Emergence of Scale-Free Networks with Seceding Mechanism

Xian-Min Geng[1], Guang-Hui Wen[1], Shu-Chen Wan[1], and Jie-Yu Xiong[2]

[1] College of Science, Nanjing University of Aeronautics and Astronautics,
Nanjing 210016, P.R. China
xianmin.g@nuaa.edu.cn
[2] School of Chemistry and Chemical Engineering,
Jiangsu Teachers University of Technology,
Changzhou 213001, P.R. China

Abstract. In order to explore further the underlying mechanism of the scale-free networks, we study stochastic secession as a mechanism for the creation of complex networks. In this evolution the network growth incorporates the addition of new links between existing nodes, the deleting and rewiring of some existing links, and the stochastic secession of nodes. To random growing networks with preferential attachment, the model yields scale-free behavior for the degree distribution. Furthermore, we get the analytical expression of the power law degree distribution with scaling exponent γ ranges from 1.1 to 9. The analytical expressions are in good agreement with the numerical simulation results.

Keywords: Complex network, Seceding mechanism, Scale-free behavior.

1 Introduction

Many social, biological, and communication systems can be well described by complex networks, where the vertices are the elements of system and the edges represent the interactions among them [1-6]. Recently, many researchers have used their own methods to describe the collective behaviors of these networks and empirical results showed that the degree distribution in these networks decays as a power law follows $P(k) \sim k^{-\gamma}$, where γ is the scaling exponent of the networks. These networks are called scale-free networks [1].

In many cases, the scaling exponent γ of such a distribution is larger than 2 and a wide array of models aimed at exploring the responsible mechanism for this behavior have been presented [1,3,4,5,8,9,10]. In contrast, scale-free networks with an scaling exponent $\gamma < 2$ have received relatively little attention in physics and mathematics literature [7], despite their widespread appearance in real world, typical examples including the peer-to-peer Gnutella networks [11,12], e-mail networks [13], traffic in networks [14], food webs [15], coauthorship networks in high energy physics [16] and the networks of dependency among software packages [17,18]. But, what is the mechanism responsible for these real world scale-free networks with an exponent γ ranges from 1 to 4 [1,3]? According to the BA-type growth rule, one node is added to the network at each time

J. Zhou (Ed.): Complex 2009, Part II, LNICST 5, pp. 1973–1983, 2009.

step, intending to mimic the growing process of some real systems. This mechanism gives an explicit description to the real networks growing process and yields scale-free behavior for the degree distribution with an exponent $\gamma = 3$ [1], however, the real networks' growing rule should be much more complex. For example, In the peer-to-peer Gnutella networks [11], the nodes represent the Gnutella servents [19], and the nodes will be considered connected if they can exchange data directly. According to the time evolution, Some Gnutella servents will be shut down or removed from the system. In the food webs [15,20-22], there are both additions and losses of species (vertices) at ecological and evolutionry times scales by means of immigration, emigration, speciation, and extinction [15]. Evidently, the nodes' secession from the system not only take place in the mentioned networks, but also in many other networks [23-30]. Take the internet's Autonomous System map for example, a node is an AS and a link is the relationship between two ASs. An AS adding means a new Internet Service Provider (ISP) joins the Internet. A node's session represents the permanent shutdown of corresponding AS as it is, this may happen due to the bankruptcy of the corporation. Investigations of the evolution of real internet maps from 1997 to 2000 verified such evolution mechanism [25,28-30]. In most cases, the node's secession is also accompanied with the deletion of all edges incident on it [26,27,31]. Nevertheless, models that combine birth and death (addition and secession of nodes) have been studied to a much lesser extent. Chung and Lu [26], Cooper et al. [27] and Deng et al [31] independently, studied a dynamic model that combines the addition of nodes and edges with a uniform deletion of both nodes and edges. These birth-death models have also been found to generate graphs with power-law degree distribution, however, these models evolution just incorporated the addition of new nodes and deletion of nodes.

Motivated by the above mentioned real networks and based on the previous work [26,27,31], we present a general model for the evolution of scale-free networks in which the network evolution is coupled with the addition of new nodes with new links, the addition of new links, the rewiring and deleting of some existing links, and the deleting of an existing nodes. We have proved the model can self-organize into a scale-free networks if the parameters are chosen properly, and the scaling exponent γ of the degree distribution ranges from 1.1 to 9. Furthermore, the analytical expressions are in good agreement with the numerical simulation results.

The rest of this paper is organized as follows: In Section 2, an evolving model of scale-free networks incorporating the nodes' stochastic secession is introduced, which captures an essence of some real networks such as internet's Autonomous System. Then, in Section 3 we performed numerical simulations of the model. Finally, section 4 concluded the investigation.

2 Establishment of the Model

A series of microscopic events can be used to mimic the evolutions of the systems. In this paper,we consider the general model of network evolution with seceding mechanism, each time step, we perform one of the following five operations: the addition of a new node with some new links, the addition of new links between existing nodes, the rewiring and deleting of some already present links, and the deleting of an existing nodes. In this section, we will introduce our model and give the analytical results of the model.

2.1 The Model

The model dynamics starts from an initial configuration of m_0 isolated vertices, denote by $N(t)$ the total number of the nodes at time t, $k_i(t)$ the degree of the node i at time t.

Each time step, we perform one of the following five operations:

(1) With probability p_1 we add l new links: For this we randomly select a node as the starting point of the new link, the other end of the link is selected with probability:

$$\Pi(k_i(t), \alpha) = \frac{k_i(t) + \alpha}{\sum_j (k_j(t) + \alpha)}.$$

This process is repeated l times.

(2) With probability p_2 we rewire n links: For this we randomly select a node i and a link l_{ij} to it. Next we remove this link and replace it with a new link $l_{i'j}$ connect to $i^{'}$, the node $i^{'}$ is selected with the preferential probability $\Pi(k_{i'}(t), \alpha)$. This process is repeated n times.

(3) With probability p_3 we delete c old links: One end of the link is chosen randomly, while the other end is chosen with the anti-preferential probability:

$$\Pi^*(k_i(t), \alpha) = \frac{1}{N(t) - 1}(1 - \frac{k_i(t) + \alpha}{\sum_j (k_j(t) + \alpha)}).$$

$N(t)$ is the total number of the nodes at time t, $(N(t) - 1)^{-1}$ is the normalized coefficient for the probability such that $\sum_i \Pi^*(k_i(t), \alpha) = 1$. Deleting links according to the anti-preferential probability is consistent with the "rich-get-rich" and "poor-get-poor" phenomenon in some real networks. This process is repeated c times.

(4) With probability p_4 we delete a node: For this we delete a node existing in $G(t)$ with anti-preferential probability $\Pi^*(k_i(t), \alpha)$ and remove all the links incident on it.

(5) With probability p_5 we add a new node: We add a new node has $m(\leq m_0)$ new links to the system, each new link with probability $\Pi(k_i(t), \alpha)$ connect to the node i already present in the network. This operation incorporating the fact new links preferentially point to the popular nodes which has a high number of connections.

Since our goal is to investigate the generic mechanism of network evolution, we use bidirectional links. In the model, the probabilities p_1,p_2,p_3,p_4 and p_5 can be varied in the interval $[0,1]$ and $\sum_i p_i = 1$. Note that we choose the probability $\Pi(k_i(t), \alpha)$ proportional to $k_i + \alpha$, and the constant $\alpha \geq 0$ called the efficiency parameter of the network which governs the probability for "young" sites to get new links.

Now we investigate how the network model evolves, what we want to get is the analytical expression of the degree distribution.

Analytic results of the model. We denote by $N(t)$ the total number of nodes at the time t. The node i was added to the system at the time t_i, and $k_i(t)$ is the degree of the node i at the time t. Furthermore, in order to keep the growth of the network, we assume $p_5 m + p_1 l > p_3 c$ and $p_5 > p_4$.

For enough large t, we can get that

(a) $N(t) = m_0 + (p_5 - p_4)t \approx (p_5 - p_4)t$.

(b) $k_i(t_i) = m$. If $s < t_i$, then $k_i(s) = 0$.

(c) We calculate the sum degree $\sum_j k_j(t)$ of the network at time t. If we delete a node at time t, let $D(t)$ is the mean degree of the deleting node. We may get

$$D(t) = \sum_i k_i(t) \frac{1}{N(t)-1}\left(1 - \frac{k_i(t)+\alpha}{\Sigma_j (k_j(t)+\alpha)}\right).$$

From the following inequalities:

$$\max_j k_j(t) \leq t + m_0,$$

$$(\min_j k_j(t)) \sum_i (k_i(t)+\alpha) \leq \sum_i k_i(t)(k_i(t)+\alpha) \leq (\max_j k_j(t)) \sum_i (k_i(t)+\alpha),$$

we have

$$\frac{1}{N(t)-1}\left(\sum_i k_i(t) - \max_j k_j(t)\right) \leq D(t) \leq \frac{1}{N(t)-1}\left(\sum_i k_i(t) - \min_j k_j(t)\right).$$

Hence, $D(t) \sim O(\frac{1}{N(t)-1} \sum\limits_i k_i(t))$. Here, we have used the fact for enough t, $\frac{\max_i k_i(t)}{N(t)-1} \leq 1$. From $\sum\limits_j k_j(t) = \sum\limits_j k_j(t-1) + 2(p_5 m + p_1 l - p_3 c) - p_4 D(t)$, we have

$$\begin{aligned}\sum_j k_j(t) &\approx \sum_j k_j(t-1) + 2(p_5 m + p_1 l - p_3 c) \\ &\approx 2(p_5 m + p_1 l - p_3 c)t.\end{aligned}$$

Namely, $\sum\limits_j k_j(t)$ has the same order with t.

(d)

$$\begin{aligned}\sum_j (k_j(t)+\alpha) &= \sum_j k_j(t) + N(t)\alpha \\ &\approx (2(p_5 m + p_1 l - p_3 c) + (p_5 - p_4)\alpha)t\end{aligned}$$

(e)

$$\Pi(k_i(t),\alpha) \approx \frac{k_i(t)+\alpha}{(2(p_5m+p_1l-p_3c)+(p_5-p_4)\alpha)t}$$
$$\Pi^*(k_i(t),\alpha) \approx \frac{1}{(p_5-p_4)t}(1-\frac{k_i(t)+\alpha}{(2(p_5m+p_1l-p_3c)+(p_5-p_4)\alpha)t})$$

In the model the probability that a node i changes its connectivity $k_i(t)$ depends only on $k_i(t)$ and the quantities characterizing the whole network (the parameters p_1,p_2,p_3,p_4,p_5,m,n,l,c,α and the number of nodes and links). We assume that $k_i(t)$ changes continuously, and the probability $\Pi(k_i(t),\alpha)$ can be interpreted as the rate at which k_i changes. Consequently, the processes (1)-(5) all contribute to $k_i(t)$, each being incorporated in the continuum theory as follow.

(1) Addition of l new links with probability p_1:

$$(\frac{\partial k_i(t)}{\partial t})_{(1)} = p_1\frac{l}{N(t)} + p_1 l(1-\frac{1}{N(t)})\frac{k_i(t)+\alpha}{\sum_j k_j(t)+\alpha}. \tag{1}$$

The first term on the right-hand side corresponds to the random selection of one end of the new link, while the second term reflects the preferential attachment $\Pi(k_i(t),\alpha)$ used to select the other end of the link.

(2) Rewiring of n links with probability p_2:

$$(\frac{\partial k_i(t)}{\partial t})_{(2)} = p_2\frac{-n}{N(t)} + p_2 n(1-\frac{1}{N(t)})\frac{k_i(t)+\alpha}{\sum_j k_j(t)+\alpha}. \tag{2}$$

The first term incorporates the decreasing connectivity of node from which link was removed, and the second term represents the increasing connectivity of the node that link is reconnected to. The total connectivity does not change during the rewiring process.

(3) Deletion of c links with probability p_3:

$$(\frac{\partial k_i(t)}{\partial t})_{(3)} = p_3\frac{-c}{N(t)} + p_3 c(1-\frac{1}{N(t)})\Pi^*(k_i(t),\alpha). \tag{3}$$

The first term corresponds to the random selection of one end from which the link was removed, while the second term reflects the anti-preferential probability $\Pi^*(k_i(t),\alpha)$ used to select the other end of the link.

(4) Deletion of a node in the system with probability p_4:

$$(\frac{\partial k_i(t)}{\partial t})_{(4)} = -p_4\sum_{j\neq i}\Pi^*(k_i(t),\alpha)\frac{k_j(t)}{N(t)-1} - p_4 k_i(t)\Pi^*(k_i(t),\alpha). \tag{4}$$

The first term represents the decreasing connectivity of the node i by deleting its neighbors, while the second term reflects the anti-preferential probability $\Pi^*(k_i(t),\alpha)$ used to delete the node i.

(5)

$$(\frac{\partial k_i(t)}{\partial t})_{(5)} = p_5 m \frac{k_i + \alpha}{\sum\limits_j (k_j + \alpha)}. \tag{5}$$

Each time step t, we perform one of the above five operations, so $k_i(t)$ satisfies the following dynamical equation:

$$\begin{aligned} \frac{\partial k_i(t)}{\partial t} &= (\frac{\partial k_i(t)}{\partial t})_{(1)} + (\frac{\partial k_i(t)}{\partial t})_{(2)} \\ &\quad + (\frac{\partial k_i(t)}{\partial t})_{(3)} + (\frac{\partial k_i(t)}{\partial t})_{(4)} + (\frac{\partial k_i(t)}{\partial t})_{(5)} \\ &= a\frac{k_i(t)}{t} + b\frac{1}{t}, \end{aligned} \tag{6}$$

where

$$a = \frac{p_5 m + p_1 l + p_2 n}{2(p_5 m + p_1 l - p_3 c) + (p_5 - p_4)\alpha} - \frac{p_4}{(p_5 - p_4)}, \tag{7}$$

$$b = \frac{(p_5 m + p_1 l + p_2 n)\alpha}{2(p_5 m + p_1 l - p_3 c) + (p_5 - p_4)\alpha} + \frac{p_1 l - p_2 n - 2p_3 c}{p_5 - p_4} - \frac{2p_4(p_5 m + p_1 l - p_3 c)}{(p_5 - p_4)^2} \tag{8}$$

For enough large t and the initial condition $k_i(t_i) = m$, equation (6) has the following solution:

$$k_i(t) = (m + \frac{b}{a})(\frac{t}{t_i})^a - \frac{b}{a}. \tag{9}$$

We may choose parameters properly such that $a > 0$, $am + b > 0$ and $p_5 m + p_1 l > p_3 c$, then connectivity distribution $P(k)$ can be determined analytically. Defining the unit of time the model as one growth/rewire/new link deleting node/link attempt, then t_i follows the uniform distribution over interval $(0, (p_5 - p_4)t)$, we get

$$P(k_i(t) < k) = 1 - \frac{1}{p_5 - p_4}(\frac{am + b}{ak + b})^{\frac{1}{a}}. \tag{10}$$

So we get degree distribution

$$P(k) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{1}{p_5 - p_4}(am + b)^{\frac{1}{a}}(ak + b)^{-\gamma} \qquad (t \to \infty), \tag{11}$$

where

$$\gamma = 1 + \frac{1}{a}. \tag{12}$$

To be able to calculate $P(k)$ we must have

$$0 < \frac{1}{p_5 - p_4}(am + b)^{\frac{1}{a}} < 1, \tag{13}$$

and

$$0 < \frac{1}{p_5 - p_4}(am+b)^{\frac{1}{a}}(ak+b)^{-\gamma} < 1, \tag{14}$$

therefore, we let $k \geq \max\left(\frac{am+b}{a(p_5-p_4)^a}, \frac{1}{a}\left(\frac{am+b}{(p_5-p_4)^a}\right)^{\frac{1}{a+1}} - \frac{b}{a}\right)$.

Equations (6) and (11) are valid only when $a > 0$, $am+b > 0$ and $p_5m+p_1l > p_3c$, which for fixed p_5 and p_4, translates into $p_5 - p_4 > 0$. For $p_5 - p_4 > 0$, $P(k)$ is given by Eq. (11), thus the degree distribution is scale-free. For $p_5 - p_4 = 0$, Eq. (6) and Eq. (11) are not valid, the continuum theory fails to predict the behavior of the system.

For the different networks, the parameters are different. The above analysis indicates that the model can self-organizes into a scale-free network, if the parameters are chosen properly. Particularly, we can get the following cases:

(1). If $p_4 = 0$ this model reduces to the scale-free model investigated in Ref. [10], we get $\gamma = 1 + \frac{p_5m+p_1l+p_2n}{p_5\alpha+2(p_5m+p_1l-p_3c)}$ which is the same as the result of Ref. [10].

(2). If $p_3 = 0$, $p_4 = 0$, $n = l = m$ and $\alpha = 1$, this model is the same as the scale-free model studied in Ref. [8] and the exponent $\gamma = 1 + \frac{p_5+2m(p_5+p_1)}{m}$ which is the same as we get from the Ref.[8].

(3). If $p_1 = p_2 = p_3 = p_4 = 0$, $\alpha = 0$, then the model is topologically equivalent to the Barabási-Albert model [1] and the scaling exponent $\gamma = 3$ is recovered.

Scale-free regime. In this model, while a power-law tail is present in any point of this regime, the scaling is different from that predicted by the simpler scale-free model [1,8]. Furthermore, the exponent γ, characterizing the tail of $P(k)$ for k, changes continuously with p_1,p_2,p_3,p_4,p_5,m,n,l,c and α, predicting a range of exponent between 1.1 and 9. This allows us to account for the wide variations seen in real networks, for which γ varies from 1.09 to 4.0 [1,3].

3 Numerical Simulation

Along with the previous analytical discussion, we have performed numerical simulations of the presented model in order to investigate its topological properties with a direct statistical analysis. Through performing the following four simulations: Fig. 1, we performed numerical simulations of networks generated by fixing $t = 100000$, $m_0 = 100$, $m = 10$, $l = 1$, $n = 1$, $c = 1$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.05$, $p_4 = 0.05$, $p_5 = 0.5$, $\alpha = 1$. The analytical expressions are well consistent with the simulation results, and we get the scaling exponent $\gamma = 3.6175$ by Eq.(12). Next, we changed some parameters of the model, the results are reported in Fig. 2, Fig. 3 and Fig. 4, respectively. The analytical expressions are in good agreement with the numerical simulation results.

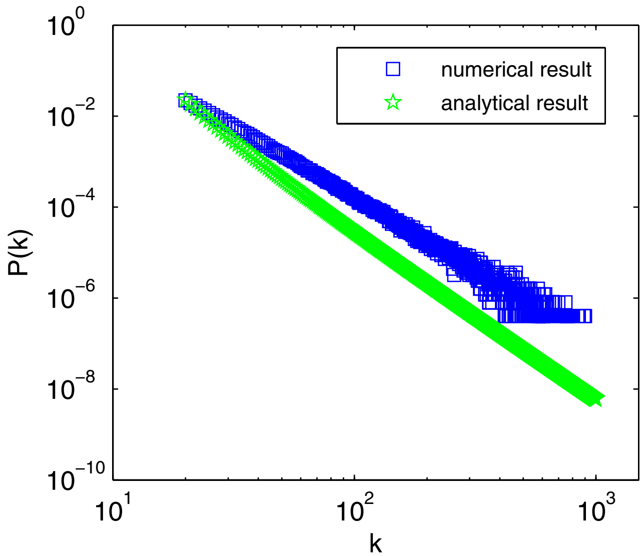

Fig. 1. Probability distribution of degree $P(k)$. The data are averaged over 50 networks of evolution time $t = 100000$, $m_0 = 100$, $m = 10$, $l = 1$, $n = 1$, $c = 1$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.05$, $p_4 = 0.05$, $p_5 = 0.5$, $\alpha = 1$, scaling exponent γ=3.6175 by Eq.(12).

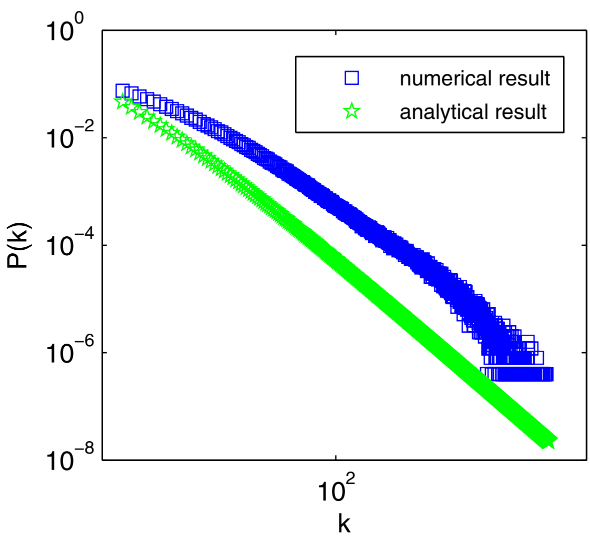

Fig. 2. Probability distribution of degree $P(k)$. The data are averaged over 50 networks of evolution time $t = 100000$, $m_0 = 100$, $m = 10$, $l = 10$, $n = 5$, $c = 1$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.05$, $p_4 = 0.05$, $p_5 = 0.5$, $\alpha = 1$, scaling exponent γ=3.4464 by Eq.(12).

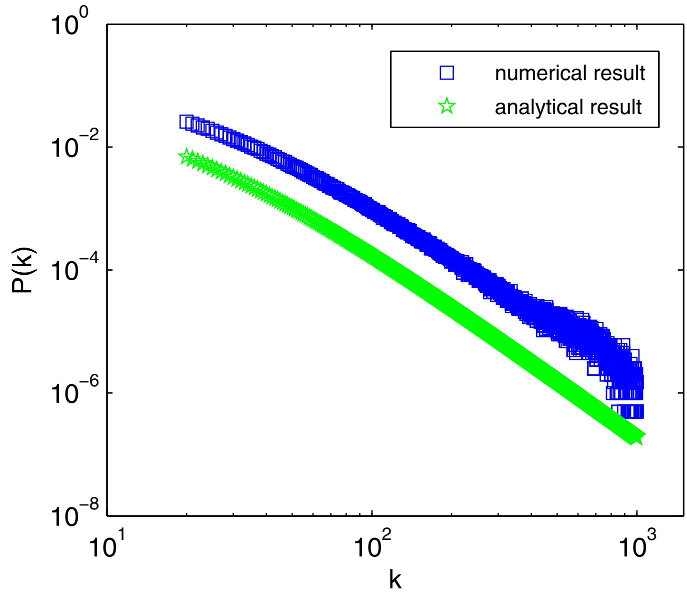

Fig. 3. Probability distribution of degree $P(k)$. The data are averaged over 50 networks of evolution time $t = 100000$, $m_0 = 100$, $m = 10$, $l = 10$, $n = 3$, $c = 1$, $p_1 = 0.4$, $p_2 = 0.15$, $p_3 = 0.03$, $p_4 = 0.02$, $p_5 = 0.4$, $\alpha = 1$, scaling exponent γ=3.1499 by Eq.(12).

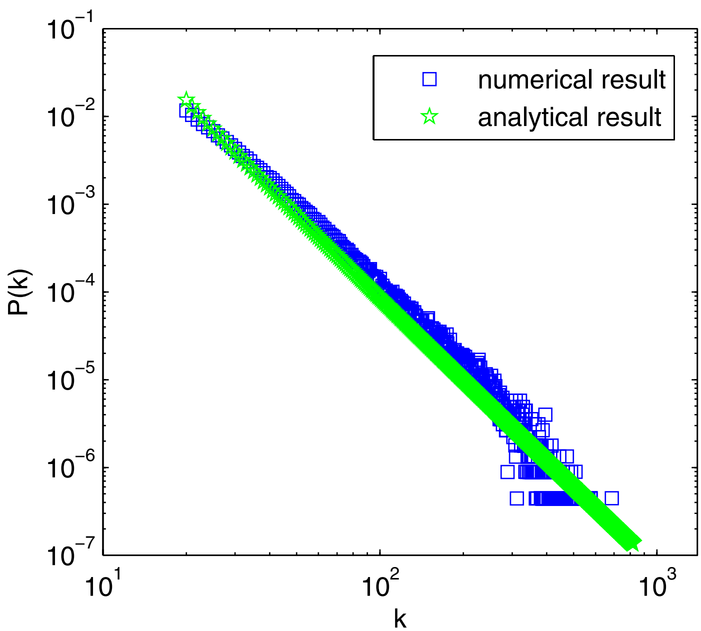

Fig. 4. Probability distribution of degree $P(k)$. The data are averaged over 50 networks of evolution time $t = 100000$, $m_0 = 100$, $m = 5$, $l = 5$, $n = 5$, $c = 1$, $p_1 = 0.25$, $p_2 = 0.20$, $p_3 = 0.05$, $p_4 = 0.05$, $p_5 = 0.45$, $\alpha = 1$, scaling exponent γ=3.0348 by Eq.(12).

4 Concluding Remarks

In this paper, we have introduced a general model of network evolution which gives more realistic descriptions of the local processes than Barabási-Albert model. The model incorporate the five local processes: the addition of a new node with new links, new links between old nodes, the rewiring and the deleting of some existing links, and nodes' stochastic secession. It has been found that, with the introduction of nodes' secession, network structure was significantly transformed. To growing networks with preferential attachment, the model self-organized into the scale-free network with an exponent γ ranges from 1.1 to 9, which recovered most of the real networks.

Acknowledgements

The authors thank Academician Zhiming Ma deeply for his much helpful suggestions.

References

1. Barabási, A.-L., Albert, R.: Emergence of scaling in random networks. Science 286, 509–512 (1999)
2. Strogatz, S.H.: Exploring complex networks. Nature 410, 268–276 (2001)
3. Albert, R., Barabási, A.L.: Statistical mechanics of complex networks. Rev. Mod. Phys. 74, 47–97 (2002)
4. Dorogovtsev, S.N., Mendes, J.F.F.: Evolution of networks. Adv. Phys. 51, 1079–1187 (2002)
5. Newman, M.E.J.: The structure and function of complex networks. SIAM Rev. 45, 167–256 (2003)
6. Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., Hwang, D.-U.: Structure and Dynamics of Complex Networks. Phys. Rep. 424, 175 (2006)
7. Seyed-allaei, H., Bianconi, G., Marsili, M.: Scale-free networks with an exponent less than two. Phys. Rev. E 73 (2006)
8. Albert, R., Barabási, A.L.: Topology of Evolving Networks: Local Events and Universality. Phys. Rev. Lett. 85, 5234–5237 (2000)
9. Chen, Q.H., Shi, D.H.: The modeling of scale-free networks. Physica A 335, 240 (2004)
10. Geng, X.M., Li, Q.: Random Models of Scale-Free Networks. Physica A 356, 554 (2004)
11. Gnutella network is an application that runs on a personal computer and shares files with other users across the Internet. The networks work by connecting individual computers together to share files instead of having to go through a central server
12. Jovanovic, M., Annexstein, F., Berman, K.: Tech. Rep., University of Cincinnati (2001), `http://www.ececs.uc.edu/~annexste/Papers/scalabilityissues.ps`
13. Ebel, H., Mielsch, L.I., Bornholdt, S.: Scale-free topology of e-mail networks. Phys. Rev. E 66, 035103 (2002)
14. Toroczkai, Z., Bassler, K.: Jamming is limited in scale-free systems. Nature 428, 716 (2004)

15. Dunne, J.A., Williams, R.J., Martinez, N.D.: Food-web structure and network theory: The role of connectance and size. Pro. Natl. Acad. Sci. USA 99, 12917 (2002)
16. Newman, M.E.J.: Scientific collaboration networks. I. Network construction and fundamental results. Phys. Rev. E 64, 016131 (2001)
17. Newman, M.E.J.: Mixing patterns in networks. Phys. Rev. E, 67, 026126 (2003)
18. Valverde, S., Solé, R.V.: Hierarchical Small Worlds in Software Architecture, e-print: cond-mat/0307278
19. `http://www9.limewire.com/developer/gnutell_protocol_0.4.pdf`
20. Montoya, J.M., Solé, R.V.: Small World Patterns in Food Webs. J. Theor. Biol. 214, 405–412 (2002)
21. Camacho, J., Guimerà, R., Amaral, L.A.N.: Robust Patterns in Food Web Structure. Phys. Rev. Lett. 88, 228102 (2002)
22. Amaral, L.A.N., Scala, A., Barthélémy, M., Stanley, H.E.: Classes of small-world networks. Pro. Natl. Acad. Sci. USA 97, 11149 (2000)
23. Lawrence, S., Giles, C.L.: Searching the World Wide Web. Science 280, 98–100 (1998)
24. Huberman, B.A., Adamic, L.A.: Internet: Growth dynamics of the World-Wide Web. Nature 406, 131 (1999)
25. Vázquez, A., Pastor-Satorras, R., Vespignani, A.: Large-scale topological and dynamical properties of the Internet. Pres. Rev. E 65, 066130 (2002)
26. Chung, F., Lu, L.: Coupling Online and Offline Analyses for Random Power Law Graphs. Internet Math. 1, 409–461 (2004)
27. Cooper, C., Frieze, A., Vera, J.: Random Deletion in a Scale-Free Random Graph Process. Internet Math. 1, 463–483 (2004)
28. Cheng, G.R., Fan, Z.P., Li, X.: Modelling the complex Internet topology. In: Vattay, G., Kocarev, L. (eds.) Complex Dynamics in Communication Networks, pp. 213–235. Springer, Heidelberg (2005)
29. Goh, K.-I., Kahng, B., Kim, D.: Fluctuation-Driven Dynamics of the Internet Topology. Phys. Rev. Lett. 88, 108701 (2002)
30. Chen, Q., Chang, H., Govindan, R., Jamin, S.: The origin of power laws in Internet topologies revisited. In: Proceedings of the 21st Annual Joint Conference of the IEEE Computer and Communications Societies, pp. 608–617. IEEE Computer Society, Los Alamitos (Silver Spring 2002)
31. Deng, K., Zhao, H.P., Li, D.J.: Statistical Mechanics and its Applications. Physica A 379, 714 (2007)