Collective Aggregation Pattern Dynamics Control via Attractive/Repulsive Function

Michael Z.Q. Chen[1,2], Zhao Cheng[3,⋆], Hai-Tao Zhang[4,5], Tao Zhou[6,7], and Ian Postlethwaite[1]

[1] Department of Engineering, Leicester University, Leicester LE1 7RH, U.K.
mc274@le.ac.uk, michael.chen@cantab.net, ixp@le.ac.uk
[2] Department of Electronic Engineering, City University of Hong Kong, Tat Chee Avenue, Hong Kong, P.R.C.
[3] Department of Electrical & Computer Engineering, Temple University, Philadelphia PA 19122, U.S.A.
zhao.cheng@temple.edu
[4] Department of Control Science & Engineering, Huazhong University of Science and Technology, Wuhan 430074, P.R.C.
zht@mail.hust.edu.cn
[5] Department of Engineering, Cambridge University, Cambridge CB2 1PZ, U.K.
[6] Department of Modern Physics, University of Science and Technology of China, Hefei 230026, P.R.C.
zhutou@ustc.edu
[7] Department of Physics, University of Fribourg, Chemin du Muse, CH-1700 Fribourg, Switzerland

Abstract. In the coordinated collective behaviors of biological swarms and flocks, the attractive/repulsive (A/R) functional link between each pair of particles plays an important role. By changing the slope of the A/R function, a dramatic transition between different aggregation patterns surfaces. With a high value of the slope, the particle aggregation shows a liquid-like pattern in which the outer particles are sparsely distributed while the inner ones densely. In addition, the particle density is reduced from the outside to the inside of each cluster. By comparison, when the slope decreases to a sufficiently low value, the particle aggregation exhibits a crystal-like pattern as the distance between each pair of neighboring particles remains constant. Remarkably, there is an obvious spinodal in the curve of particle-particle distance variance versus the slope, indicating a transition between liquid-like and crystal-like aggregation patterns. Significantly, this work may reveal some common mechanism behind the aggregation of physical particles and swarming of organisms in nature, and may find its potential engineering applications, for example, UAVs and multi-robot systems.

Keywords: Swarm/school, multi-agent systems, attractive/repulsive functions.

⋆ Corresponding author.

J. Zhou (Ed.): Complex 2009, Part II, LNICST 5, pp. 2064–2077, 2009.

1 Introduction

Collective behavior of many dynamic network agents has attracted much attention in recent years, in particular, from physicists, biologists, mathematicians, and social scientists. One of the most remarkable characteristics of systems such as a flock of birds, a school of fish, or a swarm of locusts is the emergence of *ordered state* in which the particles move in the same direction [2,12,34], despite the fact that the interactions are (presumably) of short range. Moreover, this issue can be further generalized to a consensus problem [25,38], i.e., groups of self-propelled particles agreeing upon certain quantities of interest such as position, temperature, and voltage. Distributed computation based on solving consensus problems has direct implications on sensor network data fusion, swarms/flocks, unmanned air vehicles (UAVs), attitude alignment of satellite clusters, congestion control of communication networks, multi-agent formation control, and so on [1,3,5,22,37].

A basic but popular flocking simulation model was proposed by Reynolds [28], where three heuristic rules are prescribed, (i) *separation*: steer to avoid crowding and collision; (ii) *alignment*: steer towards the average heading; (iii) *cohesion*: steer to move towards the average position. These rules have been proven effective and are often used in the design of bio-group dynamic models (see, e.g., [10,25,26]).

Vicsek *et al.* [34] proposed a dynamic model to describe the collective motion of self-propelled particles, which has been drawing more and more attention recently and gaining increased popularity from both physics and engineering communities [2,8,9,10,12,14,15,21]. In the *Vicsek model* each particle tends to move in the average direction of its neighbors while being simultaneously subjected to noise. As the amplitude of the noise increases the system undergoes a transition from an *ordered state*, in which the particles move in the same direction, to a *disordered state*, in which the movement directions are uniformly distributed. In 2003, Jadbabaie *et al.* [13] provided mathematical convergence conditions for the Vicsek model, i.e., all the individuals should be linked at least during some time intervals. Grégoire and Chaté [12] modified the Vicsek model by changing the way the noise is introduced, which switched the state transition from second to first order. Zhang *et al.* [36] introduced predictive mechanisms into the Vicsek model. Shimoyama *et al.* [31] introduced a model in which the state of particles includes position, velocity, and acceleration. Forces between the particles involve the acceleration instead of the movement direction. In Ref. [31], the gravitational center of the whole group is known to each particle and acts as a global navigation, which seems infeasible in real systems.

The Gazi and Passino A/R model [10], embedded with a similar mechanism to the inter-molecular force, is derived from the biological flocks/swarms behaviors. Thus far, the general understanding is that the swarming behaviors result from an interplay between a long-range attraction and a short-range repulsion between the individuals [6,35]. In [6], Breder suggested a simple model composed of a constant attraction term and a repulsion term which is inversely proportional to the square of the distance between two members, whereas in [35] Warburton

and Lazarus studied the effects on cohesion of a family of attraction/repulsion functions. Moreover, in the physics communiuty, a large volume of literature on systems with interactive particles have also adopted functions of attractive and repulsive forces to investigate the dynamics of the system [4,16,19,23,29].

With the A/R model [10], Gazi & Passino proved that the individuals will form a bounded cohesive swarm in a finite time. One year later, by adding another factor, i.e., attraction to the more favorable regions (or repulsion from the unfavorable regions), they generalized their former model into a social foraging swarm model [11]. Under some suitable circumstance, agents in this modified model are apt to move to the more favorable regions. The A/R model has been adopted by physicists and biologists to model self-driven particles and biological flocks [7,9,17,18,30,39]. In 2004, Moreau [21] presented a linearized model of flocks, and proved that the flock is uniformly and globally cohesive to a bounded circle if and only if there exists an agent (the "leader") connecting to all other agents, directly or indirectly, over an arbitrary time interval.

Tanner *et al.* [33] proposed a centralized algorithm that leads to irregular collapse and a distributed one that leads to irregular fragmentation. In 2006, Olfati-Saber [24] developed a theoretical framework for flocking by proposing three algorithms. The first algorithm embodies Reynolds's three rules, which leads to regular fragmentation (not necessarily flocking). The second algorithm incorporates a navigational feedback mechanism into every agent to achieve the flocking behavior. The third algorithm tackles the obstacle avoidance problem. This framework is becoming the basis for other research on flocking, e.g., [32].

In this paper, we propose a modified Olfati-Saber model. A transition between distinct aggregation patterns surfaces by changing the slope of the A/R function crossing the horizontal axis. With a high value of the slope, the particle aggregation shows a liquid-like pattern in which the outer particles are sparsely distributed while the inner ones densely, and the particle density is reduced from the outside inwards. By comparison, when the slope is a sufficiently low value, the particle aggregation exhibits a crystal-like pattern as the distance between each pair of neighboring particles remains constant. There is an obvious spinodal in the curve of inter-particle distance variance versus the slope, indicating a transition between the liquid-like and crystal-like patterns.

The rest of the paper is organized as follows. Section 2 presents a modified Olfati-Saber model. Section 3 presents the aggregation patterns that emerge in the modified Olfati-Saber model by setting different parameter values. Section 4 presents some analytical interpretations. Finally, the conclusion is given in Section 5.

2 A Modified Olfati-Saber Model

In this section, we will present a modified Olfati-Saber model. Consider a group of N dynamic particles with equation of motion

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = u_i, \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where q_i is the position of i^{th} particle, p_i is the velocity of ith particle, and u_i is the acceleration. The acceleration includes three parts:

$$u_i = u_i^{ar} + u_i^{cons} + u_i^{target},$$

where u_i^{ar}, u_i^{va}, and u_i^{target} denote attraction/repulsion, velocity consensus (alignment), and target field, respectively.

2.1 Attraction/Repulsion

The attraction/repulsion term in the acceleration is expressed as

$$u_i^{ar} = \sum f(q_j - q_i) n_{ij},$$

where n_{ij} is a vector connecting q_i to q_j. The strength of this effect is related to the distance, x, between two particles. The potential field function between non-polar particles can be approximated by $\varepsilon_p = Ax^{-m} - Bx^{-n}$, where m and n are integers.

Here, we approximate the A/R function as follows: we use an exponential function for $x \in [0, b)$ (repulsive) and a second-order polynomial for $x \in [b, \infty)$ (attractive) such that

$$f(x) = \begin{cases} Ax^2 + Bx + C, & x \in [0, b), \\ \frac{d}{c}(x-b)\exp\left(-\frac{(x-b)^2}{c}\right), & x \in [b, \infty). \end{cases} \tag{2}$$

The second-order polynomial $Ax^2 + Bx + C$ must satisfy the following equations to ensure the continuity and differentiability of $f(x)$ at $x = b$:

$$f(x) \mid_{x \to b^-} = f(x)_{x=b},$$
$$f'(x) \mid_{x \to b^-} = f'(x)_{x=b}.$$

Some examples of such A/R functions are shown in Fig. 1.

The strength of attraction decreases exponentially. When the coefficient c changes, the scale of attraction, d_{cut}, varies accordingly. The relation could be approximately described as $d_{cut} = c$ (c can be from 5 to 18), beyond which the attraction is negligible.

2.2 Velocity Consensus/Alignment

Each particle will align with its neighbors. The effect is composed of the velocity difference $(p_j - p_i)$ as:

$$u_i^{cons} = \rho\big(\|q_j - q_i\|\big)(p_j - p_i),$$

where is p a scaling coefficient:

$$\rho(x) = \begin{cases} (1 + \cos(\pi x))/2, & x \in [-1, 1], \\ 0, & \text{otherwise.} \end{cases}$$

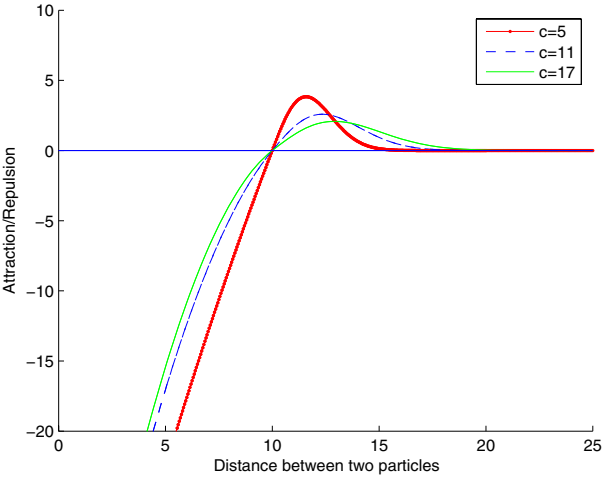

Fig. 1. (Color online) Examples of the attractive/repulsive functions in Eq. (2)

2.3 Target Field

The action of target field is a stablizing effect, influencing all particles in a group evenly and continuously. It will remain the same no matter how the state (velocity and position) of a particle changes. Its position-related and velocity-related terms are provided as below:

$$u_i^{target} = \underbrace{k_1 \frac{q_o - q_i}{\|q_o - q_i\|}}_{\text{position term}} + \underbrace{k_2 \frac{p_o - p_i}{\|p_o - p_i\|}}_{\text{velocity term}},$$

where k_1 and k_2 are the strengths of the position term and velocity term, respectively. In a gravitational field, k_1 is the acceleration due to gravity and p_o points to the center of the earth.

This model is similar to the Olfati-Saber α-lattice model [24], using the same equation of motion (see Eq. (1)). The two models are different in the definition of attraction. In the present model, strength of attraction is exponential and infinite. In the Olfati-Saber model, however, the attraction is relatively complex and with finite cut-off. In addition, the present model uses only the A/R function to stabilize the system, without any feedback (centralized) force or velocity consensus (alignment), which is also different from the Olfati-Saber model.

3 Aggregation Patterns

According to the three basic rules between particles, we design the following simulations. In a two-dimensional square with periodic boundary conditions, N particles are generated, whose initial locations and velocities are selected randomly. Then all particles in this region evolve according to Eq. (1) and the

states of all particles are updated synchronously. For all the simulation results presented here, we use only attraction/repultion forces.

From the simulations, some interesting phenomenon emerged: when the parameter c in the Eq. (2) is changed, a transition between different aggregation patterns, namely pseudo-liquid and pseudo-crystal patterns, surfaces.

3.1 Pseudo-liquid Patterns

In the square with periodic boundary conditions, there are N randomly distributed particles. After a certain period, some small clusters of particles are formed. Due to the 'surface tension', these particles will gradually contract to form a circle with increasing density from exterior to interior, as shown in Fig. 2. Such patterns are quite similar to the formation of molecules in liquid phase and hence called pseudo-liquid patterns.

Interestingly, when two groups encounter each other, they will merge into a larger circle-shaped cluster. The final shape of the new larger cluster has nothing to do with the original orientations and velocities of the two clusters before merging with each other.

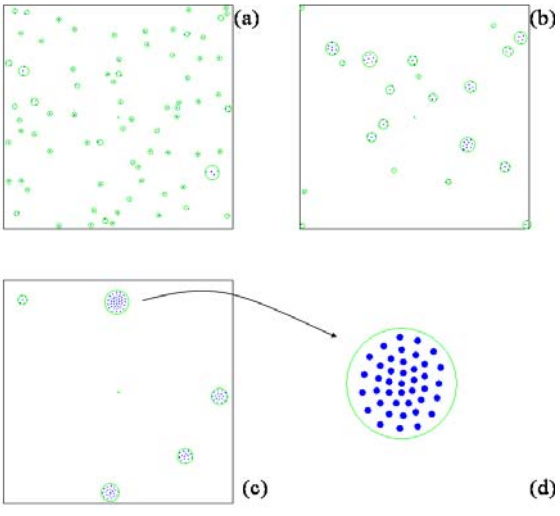

Fig. 2. (Color online) Pseudo-liquid pattern of 100 particles with $a = 50$, $b = 8$, $c = 10$, and $d = 20$. (a) Initial positions. (b) Positions after 800 steps. (c) Positions after 1600 steps. (d) Aggregation pattern of one pseudo-liquid cluster. The green circle surrounding the particles is a sign to mark the cluster.

3.2 Pseudo-crystal Patterns

In the same setting as Section 3.1, small clusters of particles emerge after a period of time. This time, however, these clusters have a more even distribution

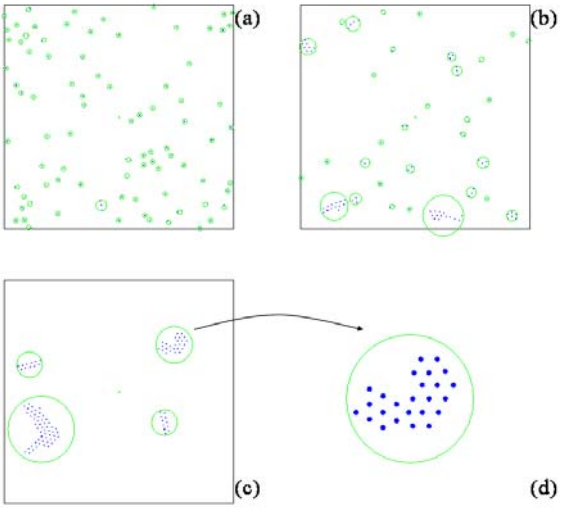

Fig. 3. (Color online) Pseudo-crystal pattern of 100 particles with $a = 50$, $b = 8$, $c = 5$, and $d = 20$. (a) Initial positions. (b) Positions after 800 steps. (c) Positions after 1600 steps. (d) The Aggregation pattern of one pseudo-crystal cluster.

of particles compared with the pseudo-liquid pattern, as shown in Fig. 3. Such patterns are quite similar to the formation of molecules in crystal phase and hence called pseudo-crystal patterns.

When two clusters encounter, a larger cluster is formed. This new larger cluster will not be round in shape. The final shape of the new cluster is dependent on the original orientations and velocities of the two clusters prior to merging with each other.

3.3 Transition between Pseudo-liquid and Pseudo-crystal Patterns

Between the two patterns, there exists a certain unstable pattern, like a transitional phase, as shown in Fig. 5(c). In this pattern, there is the crystal-like irregular shape together with the liquid-like round features. In general, the internal distribution of particles is uniform, but with exterior irregular shapes. Further interpretation is given in Section 4.

4 Analysis

We believe that the determining factor causing the two above-mentioned aggregation patterns is the distance of attraction. The reason can be explained as follows. When the effective distance of attraction is short, a particle will only affect the adjacent particles. As a result, the distribution of particles will be uniform and the distance between two particles will not be affected by the size of the group (see Fig. 5(b)). By comparison, when the effective distance of attraction

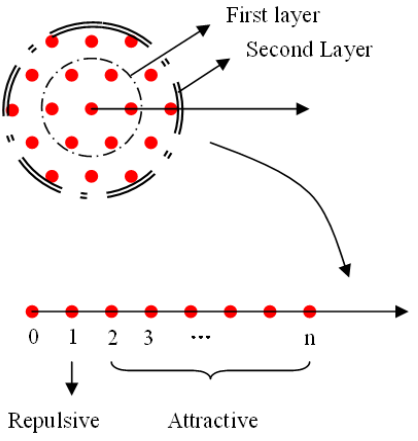

Fig. 4. (Color online) Particle distribution and inter-particle forces within a cluster

is long enough, particles will be attracted by not only the adjacent particles but also some particles further away. As a result, the inner particles will be pressed closer to their neighbors and the outer particles enjoy larger separations as the pressure among them is smaller (see Fig. 5(a)). The outer particles will also form a circle/sphere to surround the group, which is similar to the surface tension in liquid.

Assume that particles are in a dense arrangement and the distance between any two neighboring particles is constant b, we can approximate the relation between the group radius r and group population N as

$$r = b\sqrt{(N/3)}, \quad N > 150. \tag{3}$$

The calculation is as follows. Particles are in a dense arrangement (see Fig. 4), the relation between N and the number of layers, n, is $3n^2 - 3n + 1 = N$. When $n > 7$ ($N > 150$), N approximates $3n^2$. We obtain Eq. (3) by $n = \sqrt{(N/3)}$ and $r = nb$.

As for attractive and repulsive forces, particle in the center of a group is the key to stabilizing the group. Stability of an ideally distributed group is discussed here. A particle in the center of the group is named Particle 0. Particle 1 is randomly chosen, which is neighbor of Particle 0. Draw a line that goes from Particle 0 to Particle 1, along which lies Particle 2, Particle 3, etc. F_a represents the strength of the attraction exerted on Particle 0 while F_r represents strength of repulsion (see Fig. 4). The neighboring Particle 1 will exert a repulsive force to Particle 0. To keep this structure from collapsing, all the other neighbors of Particle 0 (from Particle 2 to Particle n) will exert attractive forces. As for Eq. (2), $F_r = Ax^2 + Bx + C$, x here is the distance between Particle 0 and Particle 1, and $F_a(k) = \sum_{k=1}^{n} f(kb)$. Now that we know $\int_b^\infty f(x)dx = d/2$, when $n \to \infty$, it can be shown that

$$\frac{d}{2}\left(1-\frac{6}{c}+\frac{4}{c}e^{5/c}+\frac{2}{c}e^{8/c}\right)e^{-9/c} \leqslant F_a \leqslant \frac{d}{2}\left(1+\frac{4}{c}e^{5/c}+\frac{2}{c}e^{8/c}\right)e^{-9/c},$$

which means F_a has a finite value. Only when F_a is a finite value, F_a could equal F_b and the group is stable.

After some analysis of the stable particle groups, we regard the distance between neighboring particles and each particle's distance from the group center as two parameters and draw a scatter figure with these two parameters (see Fig. 6). Fig. 6(a) shows the group in the "pseudo-liquid" pattern. The neighboring particle distance is correlated with its position, the further away (from the group center) the particle position, the larger the neighboring particle distance. The neighboring distance is discrete, which means that the distances cluster around several different positions. This is because the formed group (see Fig. 5(a)) has round shape with several circles, and the particles in the same circle will have the

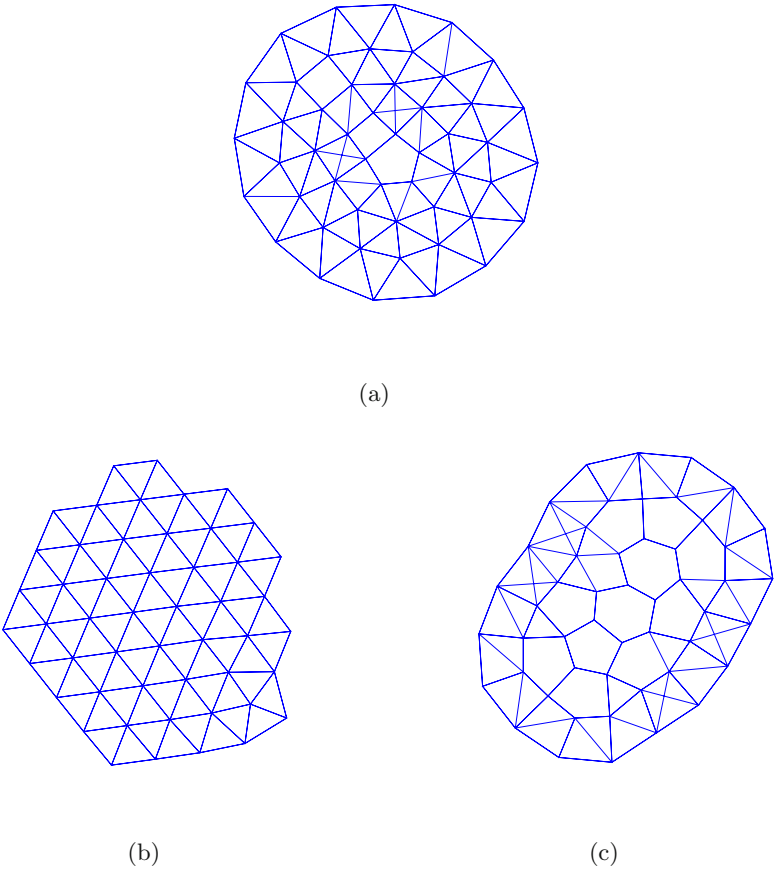

Fig. 5. Aggregation patterns. The number of particles, N, is 50 with $a = 50$, $b = 8$, and $d = 20$. (a) The "pseudo-liquid" pattern with $c = 10$. (b) The "pseudo-crystal" pattern with $c = 5$. (c) The transitional pattern with $c = 8$.

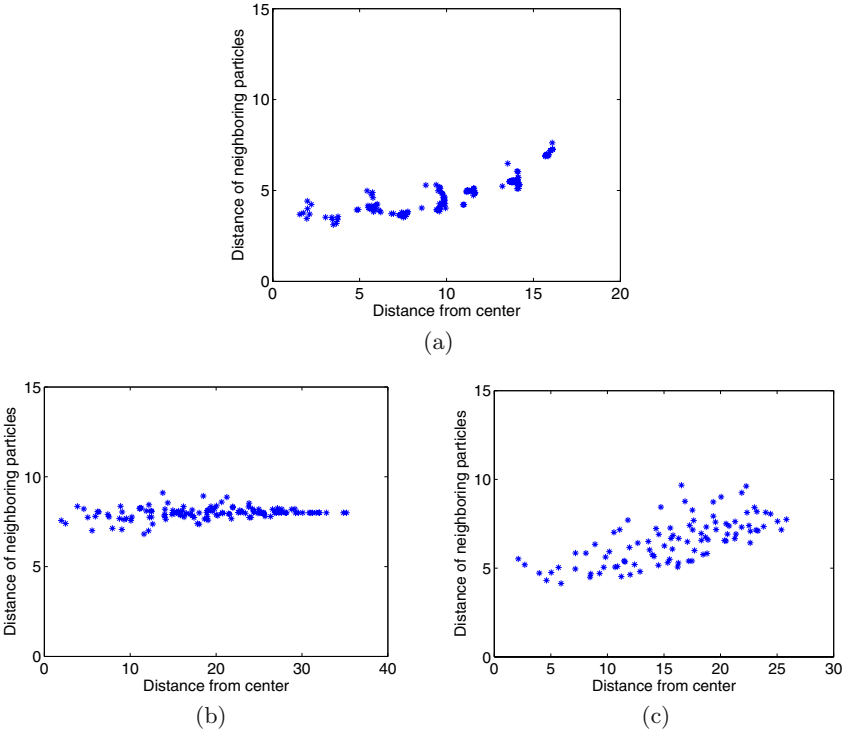

Fig. 6. (Color online) The distance of neighboring particles in each pattern. The x-axis is the distance from group center and the y-axis indicates the distance between the neighboring particles. The group population, N, is 50 with $a = 50$, $b = 8$, and $d = 20$. (a) In the "pseudo-liquid" pattern with $c = 10$, the distance between two particles increases with the increasing distance from the group center. (b) In the "pseudo-crystal" pattern with $c = 5$, the distance between two particles remains stable while the distance from group center changes. (c) In the transitional pattern with $c = 8$, the distance between particles increases with the distance from the group center. However, for the same distance from the group center, the distance between two neighboring particles is randomly distributed.

same distances due to symmetry, while the particles in different circles will have different distances because particles at different positions have different pressure. Fig. 6(b) shows the group in the "pseudo-crystal" pattern. The distances between neighboring particles are independent of the distances from the group center. Fig. 6(c) shows the group in "pseudo-crystal and pseudo-liquid transition" pattern. This has the features of both the pseudo-liquid and pseudo-crystal patterns, i.e., its neighboring particle distance are correlated with the particle positions and the distribution is more random compared with the first two.

As seen from Fig. 7, the standard deviation of the distances between neighboring particles has two stable regions: when the cut-off distance, d_{cut}, is smaller than 12, the standard deviation stays around 0.4, which implies "pseudo-crystal"

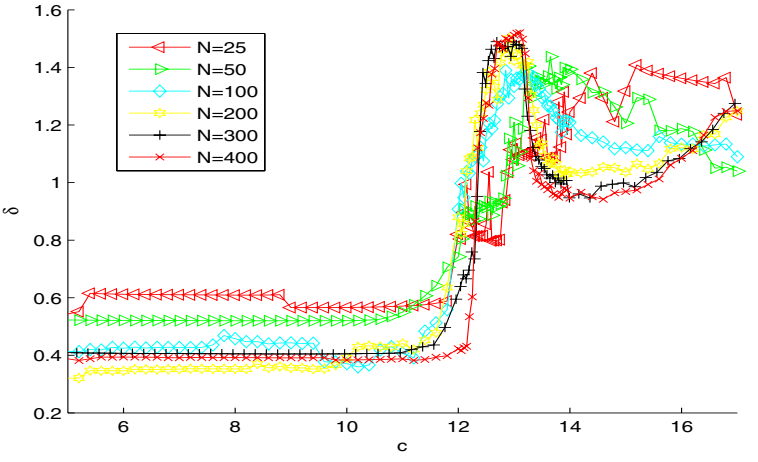

Fig. 7. (Color online) The standard deviation of neighboring particle distance with $a = 50$, $b = 10$, and $d = 20$. The number of particles, N, varies from 25 to 400.

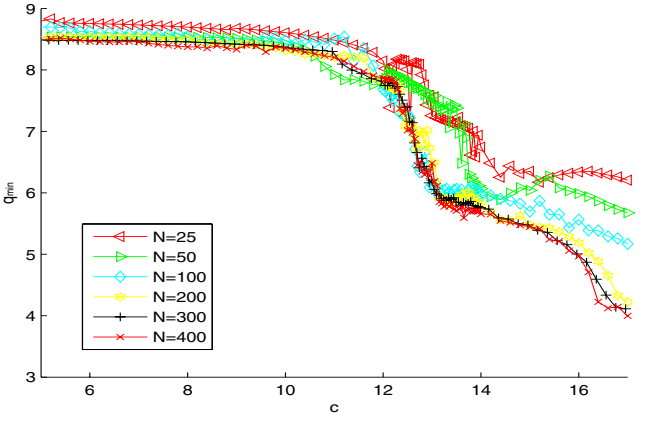

Fig. 8. (Color online) The smallest distance among particles with $a = 50$, $b = 10$, and $d = 20$. The number of particles, N, varies from 25 to 400.

pattern; when the distance rises (more than 14), the standard deviation remains at a new value (maintained at around 1.2). The minimum distance between the particles begins to decline (see Fig. 8), which is the "pseudo-liquid" pattern; between the two patterns, there is the transitional pattern, having features of both crystal and liquid (see also Fig. 5) except that the standard deviation rises shapely in this pattern.

5 Conclusion

In this paper, we have investigated the role of the slope of the A/R function in the aggregation patterns. With a steeper slope of the A/R function crossing

the horizontal-axis, the particle aggregation shows a liquid-like pattern in which the outer particles are distributed sparsely while the inner ones are densely distributely. In addition, the particle density is reduced from the outside inwards for each cluster. By comparison, however, when the slope decreases to a sufficiently low value, the particle aggregation exhibits a crystal-like pattern as the distance between each pair of neighboring particles remains constant. An obvious spinodal has been observed in the curve of inter-particle distance variance with respect to the slope, strengthening the transition between liquid-like and crystal-like aggregation patterns.

For biological sciences, the contribution of this work lies in its potential to explain some natural aggregation patterns, e.g., when under attack by predators, strong antelopes will form a circle surrounding the weak. From the engineering application point of view, designing different A/R functions for different aggregation patterns can be useful for various tasks.

Acknowledgment

M.Z.Q. Chen acknowledges the support of City University of Hong Kong under Grant No. 7002274. H. T. Zhang acknowledges the support of the National Natural Science Foundation of China (NNSFC) under Grant No. 60704041, and the Research Fund for the Doctoral Program of Higher Education (RFDP) under Grant No. 20070487090. T. Zhou acknowledges the support of NNSFC under Grant No. 10635040.

References

1. Akyildiz, I.F., Su, W., Sankarasubramaniam, Y., Cayirci, E.: Wireless sensor networks: a survey. Computers Network 38, 393–422 (2002)
2. Aldana, M., Dossetti, V., Huepe, C., Kenkre, V., Larralde, H.: Phase Transitions in Systems of Self-Propelled Agents and Related Network Models. Phys. Rev. Lett. 98, 095702 (2007)
3. Arai, T., Pagello, E., Parker, L.E.: Guest editorial advances in multirobot systems. IEEE Trans. Robotics and Automation 18, 655–661 (2002)
4. Archer, A.J., Wilding, N.B.: Phase behavior of a fluid with competing attractive and repulsive interactions. Phys. Rev. E$\tilde{7}$6, 031501 (2007)
5. Bertsekas, D.P., Tsitsiklis, J.N.: Comment on Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans. Automat. Contr. 52, 968–969 (2007)
6. Breder, C.M.: Equations descriptive of fish schools and other animal aggregations. Ecology 35, 361–370 (1954)
7. Chuang, Y.L., Orsogna, M.R.D., Marthaler, D., Bertozzi, A.L., Chayes, L.S.: State transitions and the continuum limit for a 2d interacting, self-propelled particle system. Physica D 232, 33 (2007)
8. Couzin, L.D., Krause, J., Franks, N.R., Levin, S.A.: Effective leadership and decision-making in animal groups on the move. Nature 433, 513 (2005)

9. D'Orsogna, M.R., Chuang, Y.L., Bertozzi, A.L., Chayes, L.S.: Self-Propelled Particles with Soft-Core Interactions: Patterns, Stability, and Collapse. Phys. Rev. Lett. 96, 104302 (2006)
10. Gazi, V., Passino, K.M.: Stability Analysis of Swarms. IEEE Trans. Automat. Contr. 48, 692–697 (2003)
11. Gazi, V., Passino, K.M.: Stability analysis of social foraging swarms. IEEE Trans. Syst., Man, and Cybernetics 34, 539–557 (2004)
12. Grégoire, G., Chaté, H.: Onset of collective and cohesive motion. Phys. Rev. Lett. 92, 025702 (2004)
13. Jadbabaie, A., Lin, J., Morse, A.S.: Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans. Automat. Contr. 48, 988–1001 (2003)
14. Li, W., Wang, X.F.: Adaptive velocity strategy for swarm aggregation. Phys. Rev. E 75, 021917 (2007)
15. Li, W., Zhang, H.T., Chen, M.Z.Q., Zhou, T.: Singularities and symmetry breaking in swarms. Phys. Rev. E 77, 021920 (2008)
16. Lev, B.I.: Exactly solvable three-dimensional lattice model with attractive and repulsive interactions. Phys. Rev. E 58, 2681 (1998)
17. Liu, X., Wang, J., Huang, L.: Stabilization of a class of dynamical complex networks based on decentralized control. Physica A 383, 733 (2007)
18. Liu, X., Wang, J., Huang, L.: Global synchronization for a class of dynamical complex networks. Physica A 386, 543 (2007)
19. Melzer, A., Schweigert, V.A., Piel, A.: Transition from Attractive to Repulsive Forces between Dust Molecules in a Plasma Sheath. Phys. Rev. Lett. 83, 3194 (1999)
20. Miller, M.B., Bassler, B.L.: Quorum sensing in bacteria. Ann. Rev. Microbiol. 55, 165–199 (2001)
21. Moreau, L.: Stability of multiagent systems with time-dependent communication links. IEEE Trans. Automat. Contr. 50, 169–182 (2005)
22. Ogren, P., Fiorelli, E., Leonard, N.E.: Cooperative control of mobile sensor networks: adaptive gradient climbing in a distributed environment. IEEE Trans. Automat. Contr. 49, 1292–1302 (2004)
23. Ohnesorge, F., Binnig, G.: True Atomic Resolution by Atomic Force Microscopy Through Repulsive and Attractive Forces. Science 260, 1451 (1993)
24. Olfati-Saber, R.: Flocking for Multi-Agent Dynamic Systems: Algorithms and Theory. IEEE Trans. Automat. Contr. 51, 401–420 (2006)
25. Olfati-Saber, R., Murray, R.: Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Automat. Contr. 49, 1520–1533 (2004)
26. Ren, W., Beard, R.W.: Consensus seeking in multi-agent systems under dynamically changing interaction topologies. IEEE Trans. Automat. Contr. 50, 655–661 (2005)
27. Ren, W., Beard, R.W., Arkins, E.M.: Information consensus in multivehicle cooperative control. IEEE Control System Magazine 71, 71–82 (2004)
28. Reynolds, C.W.: Flocks, herds and schools: A distributive behavioral model. Computer Graphics 21, 25–34 (1987)
29. Schlottmann, P.: Integrable two-band model with attractive and repulsive interactions. Phys. Rev. Lett. 68, 1916 (1992)
30. Shi, H., Wang, L., Chu, T.: Virtual leader approach to coordinated control of multiple mobile agents with asymmetric interactions. Physica D 213, 51 (2006)

31. Shimoyama, N., Sugawara, K., Mizuguchi, T., Hayakawa, Y., Sano, M.: Collective motion in a system of mobile elements. Phys. Rev. Lett. 76, 3870–3873 (1996)
32. Su, H., Wang, X.F., Lin, Z.: Flocking of Multi-Agents with a Virtual Leader. IEEE Trans. Automat. Contr. (to appear)
33. Tanner, H.G., Jadbabaie, A., Pappas, G.J.: Stable flocking of mobile agents, Part I: Fixed topology. In: Proc. IEEE Conference on Decision and Control, Maui, Hawaii, USA, pp. 2010–2015 (2003)
34. Vicsek, T., Czirók, A., Ben-Jacob, E., Cohen, I., Shochet, O.: Novel type of phase transition in a system of self-driven particles. Phys. Rev. Lett. 75, 1226–1229 (1995)
35. Warburton, K., Lazarus, J.: Tendency-distance models of social cohesion in animal groups. J. Theoret. Biolo. 150, 473–488 (1991)
36. Zhang, H.T., Chen, M.Z.Q., Stan, G.-B., Zhou, T., Maciejowski, J.M.: Collective behavior coordination with predictive mechanisms. IEEE Circuits and Systems Magazine 3, 65–87 (2008)
37. Zhang, H.T., Chen, M.Z.Q., Zhou, T.: Predictive protocol of flocks with small-world connection pattern. Phys. Rev. E (to appear)
38. Zhang, H.T., Chen, M.Z.Q., Zhou, T., Stan, G.-B.: Ultrafast consensus via predictive mechanisms. Europhys. Lett. 83, 40003 (2008)
39. Zimmer, C.: From ants to people, an instinct to swarm, The New York Times, November 13 (2007)