

# An Application on Merton Model in the Non-efficient Market

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**Abstract.** Merton Model is one of the famous credit risk models. This model presumes that the only source of uncertainty in equity prices is the firm's net asset value. But the above market condition holds only when the market is efficient which is often been ignored in modern research. Another, the original Merton Model is based on assumptions that in the event of default absolute priority holds, renegotiation is not permitted, liquidation of the firm is costless and in the Merton Model and most of its modified version the default boundary is assumed to be constant which don't correspond with the reality. So these can influence the level of predictive power of the model. In this paper, we have made some extensions on some of these assumptions underlying the original model. The model is virtually a modification of Merton's model. In a non-efficient market, we use the stock data to analysis this model. The result shows that the modified model can evaluate the credit risk well in the non-efficient market.

**Keywords:** Merton Model; market efficiency; probability of default.

## 1 Introduction

The credit risks influence a national macroscopic decision-making and the economic development, even affect the normal steady movement of the global economic, therefore, credit risk management has been important to banks, and become the one of the important problem on Economics.

Merton Model is one of the famous credit risks models, In the available credit risk models, Merton's equity-based approach is considered as a pioneering tool for measuring default risks. This model presumes that the only source of uncertainty in equity prices is the firm's net asset value. But the above market condition holds only when the market is efficient which is often been ignored in modern research [1]. Some scholars believed that a realistic market is different from the efficient market, and the original Merton Model is based on assumptions that in the event of default absolute priority holds, renegotiation is not permitted, liquidation of the firm is costless and in the Merton Model and most of its modified version the default boundary is assumed to be constant which don't correspond with the reality. So these can influence the level of predictive power of the model. So the Merton model used in a non-efficient

market/a real market should be modified. In this paper, we have extended the standard Merton approach to estimate the default probability of the firm, introducing randomness to the default point and allowing a fractional recovery when default occurs, and have used the stock data in the USA to analysis this question.

## 2 The Original Merton Model

Over the last 4 decades, a large number of structural models which is one broad category of credit risk models have been developed to estimate and price credit risk. The philosophy of these models, which goes back to Black-Scholes and Merton, is to consider corporate liabilities (equity and debt) as contingent claims on the assets of the firms. Models that adopt contingent claim analysis are also referred to as Merton type models. This model hypothesis that whether the company defaults completely decided by the company asset value, when the company asset value drops to some trigger point , the default event would occur. The Merton model does not rely on the rating by some external agencies and inputs of the model are equity prices which are determined by the markets and easily observable by everyone , and for the stock price often renews, therefore this model can be also provide an indication which relates with the realistic situation of the company momentarily.

The Merton model [2] (1974) suppose the market is effective, and is based on the work of Black and Scholes (1973) on option pricing and offers a framework for valuing debt issued by a firm. The model is designed in two distinguished layouts: first, the risky debt is priced in terms of the market value of firm's assets and its volatility; second, even though the market value of firm's asset and its volatility are unobservable, they are estimated from the value of firm's equities and its volatility. These insights lead to develop a coherent framework for measuring credit risk.

The firm issues two classes of securities: equity and debt. The equity receives no dividends. The debt is a pure discount bond where a payment of  $B$  is promised at date  $T$ , and the default boundary is assumed to be constant. If at date  $T$  the firm's asset value ( $V_T^A$ ) exceeds the promised payment  $B$ , the debt-holders are paid the promised amount and the shareholders receive the residual asset value, ( $V_T^A - B$ ). If the asset value is insufficient to meet the debt-holders' claims the firm defaults, the debt-holders receive a payment equal to the asset value, and the shareholders get nothing. The payoff of the debt-holders at time  $T$  can be expressed as below:

$$B - \text{Max}\{B - V_T^A, 0\} \quad (1)$$

The first term of (1),  $B$ , represents the payoff from investing in a default-free zero coupon bond maturing at time  $T$  with face value  $B$ . The second term,  $B - \text{Max}\{B - V_T^A, 0\}$ , is the payoff from a short position in a put option on firm's asset with a strike price  $B$  and maturity date  $T$ . Thus, the risky debt is equivalent to

the combination of: (i) a default free bond paying  $B$  at time  $T$  and (ii) a put option on the firm's asset with strike price  $B$  and maturity  $T$ . Consequently, pricing the risky debt involves the following steps. Step 1: valuing the put option using Black–Scholes pricing formula; step 2: subtracting the value of the put option from the value of risk free debt.

The firm's asset value ( $V_T^A$ ) and its volatility are the determinants of the value of the put option, but both are unobservable. However, it is possible to use prices of traded securities issued by the firm to estimate these variables. The first step in this procedure is to deduce functional relations between a firm's underlying assets and its equity. To this end, we can state the payoff of the equity-holder at date  $T$ :

$$\begin{cases} V_T^A - B, \text{if } V_T^A \geq B \\ 0, \text{otherwise} \end{cases} \tag{2}$$

$$\begin{cases} B, \text{if } V_T^A \geq B \\ V_T, \text{otherwise} \end{cases} \tag{3}$$

It can be observed that (2) is simply the payoff from holding a long position in a call option on firm's assets with strike price  $B$  and maturity  $T$ . Using Black–Scholes formula for pricing call option we can derive a relationship between market value of firm's equity and the market value of assets.

It is apparent that the model described above is the genesis for understanding the link between the market value of the firm's assets and the market value of its equity. It can also be applied for estimating the default probability of a firm. The default probability is the probability that the firm will fail to service obligations. In that scenario, shareholders will not exercise their call option to buy the assets of the firm for  $B$  at time  $T$ . Therefore, The risk-neutral probability,  $P_T$ , that the firm will default by time  $T$  is the probability that shareholders will not exercise their call option to buy the assets of the firm for  $B$  at time  $T$ .

Supposes the firm's assets value obey in geometry Brownian movement.

$$dV_t = \mu_v V_t dt + \sigma_v V_t dW_t \tag{4}$$

The firm's asset volatility,  $\sigma_v$ ; the expectation rate of return of the firm's asset,  $\mu$ ; the standard Brownian movement,  $\{W_t\}$ .

We can from the random analysis theory get the result:

$$V_T = V_t \exp \left[ \left( \mu - \frac{1}{2} \sigma_v^2 \right) (T - t) + \sigma_v (W_T - W_t) \right] \tag{5}$$

And that

$$\begin{aligned}
 P[V_T < B] &= P\left[\log V_t + \left(\mu - \frac{1}{2}\sigma_v^2\right)(T-t) + \sigma_v(W_T - W_t) < \log B\right] \\
 &= P\left[\sigma_v(W_T - W_t) < -\log \frac{V_t}{B} - \left(\mu - \frac{1}{2}\sigma_v^2\right)(T-t)\right] \\
 &= \Phi\left(-\frac{\ln\left(\frac{V_t}{B}\right) + \left(\mu - \frac{1}{2}\sigma_v^2\right)(T-t)}{\sigma_v\sqrt{T-t}}\right) \tag{6}
 \end{aligned}$$

The equation (5) shows the probability of default of the firm is related to a payment of  $B$ ; the payment time  $(T - t)$ , the firm’s asset volatility,  $\sigma_v$ ; the expectation rate of return of the firm’s asset,  $\mu$ ; and the firm’s first asset value  $V_t^A$ .

### 3 The Modified Merton Model

The first limitation of the Merton Model is that the only source of uncertainty in equity prices is the firm’s net asset value. But the above market condition holds only when the market is efficient. In a non-efficient market, Behaviors of investors in the equity market are generally governed by the market sentiment. As an example, post-election uncertainty or uncertainty in policies of newly elected government may induce a panic among investors which subsequently may lead a major downfall in equity prices. Thus any upturn/downturn in equity prices might be a consequence of any of the hundreds unforeseen events such as frauds or war or draught or hike/fall in oil prices, etc. These events are not predictable; however, playing on market sentiment they change the overall supply/demand and consequently disrupt the stability of the market. Another kind of factors that regulates the behavior of rational investors is firm-specific and does not depend on market behavior. The firm-specific factors, e.g. strike, rise/fall of sale, etc. induce an upward or downward shift in the net asset value of the firm which lead to a change in the equity price. Thus equity price changes might be consequent upon alteration of the firm’s financial strength or any exogenous factor that plays on the market sentiment or, otherwise, any combined effect of the above two factors. These two factors are uncorrelated when any shift in the firm’s net asset value is not governed by the market sentiment and vice-versa. Therefore, we can assume that equity  $E$ , of a firm is a linear combination of effects of two factors  $F$  and  $M$ , where  $F$  is the factor specific to the firm and  $M$  is the factor relates to the market. Thus,

$$E = \alpha F + (1 - \alpha)M, \quad (0 < \alpha < 1) \tag{7}$$

Where  $\alpha$  is the relative weight to the factor  $F$ . Any change in equity price is observable from the market; however, influence of either  $F$  or  $M$  on the equity price cannot be separated directly. We can segregate the effect of  $F$  and  $M$  from the equity price under certain reasonable assumptions which are described below.

The factors  $F$  and  $M$  can be viewed as two assets which are formed a portfolio,  $E$ . Therefore, the price of the equity at time  $T$ ,  $P_T^E$ , is a linear combination of the price of the asset  $F$ ,  $P_T^F$  and the price of the asset  $M$ ,  $P_T^M$ :

$$P_T^E = \alpha P_T^F + (1 - \alpha) P_T^M \tag{8}$$

Eq. (8) can be represented in terms of betas:

$$\beta_E = \alpha \beta_F + (1 - \alpha) \beta_M \tag{9}$$

Where  $\beta_I = \frac{Cov(P^I, P^M)}{Var(P^M)}$  ( $I = E, F, M$ ), More explicitly,  $\beta_I$  explicates the relationship between the price on the asset and the price of the market portfolio.

*Case 1.* The price of the asset  $F$  is uncorrelated with that of the asset  $M$ ,  $\beta_F = 0$

Therefore,

$$\alpha = 1 - \beta_E \tag{10}$$

Using the value of  $\alpha$ , the price of the asset  $F$  can be evaluated from Equation (8) as below:

$$P_T^F = \frac{P_T^E - \beta_E P_T^M}{1 - \beta_E} \tag{11}$$

If we assume that effects market sentiment is summarized into the market index,  $S_T$ ,  $P_T^M$  must have following relationship with  $S_T$ .

$$P_T^M = (S_T - \bar{S}) \tag{12}$$

So that:

$$\bar{P}^F = \frac{\bar{P}^E}{1 - \beta_E} \tag{13}$$

*Case 2.* The price of the asset  $F$  is correlated with that of the asset  $M$ ,  $\beta_F \neq 0$ . As any change in the net asset value of the firm would lead to a similar change in  $P_T^F$ , we can estimate using historical data on firm's net asset value (NAV) and the market index,  $S$ . Thus,  $\beta_F$  can be estimated using following formula:

$$\beta_F = \frac{Cov(NAV, S)}{Var(S)} \tag{14}$$

For  $\beta_M = 1$ , in this case,

$$\alpha = \frac{1 - \beta_E}{1 - \beta_F} \tag{15}$$

Using the value of  $\alpha$ , the price of the asset  $F$  can be evaluated from Eq. (9) as below:

$$P_T^F = \frac{(1 - \beta_F)P_T^E - (\beta_F - \beta_E)P_T^M}{(1 - \beta_E)} \tag{16}$$

When the historical series on firm’s net asset value (NAV) is not available, alternatively we can estimate  $\beta_F$  by the iterative procedure. Let us denote the approximation of  $\beta_F$  and  $P_T^F$  in the  $i$ th iteration as  $\beta_F(i)$ ,  $\beta_F(0) = 0$  and  $P_T^F(i)$  respectively. Accordingly,  $i$ th approximation of  $P_T^F$  as follows:

$$P_T^F(i) = \frac{(1 - \beta_F(i-1))P_T^E - (\beta_F(i-1) - \beta_E)P_T^M}{(1 - \beta_E)} \tag{17}$$

$$\beta_F(i) = \frac{Cov(P^F(i), P^M)}{Var(P^M)} \tag{18}$$

The process converges if  $|\beta_F(k) - \beta_F(k-1)| < \epsilon$ , Accordingly, we can obtain a desired degree of accuracy by considering a smaller  $\epsilon$ .

Therefore,  $\alpha P_T^F$  may equivalently be considered as a price of the equity in an efficient stock market. Consequently, Black-Scholes option pricing formula can be applied on it.

The second limitation of the original model is the equity receives no dividends [3]. However, we will allow cash dividend payments since they can affect the market value of common equity. The total amount of proposed cash dividend payments  $\delta$  is assumed to be prepaid now at time  $t = 0$ . Therefore the change in the firm’s market value at time  $t = 0$  will be:  $d(V_0) = -\delta$ .

The Merton Model is based on assumptions that in the event of default absolute priority holds, renegotiation is not permitted and liquidation of the firm’s is costless. These questionable assumptions imply full recovery in case the firm defaults, as we know that direct and indirect costs of financial distress such as lawyer fees, administration expenses or loss opportunities due to the firm’s uncertainty can result

in debtholders receiving less than the firm’s value. Additional default costs can arise also from possible deviation on the absolute priority rule, when equityholders gain at the expense of debtholders. If we will let  $R \in [0,1]$ , be a random variable expressing the recovery rate as %of the debt’s also that the firm is neither allowed to repurchase shares nor to issue any senior or equivalent claims, we get the following payoffs to equityholders and stockholders at time  $T$  :

$$\begin{cases} V_T^A - B, \text{if } V_T^A \geq B \\ 0, \text{otherwise} \end{cases} \tag{19}$$

$$\begin{cases} B, \text{if } V_T^A \geq B \\ RB, \text{otherwise} \end{cases} \tag{20}$$

The third limitation of the original model is that the default boundary is assumed to be constant. Hence, the estimation risk neutral expected default probabilities cannot capture changes in the relationship of asset value to the firm’s default point that were caused from changes in firm’s leverage. These changes are critical in the determination of actual default probability. Moreover, the simplifying assumption of a constant default boundary is the major reason that the model results in unrealistic estimated short term credit spreads that differ from those observed empirically. In order to capture the uncertainty associated with leverage, we will assume that the default barrier evolves as a geometric Brownian motion with a constant drift equal to  $\mu_B$  and a constant diffusion rate equal to  $\sigma_B$  :

$$dB_t = \mu_B B_t dt + \sigma_B B_t dW_t' \tag{21}$$

Where  $W_t'$  is a standard Brownian motion. Moreover, we will assure that the source of randomness that drives the default point  $W_t'$  is independent from the source of randomness that drives the asset value  $W_t$ . Finally, we will assure that the source of randomness that drives the default point  $W_t'$  is diversifiable in order to ensure the existence of the unique risk probability measure  $P$  .

Following our assumptions, at time horizon  $T$  the market value of firm’s assets equals

$$V_T = (V_t - \delta) \exp \left[ \left( \mu_V - \frac{1}{2} \sigma_V^2 \right) (T - t) + \sigma_V (W_T - W_t) \right] \tag{22}$$

And the market value of the default point equals

$$B_T = B_t \exp \left[ \left( \mu_B - \frac{1}{2} \sigma_B^2 \right) (T - t) + \sigma_B (W_T - W_t) \right] \tag{23}$$

Therefore, default occurs at the maturity of debt  $T$  if:

$$(V_t - \delta) \exp \left[ \left( \mu_V - \frac{1}{2} \sigma_V^2 \right) (T - t) + \sigma_V (W_T - W_t) \right] < B_t \exp \left[ \left( \mu_B - \frac{1}{2} \sigma_B^2 \right) (T - t) + \sigma_B (W_T - W_t) \right] \tag{24}$$

Taking the natural logarithm of the above inequality we have:

$$\ln \left( \frac{V_t - \delta}{B_t} \right) - \frac{(\sigma_V^2 - \sigma_B^2)(T - t)}{2} + \sigma_V (W_T - W_t) - \sigma_B (W_T - W_t) < 0 \tag{25}$$

Thus, we can rewrite the probability of default as:

$$P[V_T < B] = \Phi \left( - \frac{\ln \left( \frac{V_t - \delta}{B_t} \right) - \frac{(\sigma_V^2 - \sigma_B^2)(T - t)}{2}}{\sqrt{(\sigma_V^2 + \sigma_B^2)(T - t)}} \right) \tag{26}$$

## 4 Empirical Analysis

### 4.1 Sample

In this paper, the samples are the listed companies in the USA rated by the Standard Poor's. The importance of the stock market is growing in national economy .The SP500 becomes the index which can represent the movement on the national economy. So we choose the SP500 to be the market index. The stock transaction data time interval is from January 1, 2006 to January 1, 2007.From the Standard Poor's webpage, the firms which the credit rate above BBB have not in default. Therefore, the samples are the listed companies under the BB level on the New York stock exchange which including in the SP500 index. We sample 27 firms random which satisfying the above conditions and belonging to BB level from the OSIRIS. The comparative default probability is the data released by Standard Poor's on December 31, 2007.It is the actual result.

### 4.2 The Relativity between a Firm's Asset Value and the Market Index

In the USA, the historical series on firm's net asset value (NAV) are not available, we can't evaluate the relativity directly. so we get the  $i$ th iteration as  $\beta_F(i)$ , and  $P_T^F(i)$  based on Eq.(17) (18),then by Eviews evaluate the relativity.



### 4.3 Evaluation on Parameter of the Model

1. The firm's option value: the firm's option value=closing price of circulation stock × the number of circulation stock .
2. The firm's option volatility: the firm's option volatility is replaced by the volatility of the circulation stock option. Selects date closing price from January 1st, 2006 to January 1, 2007 to determine the date fluctuation rate, again and the year fluctuation's rate based on the relations:  $\sigma_E = \sigma'_E \sqrt{N}$  (N expresses stock transaction number of days in one year)[4]. Then the firm's asset volatility, the firm's asset and the firm's asset expectation return rate .Based on the Black-Scholes option theory and Ito's Lemma. The relation of the firm's option and the firm's asset can be evaluated from the equation below:

$$E_t = V_t \Phi(d_1) - B_t e^{-r\tau} \Phi(d_2) \tag{27}$$

$$\sigma_E = \frac{\Phi(d_1) V_t}{E_t} \sigma_V \tag{28}$$

$$d_1 = \frac{\ln\left(\frac{V_t}{B}\right) + \left(r + \frac{1}{2} \sigma_V^2\right) \tau}{\sigma_V \sqrt{\tau}}, d_2 = d_1 - \sigma_V \sqrt{\tau}, \Phi(\cdot) \text{ is the standard normal}$$

distribution. A payment of  $B$ ; the payment time  $\tau$ , the firm's asset volatility,  $\sigma_V$ ; and the firm's first asset value  $V_t$ . We get the result by the Newton algorithm.

3. The expectation rate of return of the firm's asset and the debt [5]:

$$\hat{\mu}_V = \frac{1}{T} \log \frac{V_T}{V_t} + \frac{1}{2} \sigma_V^2 \tag{29}$$

$$\hat{\mu}_B = \frac{1}{T} \log \frac{B_T}{B_t} + \frac{1}{2} \sigma_B^2 \tag{30}$$

The constant diffusion rate  $\sigma_B$  got from the history dates in the accounting table several years by Eviews.

4. The debt's term and the risk-free rate: in this paper, it is estimated that the default probability of the firms in one year, So  $T - t = 1$ . The U.S. federal funds rate is regarded as benchmark interest rates, thus we select federal fund rate 5.25% in 2006.

## 5 Empirical Findings

We get the probability of the default of the firm based on the Merton model and the modified model by the software Matlab7.1.

**Table 1.** The expected default probability and the actual default probability

Ticker symbol	Original model (%)	First modified model (%)	Further modified model(%)	Actual result(%)
SVU	1.76e-004	2.55e-004	2.59e-003	0.35
S	4.80e-003	5.74e-003	3.2e-002	0.35
TSN	6.47e-004	7.61e-004	8.10e-004	0.35
TSO	2.11e-006	2.33e-006	8.72e-006	0.35
GT	1.12e-003	3.13e-003	1.1e-002	0.35
DTV	2.17e-011	4.71e-010	4.80e-007	0.35
X	9.91e-004	4.33e-005	1.98e-003	0.35
ODP	9.10e-009	1.06e-008	1.64e-008	0.35
Q	0.1671	0.2739	0.4225	0.35
LEN	2.61e-004	3.81e-003	3.05e-003	0.35
PHM	3.30e-002	3.67e-002	8.12e-002	0.35
GME	1.30e-010	9.16e-009	4.46e-007	0.35
DVA	2.17e-006	4.93e-007	6.80e-005	0.35
EP	3.73e-002	3.90e-002	0.31	0.35
RSH	1.4381	1.3762	0.1308	0.35
MTW	4.23e-003	4.59e-003	1.85e-002	0.35
STZ	5.28e-014	7.36e-014	8.32e-014	0.35
NYT	8.66e-003	2.83e-002	0.31	0.35
LSI	4.88e-007	8.933e-007	8.90e-003	0.35
MEE	3.08e-003	6.61e-005	0.3440	0.35
FTR	6.53e-003	2.51e-002	0.10369	0.35
MYL	1.16e-007	3.86e-007	4.39e-006	0.35
AIV	1.55e-019	3.42e-014	1.13e-011	0.35
MIL	7.36e-009	3.72e-007	3.98e-007	0.35
AMT	5.34e-010	2.65e-009	7.07e-009	0.35
SWN	2.75e-007	8.11e-007	8.27e-007	0.35
RRC	3.01e-004	4.15e-004	6.77e-003	0.35

The original model presumes that the only source of uncertainty in equity prices is the firm's net asset value. But the above market condition holds only when the market is efficient. From Table 1, the probability of the default calculated by the primitive Merton model is lower than the result evaluated by the first modified model which only extend the assumption that the actual market is non-efficient, the primary cause is the domestic market is non-effective, stock price's uncertainty is influenced by the firm's asset value and the market factor, the firm's net asset value is not the only source of the uncertainty in equity prices. Secondly, by the further modified model which extend the other assumption including that the equity receives no dividends, the default boundary is assumed to be constant the expected probability of default has closed to the actual default probability further, thus the further model which extend some assumption belonging to the original model has closed to the actual market. The result shows that the modified model can evaluate the credit risk well in the non-efficient market.

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