Collective Behavior Coordination and Aggregation with Low-Cost Communication

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Abstract. An important natural phenomenon surfaces that satisfactory synchronization of self-driven particles can be achieved via remarkably reduced communication cost, especially for high density particle groups with low external noise. Statistical numerical evidence illustrates that a highly efficient manner is to distribute the communication messages as evenly as possible along the whole dynamic process, since it minimizes the communication redundancy. More surprisingly, it is discovered that there exists an abnormal region in the state diagram where moderately decreasing the communication cost can even improve the synchronization performance. Significantly, another interesting fact is found that low-cost communication can help the particles aggregate into synchronized clusters, which may be beneficial to explain the forming mechanism of individuals' aggregation phenomena over biological flocks/swarms.

Keywords: Vicsek model, communication, synchronization, state diagram.

1 Introduction

Over the last decade or so, physicists have been looking for common, possibly universal, features of the collective behaviors of animals, bacteria, cells, molecular motors, as well as driven granular objects. One of the most remarkable

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characteristics of systems such as a flock of birds, a school of fish, or a swarm of locusts is the emergence of *ordered state* in which the particles move in the same direction [1,2,3], despite the fact that the interactions are (presumably) of short range. Moreover, this issue can be further generalized to a consensus problem [4], i.e. groups of self-propelled particles agreeing upon certain quantities of interest like attitude, position, temperature, voltage, etc. Distributed computation based on solving consensus problems has direct implications on sensor network data fusion, load balancing, swarms/flocks, unmanned air vehicles (UAVs), attitude alignment of satellite clusters, congestion control of communication networks, multi-agent formation control, and so on [5,6,7,8].

Vicsek *et al.* [1] proposed a dynamic model to describe the collective motion of self-propelled particles, which has been drawing more and more attention recently and gaining increased popularity from both physics and engineering communities [2,3,9,10,11,12,13,14,15,16]. In the *Vicsek model* each particle tends to move in the average direction of its neighbors while being simultaneously subjected to noise. As the amplitude of the noise increases the system undergoes a transition from an *ordered state* in which the particles move in the same direction, to a *disordered state* in which the moving directions are uniformly distributed. Grégoire and Chaté [2] modified the Vicsek model by changing the way in which the noise is introduced into the group. By this means, the states transition is switched from second to first order. More recently, in order to stabilize flocks/swarms, Gazi-Passino [11] and Moreau [12] developed two alternative models, i.e. the Attraction/Repulsion (A/R) model and the linearized model, respectively. The former yields a cohesive swarm with bounded size in a finite time, while the latter can guarantee the convergence of all the particles' states to a common one with *complete communication*, i.e. sending messages all along. Zhang *et al.* introduced predictive mechanisms into the Vicsek model and the linearized flocking model, respectively [17,18,19]. Moreover, they have also found some singular behavior inside such mainstream flocking models [20], and developed some adaptive velocity approach to accelerate the synchronization process of flocks/swarms[21].

Based on complete communication, most of the previous models of self-propelled particles yield many attractive characteristics like convergence, ordered state, consensus, rendezvous, cohesion, robustness, etc. However, in this paper, an important phenomenon is discovered that complete communication is not the most efficient manner. For many kinds of self-propelled particle groups and natural swarms/flocks/schools, satisfactory ordered state can still be achieved with sharply reduced communication cost. More surprisingly, in the density-noise space, there exists an abnormal region where moderately reducing the communication cost can help increase the performance. A general physical picture behind our finding is as follows: in abundant natural bio-groups composed of animals, bacteria, cells and so on, each particle does not send messages throughout the whole process, but now and then in some suitable manner, which is called *partial communication.* Some close examples can be found in firefly groups, deep-sea luminous fish schools and so on. Each particle uses light signals with limited power to guide the others, and just flashes at some suitable discrete times to save energy, which yields satisfactory collective performances. Other than the above mentioned natural phenomena, our work is also partially inspired by Ref. [13], which reveals that a very small proportion of informed individuals can efficiently guide a large-scale group. In addition, we found the role of information redundancy on the present model: the higher the redundancy, the worse the synchronization performance. Therefore, a highly efficient manner is to distribute the communication messages as evenly as possible along the whole dynamic process.

On the other hand, it was also found that particles are more likely to congregate into synchronized clusters using partial communication, which is useful to understand the collective behaviors of natural flocks/swarms like obstacle-avoidance, forging and anti-intrusion, where effective and economical mechanisms are desirable to reform agents into several synchronized clusters. Aggregation in biology has numerous counterparts in physics. As an irreversible process, clustering is a typical nonthermal equilibrium phenomenon, which typically follows a power law and is essential in the construction of a statistical physics for systems far from thermal equilibrium [22,23]. It is also known that intermittent dynamics is mainly due to the relative direction of motion of different clusters [24]. From an industrial application point of view, the phenomena and strategies reported in this paper may be applicable in some prevailing engineering areas such as autonomous robot formation, sensor networks, and UAVs [5,6,7]. Since each particle in these groups has limited power to send messages, partial communication can be used to save energy and form congregation.

2 Synchronization via Low-Cost Communication

Due to its popularity, we will focus on the Vicsek model [1]. In this model, the velocities $\{\mathbf{v}_i\}$ of N particles are determined simultaneously at each time step, and the position of the ith particle is updated according to $\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)$. The velocity of a particle, $\mathbf{v}_i(t+1)$, has a constant speed v and a direction given as $\theta(t+1) = \langle\theta(t)\rangle_r + \Delta\theta$, where $\langle\theta(t)\rangle_r$ denotes the average direction of particles within a circle of radius r surrounding the particle i (including i itself), which is given as $\arctan\left[\langle\sin(\theta(t))\rangle_r / \langle\cos(\theta(t))\rangle_r\right]$ if $\langle\cos(\theta(t))\rangle_r \geq 0$, or $\arctan\left[\langle\sin(\theta(t))\rangle_r / \langle\cos(\theta(t))\rangle_r\right] + \pi$ if $\langle\cos(\theta(t))\rangle_r < 0$, where $\langle\sin(\theta(t))\rangle_r$ and $\langle\cos(\theta(t))\rangle_r$ denote the average sine and cosine values, and $\Delta\theta$ represents a random noise obeying a uniform distribution in the interval $[-\eta/2, \eta/2]$. In accordance with the Vicsek model, we use the same settings as in Ref. [1], i.e. $r = 1$, $v = 0.03$ and $N = 300$, and employ the absolute value of the average normalized velocity $v_a = |\sum_{i=1}^{N} \mathbf{v}_i|/(Nv)$ as the performance index. $v_a \simeq 0$ if the direction of motion of the individual particles is distributed randomly, while for the coherently moving phase, $v_a \simeq 1$. Note that the linear size L of a square shaped cell determines the density $\rho = N/L^2$, and in all the simulations we use 1000 independent runs and at most $M = 3000$ running steps for each run, which is sufficiently long to reach the steady state of all the following cases. The initial position and moving direction of each particle are selected randomly from the

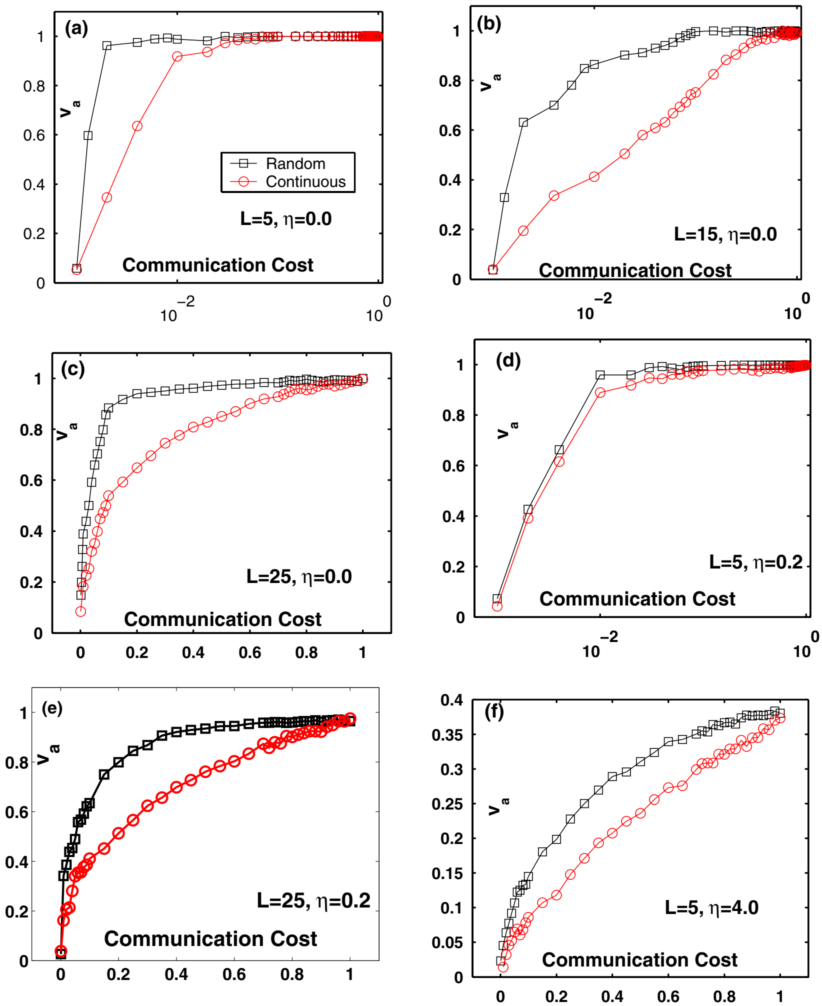


Fig. 1. (Color online) Synchronized performance index v_a with respect to communication cost p for random (squares) and continuous (circles) manners

square $[0, L] \times [0, L]$ and range $[0, 2\pi]$, respectively. The boundary conditions are periodic.

For partial communication, only part of the particles will broadcast their positions and velocities at each time step. The communication cost p is evaluated by the average number of broadcasting particles over the total number of particles at each time step. To investigate partial communication and find a highly efficient manner, we compare two communication manners, namely, *random* and *continuous*. The random communication manner (resp. the continuous communication manner) demands that each particle send $p \cdot M$ messages reporting its position and velocity randomly (resp. continuously with randomly selected

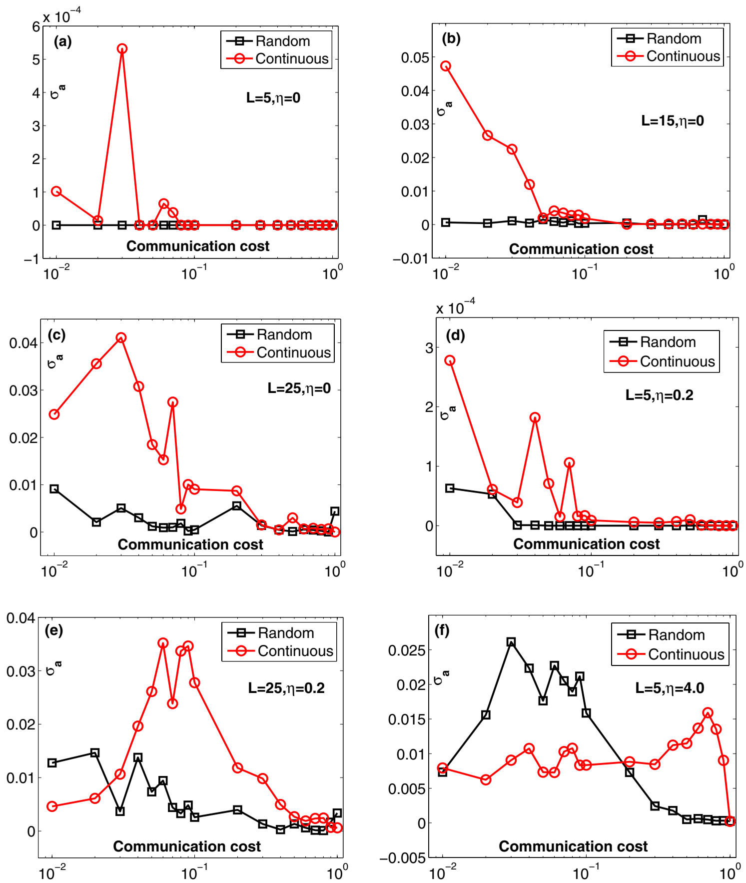


Fig. 2. (Color online) Variance of v_a with respect to different initial conditions over 1000 independent runs

beginning step). Since the cost of communication is the major concern here, we fix the number of messages that each particle can send throughout the process.

First, these two protocols are compared for a high density particle group ($L = 5$) without noise in Fig. 1(a). The first attractive characteristic of the random manner is that satisfactory v_a can be yielded with sharply reduced p (e.g. no less than 95% of v_a of the complete communication for $p \approx 2\%$). To further investigate the influence of the density ρ on the performance v_a, we have done simulations for a medium density case ($L = 15$) and a low density case ($L = 25$) (see Fig. 1(b) and Fig. 1(c), respectively). Comparing the performances for the random communication alone across Figs. 1(a)–(c), one can observe that, when the density ρ decreases, to achieve the same satisfactory v_a a higher communication cost p is required. Similar conclusions can also be drawn for the continuous protocol. Next, we investigate the effect of external noise by adding low and high noises. As shown in Figs. 1(d)–(f), the noise has a more intensive effect on the performance than density. It can be seen that the random manner outperforms the continuous manner. Moreover, when there is no neighboring particles sending any messages around a particle, which happens frequently in the low-density and/or low communication cost cases, it can still be aware of its own velocity as $\theta(t+1) = \theta(t) + \Delta\theta(t)$.

Moreover, since the initial positions and velocities of the particles are randomly selected, to support the generality of our observation, we examine the sensitivity of v_a to the initial conditions as shown in Fig. 2(a)–(f). For high-density/low-noise cases, such as Fig. 2(a) and (d), the standard variance of v_a, denoted by σ_a, keeps at a very low level of less than 6×10^{-4}. On the other hand, although σ_a increases with increasing η or decreasing ρ , it still remains at an acceptable low level (see Fig. 2(b), (c), (e) and (f)), which will not change the tendency of v_a (see Fig. 1 (b), (c), (e) and (f)). The validity of our observation on v_a (see Fig. 1) is thus verified.

The physical reason for achieving synchronized performances via very low communication cost is: due to the high density (e.g. $L = 5$), on average there are enough particles inside the radius r of each particle, and the combination of the sparse messages of each one of the plentiful neighbors constitutes a sufficient information flow which can guide each particle to the right direction. If the density of particles is decreased (e.g. $L = 15, 25$), p should be increased to compensate for the deficiency of the neighboring guidance information. The random protocol is better since it distributes messages more evenly, thus reduce the redundancy. In contrast, in the continuous manner, when one message sent by a particle can guide its neighbors along the right direction, its subsequent messages do little to help the group performance. In this sense, these subsequent messages become redundant, and the efficiency is thus decreased substantially. Therefore, it is reasonable to deduce that the best manner is to distribute the communication messages as evenly as possible along the whole dynamic process, since it minimizes the communication redundancy.

We propose a simple noise-free model to demonstrate that more evenly distributed communication leads to better performance. As shown in Fig. 3, with a given integer Q, for strategy S_i $(1 \leq i \leq Q)$, each particle sends its messages with probability $1/(Q-i+1)$ if the current time step t satisfies the inequality $t \bmod Q \geq i-1$. It is obvious that the communication cost p equals $1/Q$ for each strategy and the communication redundancy increases with increasing i. From the performance comparison in Fig. 4(a), we can observe that v_a decreases with increasing communication redundancy. In order to clarify the conception of communication redundancy or evenness, we use the variance of the message sending-time difference sequence to quantify the communication evenness $J_e = \frac{1}{pM}\sum_{j=1}^{pM}(h_j - \bar{h})^2$ with $\bar{h} = \frac{1}{pM}\sum_{j=1}^{pM} h_j$ and h_j the sending-time difference between the $(j-1)$th and jth messages. For completeness, we define h_1 as the sending-time of the 1st message. Larger J_e implies more evenly-distributed message sequence. For the strategy S_Q, one has $h_j = \bar{h} = Q$ $(j = 1, \ldots, pM)$, the

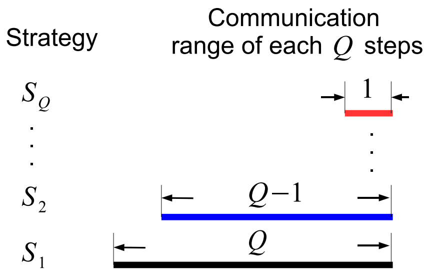


Fig. 3. (Color online) Illustration of different communication strategies S_i $(i = 1, 2, \cdots, Q)$. S_1 is the same as the random communication manner.

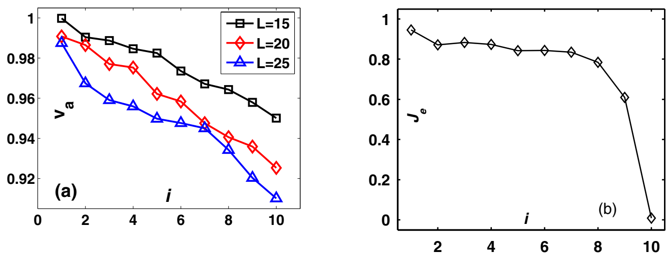


Fig. 4. (Color online) (a) Performance of strategy S_i $(i = 1, 2, \cdots, Q)$. Here we set $Q = 10$, $\eta = 0$, and (b) Evenness analysis. $M = 3000$, $Q = 10$, and this curve is the average value over 1000 independent runs.

messages of all the N individuals are concentrated simultaneously at the Qth step, $2Q$th step, ..., $\frac{M}{Q}Q$th step, which creates the greatest redundancy and leads to a zero J_e. On the contrary, for strategy S_1, the message sending-time difference sequence h_j are distributed randomly from 1 to Q, in other words, the messages of all the N individuals can be distributed randomly in the whole process, yielding the least redundancy or the greatest evenness. It is observed from Fig. 4(b) that J_e decreases with increasing i, which supports our argument.

Furthermore, a surprising phenomenon is observed that, in the case of the random manner with noise (such as $L = 25$, $\eta = 0.4$, $L = 7$, $\eta = 2$ and $L = 9$, $\eta = 2$), there exists an abnormal region where moderately reducing p might even increase v_a, which means reduced communication of a flock may improve synchronization or stability. To illustrate the abnormal phenomenon of the random manner more vividly, first we provide several typical abnormal cases and mark their corresponding abnormal values by red points in Fig. 5(a). Significantly, for the groups working in this region, each particle can use much less communication power to gain even better performance. The applaudable physical rule behind this astonishing phenomenon might be: in some suitable areas of the density-noise space, the influence of noise defeats the counterpart of

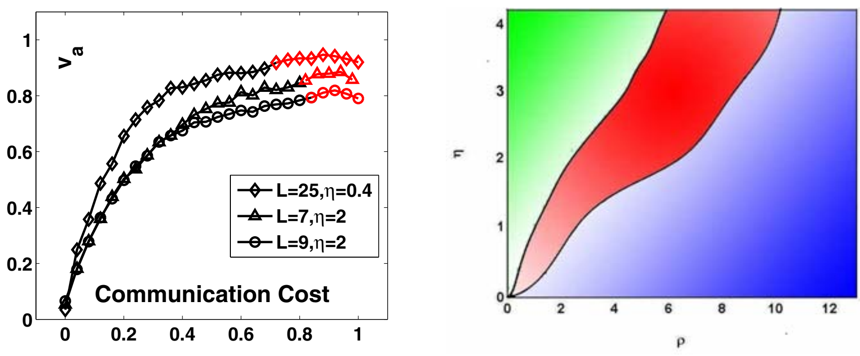


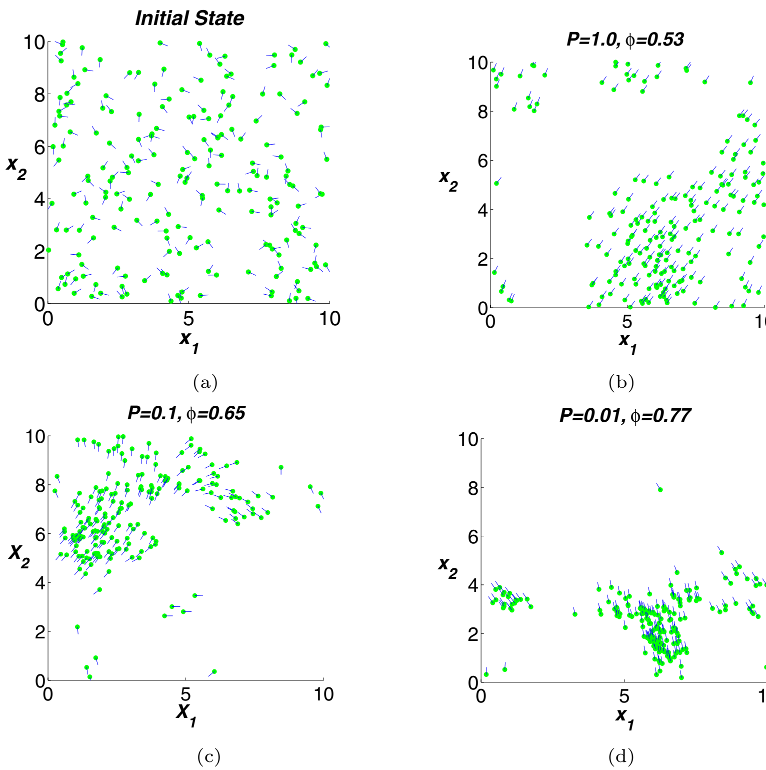
Fig. 5. (Color online) (a) Abnormal cases of random manner, red points denote the abnormal values; (b) State diagram: abnormal (red), normal (blue) and disordered (green) regions

the neighboring communications but it has not yet reached the extent of totally disordering the system dynamics, thus in some range of p (e.g. $0.5 \leq p \leq 1$) more communication means propagating more errors. Consequently, partial communication outperforms complete communication. This rule can be very useful in plentiful industrial applications, since more benefits can be achieved with less communication cost in some working conditions. However, note that this phenomenon is only found in the random communication. As to the continuous manner, its communication redundancy is too much to arouse this abnormal phenomenon.

We sketch the diagram in Fig. 5(b) where the density-noise space is divided into three regions, namely abnormal, normal and disordered regions. Here, the disordered region represents the density-noise combinations with which the performance v_a remains at a very low random value no matter what p is. Furthermore, the intensity of the color represents the likelihood of the occurrence of each phenomenon. For instance, the very inner part of the abnormal region is darker than the boundary, which means in this central part the abnormal phenomenon is more likely to occur. There is no abnormal phenomenon in noise-free cases, thus the line of $\eta = 0$ always belongs to the normal region; for $\rho = \eta = 0$, there is no particle, therefore the origin point is the only intersection of the three regions. More importantly, in the abnormal region, the intensity of the noise has been increased to defeat the influence of the neighboring communication. However, if the noise is too heavy, then the system will enter the disordered state. Using such density-noise space diagrams, one can tell whether the current working condition of the network is in the abnormal region or not. If so, one can estimate how much communication energy can be saved to yield even better performance than complete communication. In this sense, the discovery of such abnormal regions will be valuable in abundant industrial applications.

3 Aggregation via Low-Cost Communication

Apart from the analysis of the synchronization index v_a with respect to communication cost p, it is necessary to investigate the dynamical geographical bias caused by the low-cost communication mechanism. To implement this, first, the steady-state distributions of the self-propelled particle system are displayed with different communication costs $p = 1, 0.1, 0.01$ in Fig. 6. We observe in Fig. 6(b) that, in complete communication mechanism with $p = 1$, the particles are homogenously distributed. In contrast, as shown in Figs. 6(c) and (d), the particles become condensed, and all those in a given cluster move approximately in the same direction, and meanwhile, all clusters also head in similar directions. In other words, although the velocity still remains synchronized, the state of homogenously distributed particles has been broken by the low-cost communication mechanism and the particles aggregate into increasingly compacted clusters with decreasing communication cost p. In order to quantify this appealing clustering



(a) (b) (c) (d)

Fig. 6. (Color online) Condensing performance for different communication cost p with random mechanism. Here, $L = 10$, $\rho = 1$, $v = 0.05$, $\eta = 0$, $\phi = \sum_{i=1}^{L^2} |x_i - \bar{x}|/(2(L^2 - 1)\bar{x})$. All four panels show snapshots at the 2000th running step where the short black lines denote the directions of movements of the particles.

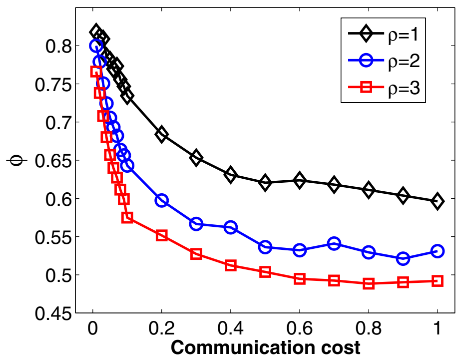


Fig. 7. (Color online) Condense performance index ϕ with respect to communication cost p with random mechanism. Here, $L = 10, \eta = 0, M = 2000$, and the ϕ curves denote average values over 1000 independent runs.

phenomenon, we propose another index $\phi = \sum_{i=1}^{L^2} |x_i - \bar{x}|/(2(L^2-1)\bar{x})$, where x_i and $\bar{x}$ represent the particle number in ith unit grid (1×1 squares) and the average particle number in each unit grid, respectively. If particles are homogenously distributed (no clustering phenomenon), index $\phi \simeq 0$. On the contrary, if all the particles are congregated into a single unit grid, ϕ equals the maximum 1. Therefore, ϕ rises with increasing condensing intensity of the whole group, which has been verified by Figs. 6(b)–(d).

To exhibit a more vivid contrast between complete and partial communication mechanisms, we examine the $\phi - p$ curves in Fig. 7. One can observe that the condensing phenomenon is intensified with decreasing p. Thereby, low-cost communication makes the particles congregate into several clusters heading in similar directions. This interesting phenomenon can be interpreted as follows. Under partial communication, a particle often cannot perceive whether it is approaching some 'dark' neighbors who have not reported their dynamics for a while until it receives the first message from those neighbors. At that moment, however, this particle has already been very close to its neighbors regardless that it begins to align the direction according to its neighboring velocity. Therefore, the condense phenomenon is much more likely to occur with low-cost communication. In other words, low-cost communication keeps a satisfactory synchronization performance while condensing the whole group into several clusters heading in similar directions. Significantly, such phenomena are compatible with existing ones that measured clustering in biological swarms [24].

4 Conclusion

In summary, we have numerically analyzed the collective dynamics of self-propelled particle groups via partial communication and found that: (i) Ordered state can be possibly achieved with fairly low communication cost. When the density is high, for noise-free or low-noise cases, just a very small proportion of communication can produce satisfactory performances; in other words, almost no benefit can be gained by increasing the communication cost when p exceeds a very small value. (ii) There exists an abnormal region in the density-noise space, in which moderately reducing the communication cost can even improve the performance. (iii) Particles are more likely to congregate into synchronized clusters heading in similar directions via low-cost communication.

To verify the universality of these conclusions, we have also applied the rule of partial communication to another two popular models of self-propelled particles, the Attraction/Repulsion model [11] and the linearized model [12], and similar conclusions have been drawn. For natural science, the contribution of this work is to explain why particles of biological flocks/swarms/schools do not send messages to others all along but just now and then during the whole process, and how they reform into clusters to facilitate some complex collective behaviors. From the industrial point of view, the value of this work is two-fold: (i) The required communication energy can be reduced sharply at the cost of a tiny decrease of synchronization performance. (ii) Congregation or clustering formations required under certain circumstances can be economically achieved by low-cost communication mechanism.

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