Hopfield's Model of Patterns Recognition and Laws of Artistic Perception

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Abstract. The model of patterns recognition or attractor network model of associative memory, offered by J.Hopfield 1982, is the most known model in theoretical neuroscience. This paper aims to show, that such well-known laws of art perception as the Wundt curve, perception of visual ambiguity in art, and also the model perception of musical tonalities are nothing else than special cases of the Hopfield's model of patterns recognition.

Keywords: Wundt curve, ambiguity, musical tonality, catastrophe theory.

1 Introduction

As is known, any science develops from empirical quantitative laws to the fundamental theory in which these laws are the special cases of the general principles. For example, development of mechanics occurred from empirical laws formulated by Galileo and Kepler to the Newton's fundamental laws of dynamics and the law of gravitation. In psychology of artistic perception there are some empirical laws for which the conventional theoretical explanation have not proposed and its relation with fundamental principles of the brain functioning is not revealed. For instance, the Wundt curve and structure and traits of musical tonalities are not explained from principles of the brain working. Our objective is to show that these empirical laws are the simple property of the Hopfield's neural network model of patterns recognition [1],[2].

2 The Wundt Curve

Wilhelm Wundt in 1874 qualitatively described hedonic value received by the person from perception of a artwork, depending on arousal potential of this work in the form of the inverted U-shaped curve. Strangely enough, but any theoretical explanation for this empirical law have not been proposed up to date It is important to note, that the Wundt curve is not described by any concrete mathematical formula with concrete numerical parameters. Numerous experimental

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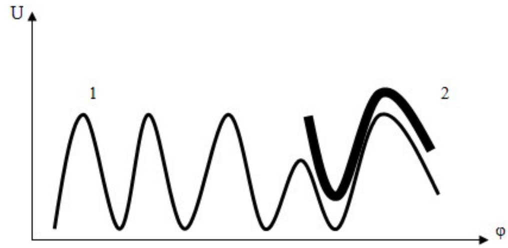

Fig. 1. Replacement of area of an attraction of one of stereotypes pattern of potential function in Hopfield's model of patterns recognition on potential function of elementary catastrophe "fold". 1 - potential function in the Hopfield's model , 2 - potential function of elementary catastrophe "fold", φ - coordinate in configuration space.

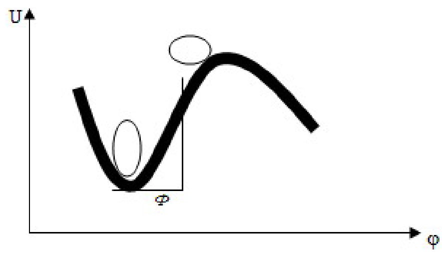

Fig. 2. The Wundt curve can be obtained as universal property of elementary catastrophe "fold". Distance characterizes a deviation of an inputted pattern from a stereotype pattern can be taken as a measure of arousal potential or novelty. 0 - a stereotype pattern, o - an inputted pattern.

date, first of all in empirical aesthetics [3],[4], confirm, that the dependence of aesthetic value of artworks from its novelty is described by inverted U-shaped curve only in its top part. The bottom branches of this curve may view asymmetrically, might be bent, etc. In order to obtain an inverted U-shaped curve let us replace the fragment of a potential function related to the area of an attraction of one stereotypes pattern in the Hopfield's model by the potential function of elementary catastrophe "fold" [5] (see fig. 1)

A distance between position of a minimum corresponding to an imprinted stereotype pattern and position of an inputted pattern we use as a measure of arousal potential or novelty (see fig. 2).

Potential function of elementary catastrophe "fold" has the following view:

$$U = \Phi^3 - (K_C - K)\Phi - C \tag{1}$$

where K - hedonic value, K_C - maximum of hedonic value, Φ_C - maximum of arousal potential, C - constant. Dependence for variables $\frac{\Phi}{\Phi_C}$ and $\frac{K}{K_C}$ looks like the following (see fig. 3).

The arousal potential is the extensive variable similar to density of substance, magnetization, etc. in phase transitions. As is known, in Hopfield's neural networks

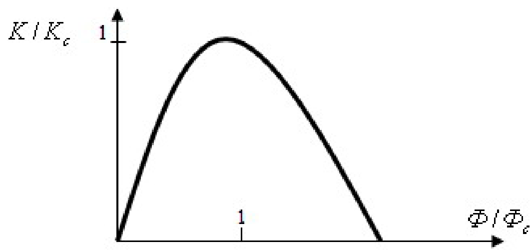

Fig. 3. Modeling of the Wundt curve by means of elementary catastrophe “fold”

the order parameter is correlation between patterns. Clearly, that the less correlation between the pattern offered by a work of art and the corresponding stereotype, the more is arousal potential. On the contrary, the hedonistic value is the intensive variable similar to temperature at phase transitions. In neural networks as a measure of temperature it is expedient to take intensity of noise.

3 Visual Ambiguity in Art

There is one more basic model of artistic perception - perception of visual ambiguity. The best known examples of ambiguous figures are specially designed patterns such Necker’s cube or “young girl-old lady”.

One of the first who specially created ambiguity in painting was Italian artist Giuseppe Arcimboldo [6]. The painting Disappearing Bust of Voltaire by Salvador Dali is an another example of this kind of visual ambiguity, when a human face is created with different human figures. The most famous example of ambiguity in painting is, of course, ambiguity “mocking-sadness” of Mona Lisa’s smile in Gioconda by Leonardo da Vinci [5],[6].

The perception of ambiguous figures has non-linear properties, and that multistable perception could be modelled by catastrophe theory methods [7],[8]. Ambiguous patterns are examples of two-state, bimodal systems in psychology. When we perceive an ambiguous figure, like the fourth picture in the row in fig. 4 , the perception switches between two interpretations, namely ‘man’s face’

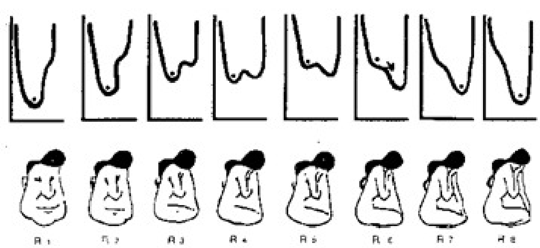

Fig. 4. Ambiguous patterns are two-state systems. Their perception one can model by using elementary catastrophe “cusp”.

or 'kneeling girl' because it is impossible for the brain to recognise both interpretations simultaneously. Just like for any bifurcative state, it is impossible to predict in the case of an ambiguous figure which interpretation will appear first. G. Caglioti from the Milan Polytechnique Institute first paid attention to the fact that ambiguous figures are a cognitive analogue of critical states in physics. Various authors pointed out that the perception of ambiguous figures has non-linear properties, and that multistable perception could be modelled by catastrophe theory methods [7],[8].

The switch between two interpretations could be described by elementary catastrophe "cusp".

$$x^3 - bx - a = 0 \tag{2}$$

where a and b are control parameters and x is the state variable. The first parameter a is called the *normal factor* and quantitatively describes the change in bias in the drawing in a "shape space" from a man's face to a woman's figure.

The second parameter b is called the *splitting factor* or *bifurcation factor* and describes how much the amount of details is presented in the ambiguous figure. The state variable x is presented as a scale from +10 ("looks a lot like a man's face") to −10 ("looks a lot like kneeling girl"). For this model we could formally represent potential function

$$V = \frac{1}{4}x^4 + b\frac{1}{2}x^2 + ax \tag{3}$$

It is worth noting that unlike in physical sciences, where the potential function is usually deduced from fundamental laws or standard theories, in mathematical models in psychology and other "soft sciences" the potential function is hypothesized and is really considered as the potential energetic function, which should be minimized. In this case it might be also considered as the Lyapunov function in Hopfield's model of patterns recognition.

4 Structure of Musical Tonality

Using Hopfield's model, we can consider pitch perception as a pattern recognition process and it gives us an ability to explain why notes with octave interval we hear as very similar. A musical note is defined by its lowest pitch, known as its 'fundamental frequency', but a note also contains higher-pitch overtones with frequencies that are some multiple of the fundamental. When we hear, for instance, notes "C" in different octaves, we recognize very similar sound patterns, keeping in mind complex overtone structure of every note. Other words, sound patterns of notes divisible by octave are the most similar among all others notes and therefore belong to the same basin of attraction and precisely by this reason we hear notes divisible by octave as very similar.

It is reasonable to suggest that three stable steps of tonality: tonic, mediant, and dominant are keynotes or attractors of neural network. Others steps of tonality: subdominant, submediant, ascending parenthesis sound, descending parenthesis sound play the role recognizable patterns, gravitating to some

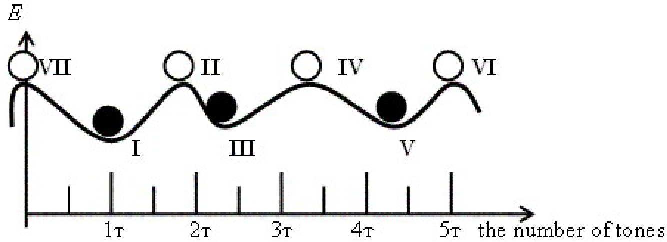

Fig. 5. Potential function E in Hopfield's model for minor tonality in Western tonal music. Black circle denotes stable step of tonality, white circle denotes unstable step.

or other keynote. As we already said, I, III, and V steps of tonalities (tonic, mediant, and dominant accordingly) are stable states. Steps II, IV, VI, and VII are unstable. The degree of their instability (the degree of their gravitation to appropriate stable state) depends on distances between unstable and stable sounds. The strongest gravitation of VII step to I step and of IV step to II step are observed.

For the lack of minor seconds intervals in a pentatonic scale there are not such strong gravitation as in a natural scale. We can depict exactly only distances between minima, but not depths of these minima. It is reasonable to suggest that the minima depth of tonal energetic function is extremely personal for human beings and reflects a musical abilities (giftedness) of a person. The more a person has a gift for music, the more depth of valleys has appropriate energetic function. A person who is devoid of music ability has energetic function with shallow valleys.

5 Concluding Remarks

Our senses and the brain itself must operate near the critical point because we originated and evolved in a critical world [9]. Only at critical point the brain can create new forms of behaviour. As we have shown above, perception of art and music takes place close to critical state. Therefore art may be considered as an essential tool for maintenance of the brain close to critical point that gives human beings possibilities for better adaptation.

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