Composing Music with Complex Networks

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Abstract. In this paper we study the network structure in music and attempt to compose music artificially. Networks are constructed with nodes and edges corresponding to musical notes and their co-occurrences. We analyze sample compositions from Bach, Mozart, Chopin, as well as other types of music including Chinese pop music. We observe remarkably similar properties in all networks constructed from the selected compositions. Power-law exponents of degree distributions, mean degrees, clustering coefficients, mean geodesic distances, etc. are reported. With the network constructed, music can be created by using a biased random walk algorithm, which begins with a randomly chosen note and selects the subsequent notes according to a simple set of rules that compares the weights of the edges, weights of the nodes, and/or the degrees of nodes. The newly created music from complex networks will be played in the presentation.

1 Introduction

The study of complex networks in physics has aroused a lot of interest across a multitude of application areas. A key finding is that most networks involving man-made couplings and connection of people are naturally connected in a scale-free manner, which means that the number of connections follows a power-law distribution [1]. Scalefree power-law distribution is a remarkable property that has been found across of a variety of connected communities [2]–[8] and is a key to optimal performance of networked systems [9].

Across cultures, and between individuals, certain musical pieces are consistently rated more favorably than others and the mathematical analysis of musical perception has a long history [10]. One fundamental question of interest is whether these different music share similar properties, and the implication of this question is whether a common process/rule exists in the human brain that is responsible for composing music. To answer this question, our approach is to employ a data-driven transformation to represent a musical score as a complex network. In particular we analyze a few distinct types of music, including classical music and Chinese pop. Specifically we treat a piece of music as a complex network and to evaluate the properties of the resulting network, such as degree distribution, mean degree, mean distance, clustering coefficient, etc. The purpose is to find out if different music would display uniformity or disparity

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in terms of network structure. Our results demonstrate, quite surprisingly, that different music types actually share remarkably similar properties. Our final task in this paper is to make an attempt to create "reasonably good" music from the network that has been formed from given compositions such as those of Bach and Mozart. We basically find that if the same network property is retained, it is possible to compose music artificially and the remaining open problem is the choice of a particular sample from a large number of possible compositions. In composing a music, from a system's viewpoint, our human brain would have automatically performed a processing step that allows only compositions that satisfy certain network properties to emerge and finally pick the best composition according to the composer's subjective choice. Of course, we do not know exactly how the brain does that. As an interim approach, some rudimentary rules may be incorporated when selecting compositions.

2 Review of Networks

A network is usually defined as a collection of "nodes" connected by "links" or "edges" [2]. If we consider a network of musical notes, then the nodes will be the individual musical notes and a link between two nodes denotes that the two musical notes are neighbors in the score. The number of links emerging from and converging at a node is called the "degree" of that node, usually denoted by k. So, we have an average degree for the whole network. The key concept here is the distribution of k. This concept can be mathematically presented in terms of probability density function. Basically, the probability of a node having a degree k is $p(k)$, and if we plot $p(k)$ against k, we get a distribution function. This distribution tells us about how this network of musical notes are connected. Recent research has provided concrete evidence that networks with man-made couplings and/or human connections follow power-law distributions, i.e., $\log(p(k))$ vs $\log(k)$ being a straight line whose gradient is the characteristic exponent [3]–[8]. Such networks are termed *scalefree networks.*

3 Network Construction Based on Co-occurrence

A musical *note* is defined by its pitch and time value. For example, a crotchet of the middle C is considered as a note, and a quaver of the same middle C is a different note. See Fig. 1. Consider an 88-key piano keyboard. If we limit each key

Fig. 1. A crochet of middle C is a note (left), and a quaver of middle C is a different note (right). Both are considered as different nodes in a musical network.

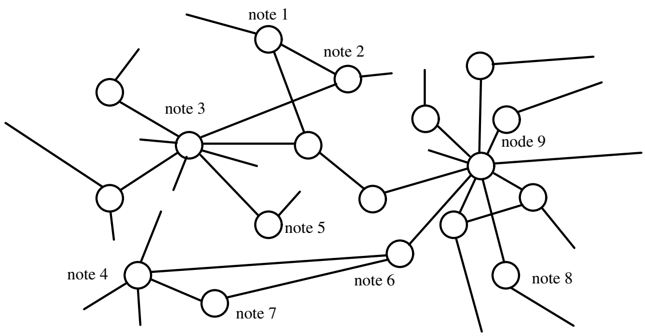

Fig. 2. A network for music, where nodes are notes and edges are connections of two consecutively played notes

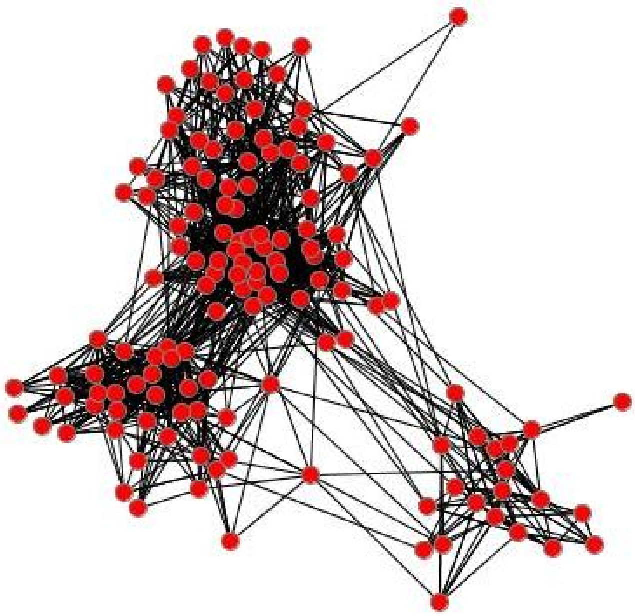

Fig. 3. Network from Bach's violin solos

to have 20 possible time values (e.g., brieve, semi-brieve, dotted minum, minum, dotted crochet, crochet, dotted quaver, quaver, dotted semi-quaver, semi-quaver, dotted demisemi-quaver, demisemi-quaver, etc. [11]), for instance, there are altogether 1760 possible notes.

For simplicity, we consider single-note scores where notes are to be played one after another, without simultaneous playing of two or more notes like a chord. Then, we may examine the way in which notes appear in the score for the purpose of constructing a complex network to represent the score.

To form a network, we need to define both *nodes* and *edges*. For the purpose of constructing a network from a musical score, we consider notes as *nodes* as explained earlier. A piece of music can be considered as a sequence of notes and hence edges can be defined by connections from one note to another chronologically. That is, if note i starts at time T and note j ends at the same time, then an edge is established from note i to note j.

Suppose there are N nodes. Then, node i is connected to node j when node i is played and followed by node j, and the connection is directed from node i to node j. Eventually, a network is formed with each node connected to a number of other nodes, as shown in Fig. 2. Of particular interest is the number of edges emerging from a node, which is defined as the *degree* of that node and is denoted by k. Also, the *distance* between two nodes, d, which reflects how closely two nodes are connected, and the *clustering coefficient, C*, which reflects on the extent of inter-connections of nodes, are also of importance. Furthermore, to probe into the structure of the network, the distribution of the degree will be considered.

In the following section we will examine the networks formed from music composed by Bach, Chopin and Mozart, as well as from Chinese pop music. A typical network formed using the method described above is shown in Fig. 3, which corresponds to Bach's violin solos.

4 Analysis

The MIDI (Musical Instrument Digital Interface) format is used here for representation of music [12]. MIDI allows music to be stored in digital forms that can facilitate repeated performance at later times. Referring to Table 1, tick n is the time mark which indicates the time an event occurs. An event is either the start or end of a musical note. For instance, pitch name 1 starts at tick 1 and ends at tick 3.

MIDI files can be created by direct conversion from the scores or from the actual real-time performance. In the case of actual real-time performance, the time duration for a note is generally imprecise resulting in a much larger number of nodes due to the wide variation of the actual time duration of the same

Table 1. Simplified MIDI file format

Time mark	Event	Note identity
Tick 1	Start	Pitch name 1
Tick 2	Start	Pitch name 2
Tick 3	End	Pitch name 1
Tick 4	End	Pitch name 2
Tick 5	Start	Pitch name 3
Tick 6	End	Pitch name 3
...	...	...

intended note. We therefore limit our study to MIDI files created by direct conversion in order to keep the musical information precise. Once the network is formed, we can compute the following parameters:

1. Length of composition, i.e., total number of notes, T
2. Number of nodes, N
3. Total number of edges, $\sum k$
4. Mean degree, $\bar{k}$
5. Mean shortest distance between nodes, $\bar{d}$
6. Network diameter, d_{max}
7. Clustering coefficient, C
8. Assortativity, R
9. Power-law exponent of
   - Edge weight distribution,[1] γ_{ew}
   - Node degree distribution,[2] γ_{nd}
   - Node weight distribution,[3] γ_{nw}

The length of the selected pieces and the number of nodes for a network can be found by simple counting. The mean degree can also be found relatively easily by taking the average over the degree values of all nodes in the network. The calculation of the mean minimum distance between nodes requires some computational effort, and in this work we have adopted the Floyd-Warshall algorithm [13]. The network diameter is the largest minimum distance between nodes. The clustering coefficient of a node is the percentage of nodes connected to it which are themselves connected. The average clustering coefficient is found with the following formulas [14,15]:

$$C_1 = \frac{3 \times \text{number of triangles in the network}}{\text{number of connected triples of nodes}}$$
$$C_2 = \frac{1}{N} \sum_i \frac{\text{number of triangles connected to node } i}{\text{number of triples connected to node } i}$$

The assortativity, R, is essentially the correlation between pairs of nodes [16], and can be calculated by

$$R = \frac{1}{\sigma_k^2} \sum_{ij} ij(e_{ij} - k_i k_j)$$

where $\sigma_k^2 = \sum_i i^2 k_i - \left[\sum_i i k_i\right]^2$, k_i is the out-degree of node i, e_{ij} is the joint probability distribution of the out degrees of nodes i and j.

The power-law exponent is the slope of the log-log plot of the distribution, assuming that it is a straight line and thus reflects a scalefree distribution.

[1] The weight of an edge is the total number of connections made via the edge.

[2] Since networks in our study are directed networks, distributions of in-degree, out-degree and total-degree are calculated separately.

[3] The weight of a node is summation of weights of all the edges connected to it. Also, in-weight, out-weight and total-weight are calculated separately.

Table 2. Network parameters calculated for selected works

Network	T	N	$\sum k$	$\bar{k}$	$\bar{d}$	$d_{\max}$	C_1	C_2	R
Bach's violin sonata	18000	301	3797	12.6	3.0	10	0.37	0.30	0.00
Bach's violin solo	18000	229	1965	8.6	3.1	9	0.34	0.37	0.13
Bach's WTC Book I	18000	589	8955	15.2	3.1	11	0.23	0.17	-0.03
Bach's WTC Book II	18000	726	14476	19.9	2.8	9	0.34	0.30	0.05
Chopin's Op.28	18000	721	13397	18.6	2.8	8	0.28	0.28	0.08
Chopin's Nocturne	18000	899	9718	10.8	3.5	16	0.11	0.15	0.08
Mozart's works	18000	509	4005	7.9	3.4	12	0.16	0.16	-0.07
Jay Chou's "Secret"	8386	340	2377	7.0	4.2	22	0.15	0.18	0.05

Table 3. Power-law exponents calculated for selected works

Networks	γ_{ew} (fitting error)	γ_{nd} (in/out/total) (fitting error)	γ_{nw} (in/out/total) (fitting error)
Bach's violin sonata	1.7 (0.0014)	0.9/1.2/1.2 (0.0044/0.0059/0.0034)	0.8/1.1/0.4 (0.0015/0.0010/0.0040)
Bach's violin solo	1.2 (0.0039)	1.0/1.0/1.0 (0.0091/0.0088/0.0053)	0.6/0.6/0.8 (0.0027/0.0019/0.0007)
Bach's WTC Book I	1.4 (0.0009)	1.1/1.2/1.0 (0.0038/0.0029/0.0029)	1.1/0.9/1.1 (0.0013/0.0012/0.0007)
Bach's WTC Book II	1.7 (0.0005)	1.1/1.0/1.0 (0.0022/0.0025/0.0020)	1.3/1.2/1.1 (0.0007/0.0007/0.0005)
Chopin's Op. 28	1.6 (0.0003)	0.8/1.1/0.9 (0.0033/0.0020/0.0018)	0.9/1.1/0.9 (0.0005/0.0003/0.0003)
Chopin's Nocturne	1.8 (0.0010)	1.2/1.3/1.1 (0.0021/0.0019/0.0018)	1.1/1.1/1.1 (0.0008/0.0010/0.0006)
Mozart's works	1.7 (0.0011)	1.2/1.1/1.1 (0.0026/0.0029/0.0023)	1.1/1.2/1.1 1(0.0007/0.0005/0.0004)
Jay Chou's "Secret"	1.8 (0.0023)	1.4/1.8/1.5 (0.0034/0.0031/0.0028)	1.3/1.5/1.3 (0.0008/0.0008/0.0006)

In our study, compositions from several composers and sources are considered, namely, selected Bach's sonatas and partitas for violin solo, Bach's "Well-Tempered Clavier" (WTC), Mozart's KV545, KV252, KV487, KV525 and KV457, Chopin's Op. 28 and Nocturnes, and Chinese pop music.

Basically, we concatenate a number of pieces of the same type of works together to form a single set, from which a MIDI file is generated. Complex networks are then constructed and the parameters are extracted for each network. Tables 2 and 3 summarize the results for the selected musical works. Some findings are worth noting.

1. The networks formed for the different musical works are found to be scalefree in their edge weight, node degree and node weight distributions, and the power-law exponents, γ, are surprisingly consistent and all fall in the range

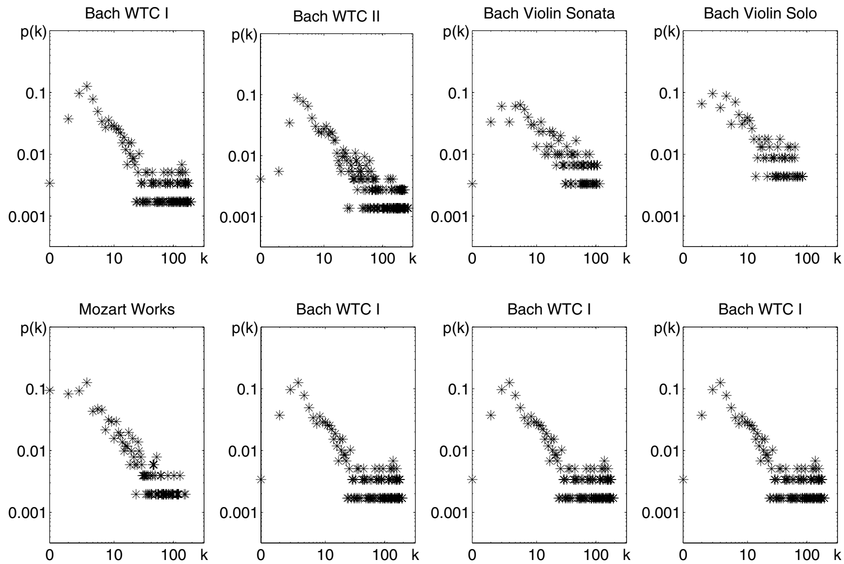

Fig. 4. Degree distributions (summation of in and out degrees) for different works. $p(k)$ versus k in log-log scale. Slopes are measured and reported as $\gamma_{\rm nd}$ in Table 2.

of 1 to 2, using a least-square-error estimation. The mean fitting errors are from 0.001 to 0.006 for sample sizes of around $N/2$ (as half of the data corresponding to very large and small k have been discarded) [17]. Fig. 4 shows the degree distributions plotted in a log-log scale.

2. With the length of the music fixed, other parameters like the number of nodes, total number of edges, and average degree vary quite significantly for different works. This shows that the same scalefree structure of music can produce significantly different kinds of music, and other parameters may play some roles in characterizing them.
3. The assortativity values of the networks are generally very small in magnitude, between 0.01 and -0.01. This shows that pairs of nodes are generally uncorrelated, and there is no preference for a node of high degree to connect to nodes of high or low degree, or vice versa.
4. Community structure has been observed in almost all music networks. A few communities of nodes of similar time durations are found, consistent with the usual observation of sections of different tempos in a piece of work, e.g., allegrissimo, allegro, andante, etc.

5 Composition from Complex Networks

The properties of the musical networks have indicated some universality across different composers and styles. For instance, the power-law degree distribution is a specific manifestation of such universality. If different composers would come up with music displaying universality, then would it be possible that music can be created artificially by preserving the same or similar network parameters?

Suppose we form a network from the works of Bach's violin solo. Then, we may create a new score using the same set of nodes (notes) and connecting one after another following an algorithm that preserves some selected network parameters. Let us focus on the preservation of the scalefree degree distribution as it seems to be the most striking common feature. We take a simple approach in connecting the nodes (generating the sequence of notes), which is based on a *biased random walk* algorithm.

Algorithm 1 (Based on Edge Weights). First, we begin with an node (note, the probability be chosen is according to its degree) in the network. The next node in the sequence will be chosen among those connected to it. According to the weight of a connecting edge,[4] we define the probability that this edge will be chosen. Then, the node connected to the chosen edge will be the next node. The process continues and thus a new score is created.

Algorithm 2 (Based on Node Degrees). Another way to create a new score is as follows. Again, we begin with an arbitrarily chosen node (note) in the network. The next node in the sequence will be chosen among those connected to it. Here, according to the degrees of all connecting nodes, we define the probabilities that these nodes will be chosen as the next node. In this way, nodes are chosen one after another. The process continues and thus a new score is created.

Algorithm 3 (Based on Node Weights). Similar to Algorithm 2, we start with an arbitrarily chosen node (note) in the network. The probability that a node connected to the starting node to be chosen as the next node is calculated from its node weight, i.e., the total edge weights of all edges connected to the node. Likewise, nodes are chosen one after another. The process continues and thus a new score is created.

Some samples of music generated from the musical networks can be downloaded from the the following website:

– http://cktse.eie.polyu.edu.hk/MUSIC/

For each algorithm, a network of the same length as the original Bach's violin solo is generated. We also preserve the proportion of different musical scales as in the original composition, and in this case, A minor : B minor : C major : D minor : E major : G minor = 1 : 6 : 1 : 3 : 3 : 1.

Table 4 shows the properties of the original network and the re-composed networks. From the table we could see that the network recomposed using Algorithm 1 "edge weight" has the closest resemblance with the original network, having the number of nodes, mean degree and clustering coefficient closest to the original Bach's piece. Also, Algorithm 1 seems to produce "more appealing" music than the other two algorithms.

[4] The weight of an edge connecting two nodes is the number of times the two nodes are connected as the music is played in the original music from which the network was generated. See also footnote 1.

Table 4. Properties of re-composed music networks

Property	Bach's violin solo	Algorithm 1	Algorithm 2	Algorithm 3
T	18000	18000	18000	18000
N	229	204	185	133
$\sum k$	1965	1749	1793	1253
$\bar{k}$	8.6	8.6	9.7	9.4
$\bar{d}$	3.1	3	2.9	2.8
$d_{\max}$	9	7	7	7
C_1, C_2	0.34, 0.37	0.37, 0.34	0.42, 0.36	0.52, 0.43
R	0.13	0.04	0.01	0.03
γ_{ew}	1.2	1.2	0.9	1
γ_{nd} (in/out/total)	1.0/1.0/1.0	0.9/1.1/1.1	0.8/0.9/1.0	0.8/0.8/1.3
γ_{nw} (in/out/total)	1/1/1	0.7/0.8/1	0.9/0.9/1	1.5/1.4/1

Remarks: As can be expected, huge possibilities exist in generating music from the above algorithms. Thus, filtering off "bad" music is important. Our initial consideration is the extent of duplication of any sequence of notes. Intuitively, a duplication-free sequence resembles a random sequence which is undesirable. Thus, we may incorporate a duplication measure in our algorithm to improve our compositions.

6 Conclusion

We have analyzed selected musical compositions in terms of co-occurrence network structures. Selected works from Bach, Chopin and Mozart, as well as from Chinese pop music, are analyzed, and networks are constructed according to the note-to-note connections of the musical scores. The networks have been found to be scalefree and their degree distributions have a similar power-law property with the values of the exponent equal to around 1.1. Such commonality suggests that the human brain composes music which naturally exhibits a scalefree degree distribution. We have therefore extended our study to reconstructing music and the basic criterion is to preserve the same power-law property. The resulting reconstructed music are still very numerous and not all sound appealing. An optimization (selection) process is needed to pick the finalist, and it will be a challenging task to study how the human brain does the selection in the process of composing music.

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