crucial events, by listening to it [18]. Within the context of stochastic music [26,27], our music composition is based on weak rather than strong chaos [28]. Thus, there might be a connection with DNA music [29], especially in the case where the DNA spectrum yields $\eta > 1$ [30], which, according to Eq. (9), should correspond to $\mu < 2$. We think that this music composition may match the brain nature and consequently realize beneficial therapeutic effects based on the Complexity Matching Phenomenon.

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