# An Approach to Enhance Convergence Efficiency of Self-propelled Agent System

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**Abstract.** In this paper, we investigate a weighted self-propelled particles system, wherein each agent's direction is determined by its spatial neighbors' directions with exponential weights concerning the neighbor numbers. In order to describe the fact that some agent with more neighbors might have much larger influence on its neighbors, we introduce a scaling exponent of the neighbor number between 0 and $\infty$. As the exponent increases, i.e., the effect of weight becomes stronger, the network of agents becomes much easier to achieve direction consensus in our simulation. Especially, when the exponent equals to 1, the convergence efficiency is enhanced.

**Keywords:** dynamic network, swarm, topological structure, convergence time, degree of consensus.

## 1 Introduction

Biological swarms are ubiquitous in nature, such as herds of quadrupeds, schooling of fish and flocking of birds, etc. From biology to physics, there has been a long standing interest in swarm research [1, 2, 3, 4]. Reynolds created the first computer simulation of flocking with three heuristic rules known as cohesion, separation and alignment [5, 6], and named it as Biod model. In 1995, Vicsek et al. performed research on the condition of alignment, and provided a simplified version of the Boid model [7]. In the Vicsek model, $N$ self-propelled agents are driven towards different directions with a constant absolute velocity in a squared zone. At each time step, every agent updates its direction according to the average direction of agents' motion in its neighborhood of radius $R$. The simulation results in the literature [7] illustrates the fact when the density of the system is high and the noise is small enough, all agents will converge to the same direction on a macroscopic scale.

In recent years, as development of research in distributed sensor network [8], unmanned aerial vehicles [9], underwater vehicles [10] and attitude alignment for clusters of satellites, etc., Vicsek model as a fundamental model of consensus has triggered widespread concern. The theoretical explanations [11, 12, 13, 14, 15, 16, 17] demonstrate the sufficient conditions for coordination of the Vicsek

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model which are given in terms of a family of undirected neighbor graphs. In the standard Vicsek model, the influencing radius and absolute velocity are fixed and invariable. The influencing radius is randomly chosen according to a power-low distribution in Ref. [18], which can make the network heterogeneous and enhance the convergence efficiency. The agent updates its direction and speed adaptively according to the degree of consensus among its neighbors in Ref. [19] while according to the local order parameter of consensus and the global consensus in Ref. [20]. All the above methods and strategies are utilized to enhance the convergence efficiency.

At present, research of complex network demonstrates that kinds of network's topological structures are inhomogeneous [21, 22], i.e., the degree of some node is much larger than others. These nodes affect the structure and the dynamic process of network a lot. For instance, in the Word Wide Web, the new web site always connects with the web sites of larger degree; in the financial network and the traffic network, some hub nodes destroyed may lead to paralysis of the whole network; in the animal swarming, very few individuals within fish schools are known to be able to influence the group [24]. In the Vicsek model, although influencing radius is the same, the neighbor number of each agent is different. Increasing the radius of some agents can make the network heterogeneous and enhance the convergence efficiency [18], however, for limited perception capability of animal and cost as well as technology of the actual multi-agent system(such as robot formation and moving sensor network), the influencing radius should be confined to a small range.

In this paper, we propose a weighted self-propelled agent system, wherein the weight is determined by an exponent form of each agent's neighbor number. The direction of each agent is updated by the weighted average directions of its neighbor, instead of its neighbors's average direction. The aim of this work is to accelerate convergence and its degree of the self-propelled agent system, according to the position and interaction of each agent in the network. The simulation results demonstrate that the convergence time and the degree of consensus increase when exponent is equal to 1. Furthermore, convergence efficiency in the self-propelled particles system is enhanced as the exponent increases.

## 2 The Vicsek Model

In the Vicsek model, a group of $N$ agents are moving in a $L \times L$ square with the same and constant speed but towards different directions. At initial time, the agents are randomly distributed, and their initial directions are also uniformly distributed in the interval $[0, 2\pi)$. At each time step, the direction of each agent is determined by the average directions of all the agents within a circle centered at the given agent, whose influencing radius is $R$. At time $t$, the position of a specific agent is updated according to:

$$x_i(t+1) = x_i(t) + v_0 e^{\mathrm{i}\theta_i(t)} \tag{1}$$

And its direction is updating as:

$$e_i^{\theta}(t+1) = e^{\mathrm{i}\triangle\theta_i(t)} \frac{\sum\limits_{j\in\Gamma_i(t+1)} e^{\mathrm{i}\theta_j(t)}}{\|\sum\limits_{j\in\Gamma_i(t+1)} e^{\mathrm{i}\theta_j(t)}\|_2} \tag{2}$$

where $\triangle\theta_i$ denotes the white noise (in this paper, we only consider $\triangle\theta_i = 0$), $e^{i\theta_i(t)}$ denotes unit directional vector, and $\Gamma_i(t+1)$ is the set of neighbors for agent $i$ at time step $t+1$.

In order to measure the degree of consensus for all the agents, an order parameter is introduced as [7, 23]:

$$V_\alpha = \frac{1}{N}\|\sum_{i=1}^{n} e^{\mathrm{i}\theta_i(t)}\|_2, 0 \leq V_\alpha \leq 1 \tag{3}$$

A larger value of $V_\alpha$ indicates a better consensus, especially when $V_\alpha = 1$, all the agents moving in the same direction. Numerical stimulation claims that, in the circumstances of high density and low noise, all the particles will definitely approach to the consensus state, namely having the same direction of velocity after definite time steps(convergence time) [24, 25].

Due to the limitation of influencing radius $R$, each agent can only communicate with a certain part of agents in the range of its radius and change its direction according to this local information. Besides, the absolute velocity $v_0$ determines the changing frequency of neighboring agents. Different influencing radius and absolute velocities will give rise to diverse convergence time and degree of consensus, which will be numerically investigated in this paper.

In this paper, two aspect of convergence efficiency are defined as:

**Definition 1.** *Convergence time $t$: when the direction of all the agents in the system become constant or the sum of change is small enough, the time steps have past is defined as convergence time.*

**Definition 2.** *Degree of consensus: To evaluate the degree of direction consensus among agents, there are two indexes as follows: the first one is shown in Equ.(3);the other is measured in terms of a ratio, that is, when the model evolves to a steady state, the proportion occupied by agents with the same direction of the total number of agents in the network.*

## 3 The New Approach

### 3.1 The Neighbor Number Weighted Model

The degree is an essential variable in complex network as the neighbor number in the self-propelled particle system. The agent with more neighbors might have much larger influences on its neighbors. Thus, we introduce a weight for each agent into Vicsek model when its direction is updated. We propose that each agent influences its neighbors according to the neighbor number of a given agent, then at time $t$, its direction is as:

$$e^{\mathrm{i}\theta_i(t+1)} = \frac{\sum\limits_{j\in\Gamma_i(t+1)} \Upsilon_j^{(1)}(t)e^{\mathrm{i}\theta_j(t)}}{\|\sum\limits_{j\in\Gamma_i(t+1)} \Upsilon_j^{(1)}(t)e^{\mathrm{i}\theta_j(t)}\|_2} \tag{4}$$

$$\sum_{j\in\Gamma_i(t+1)} \Upsilon_j^{(1)}(t) = 1, \Upsilon_j^{(1)} \geq 0, j \in \Gamma_i(t+1) \tag{5}$$

where $\Upsilon_j^{(1)}$ denotes the weight of the neighbor $j$ of agent $i$ at step $t$, and it is defined as

$$\Upsilon_j^{(1)}(t) = \frac{n_j(t)}{\sum\limits_{k\in\Gamma_i(t+1)} n_k(t)} \tag{6}$$

where $n_j(t)$ denotes the neighbor number of agent $i$ at time step $t$.

### 3.2 The Amplified Model

The neighbor number weighted model is presented in section 3.1, which can enhance the convergence efficiency shown in Fig.1 and Fig. 2. If the weight of an agent with more neighbors enlarges, the convergence efficiency will be enhanced compared with the standard Vicsek model. In order to enlarge the weight of agent with more neighbors, we propose a general equation of weight as

$$\Upsilon_j^{(\lambda)}(t) = \frac{n_j^{\lambda}(t)}{\sum\limits_{k\in\Gamma_i(t+1)} n_k^{\lambda}(t)} \tag{7}$$

When $\lambda = 0$, Eq.7 denotes the standard Vicsek model. When $\lambda > 1$, the effect of neighbor number as weight is amplified. On the other hand, when $\lambda < 1$, the effect of neighbor number as weight is weakened. When $\lambda \to +\infty$, a new model is proposed as

$$e^{\mathrm{i}\theta_i(t+1)} = \lim_{\lambda\to+\infty} \frac{\sum\limits_{j\in\Gamma_i(t+1)} \Upsilon_j^{(\lambda)}(t)e^{\mathrm{i}\theta_j(t)}}{\|\sum\limits_{j\in\Gamma_i(t+1)} \Upsilon_j^{(\lambda)}(t)e^{\mathrm{i}\theta_j(t)}\|_2} = \frac{\sum\limits_{j\in\Omega_i(t+1,1)} e^{\mathrm{i}\theta_j(t)}}{\|\sum\limits_{j\in\Omega_i(t+1,1)} e^{\mathrm{i}\theta_j(t)}\|_2} \tag{8}$$

where $\Omega(t+1,1)$ denotes the set of the maximum neighbors of the neighbors of agent $i$.

The rule of updating direction in the amplified model is as follows: the direction of each agent is determined by the average velocity direction of agents who own the most neighbors among the circle with influencing radius and centered at the given agent at each time step illustrated in Eq.8.

## 4 Simulations and Discussions

In order to compare the effect of the standard model with the weighted model, we consider $N$ agents moving in the whole complex $2D$ plane without boundary restrictions [19] instead of in a rectangle with open boundary or periodic boundary conditions [11]. Numerical results of convergence time $t$ based on Vicsek model when $\triangle\theta_i = 0$ are shown in Fig. 1(a). Fig. 1(b) shows the convergence time $t'$ when the parameter $\lambda$ is equal to 1. Apparently, the current motion protocol with the neighbor numbers as weight for updating directions can lead to faster consensus, that is, the ratio $t/t'$ is larger than 1, and it ranges from 1.3 to 2 in our numerical simulation.

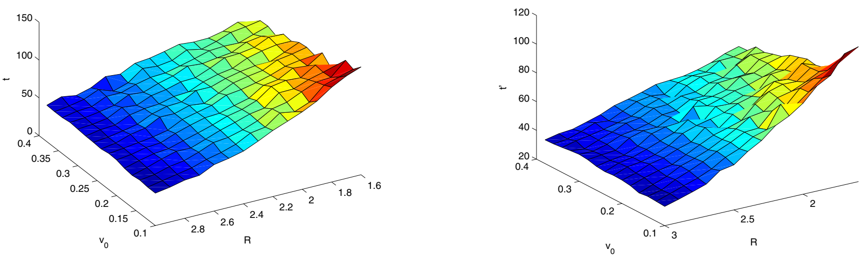


**Fig. 1.** (color online) (a) Plots of convergence time $t$ on the velocity $v_0$ and influencing radius $R$. (b) The convergence time $t'$ as a function of velocity $v_0$ and influencing radius $R$. We assume that all the agents move in a squared plane of size $L = 10$, the number of agents $N = 200$, and parameters $R$ and $v_0$ vary from 1.2 to 3 and from 0.01 to 0.1, respectively. All the data points above are obtained by averaging over 300 different realizations.

As is shown in Fig. 1, the new approach could get to consensus with higher efficiency. The self-propelled particle system required the least convergence time at the point $R = 3$ and $v_0 = 0.1$, while the most at point $R = 1.6$ and $v_0 = 0.1$.

For a fixed $v_0$, we can see that $t$ is a decreasing function of $R$, due to the fact that bigger $R$ means better connection.

For a fixed $R$, we can see that when $R$ is larger, the convergence time is an increasing function of $v_0$, while $R$ is small, the convergence time is a decreasing function of $v_0$. The law is caused by the following reasons: As is shown in Fig. 2(a), the degree of consensus is an increasing function of $R$ and a decreasing function of $v_0$ with respect to fixed $v_0$ and fixed $R$. However, the impact of $R$ is stronger than $v_0$; thereby when $R$ is smaller, it is not easier to reach consensus, and the self-propelled particle system becomes no easier to gain consensus as $v_0$ increases. Nevertheless, the agent in the self-propelled particle system with small $R$ is easy to achieve convergence. On the other hand, when $R$ is larger and as $v_0$ enlarges, the self-propelled particle system is more difficult to obtain consensus, and thus more convergence time is required.

Numerical results of the degree of consensus when $\triangle\theta_i = 0$ are shown in Fig. 2(a) and Fig. 3(a). Fig. 2(b) and Fig. 3(b) reports the results of the degree of

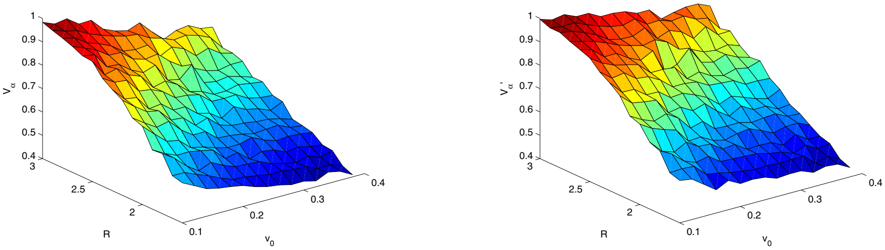


**Fig. 2.** (color online) (a) Plots of the degree of consensus $V_\alpha$ on the velocity $v_0$ and influencing radius $R$. (b) The degree of consensus $V'_\alpha$ as a function of velocity $v_0$ and influencing radius $R$. We assume that all the agents move in a squared plane of size $L = 10$, the number of agents $N = 200$, and parameters $R$ and $v_0$ vary from 1.2 to 3 and from 0.01 to 0.1, respectively. All the data points above are obtained by averaging over 300 different realizations.

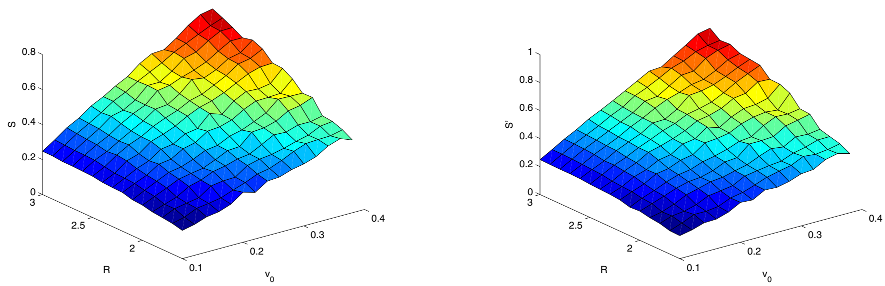


**Fig. 3.** (color online) (a) Plots of the degree of consensus $V_\alpha$ on the velocity $v_0$ and influencing radius $R$. (b) The degree of consensus $V'_\alpha$ as a function of velocity $v_0$ and influencing radius $R$. We assume that all the agents move in a squared plane of size $L = 10$, the number of agents $N = 200$, and parameters $R$ and $v_0$ vary from 1.2 to 3 and from 0.01 to 0.1, respectively. All the data points above are obtained by averaging over 300 different realizations.

consensus when the parameter $\lambda$ is equal to 1. Apparently, the current motion protocol with the neighbor numbers as weight for updating directions can lead to higher degree of consensus, that is, the ratio $V'/V$ is larger than 1.

As is shown in Fig. 4(a), convergence time $t$ decreases as the exponent $\lambda$ or influencing radius $R$ increases. In terms of a fixed exponent $\lambda$, we can see that $t$ is a decreasing function of $R$, due to the fact that increasing $R$ improves the connectivity of the network. Furthermore, for any given value of the influencing radius $R$, $t$ is a decreasing function of $\lambda$, implying that if the effect of neighbor number as weight becomes stronger, it will be faster to obtain consensus. We also computed the degree of consensus when the self-propelled particle system gets to convergence. Fig. 4(b) shows the degree of consensus $V_\alpha$ as a function of the exponent $\lambda$ with various influencing radius $R$. This implied that the degree

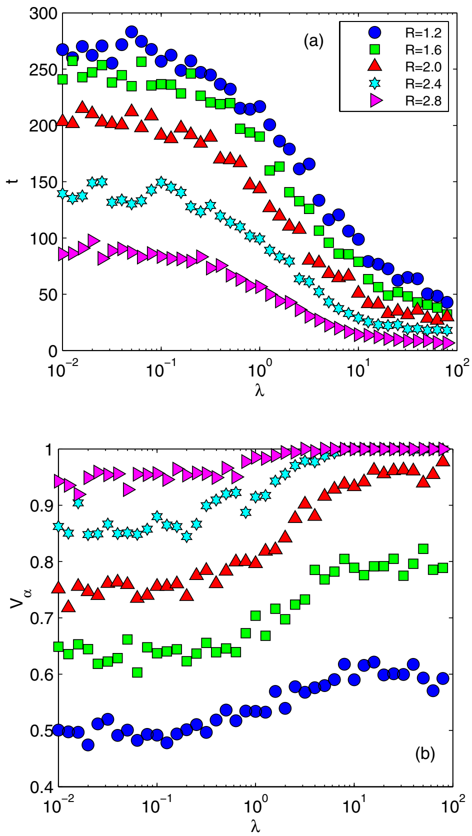


**Fig. 4.** (color online) (a) The convergence time $t$ as a function of the exponent $\lambda$ for different influencing radius $R$. (b) The degree of consensus $V_\alpha$ as a function of the exponent $\lambda$ for different influencing radius $R$. All the data points above are obtained by averaging over 300 different realizations.

of consensus is increasing as the effect of neighbor number as weight becomes stronger or network has better connectivity.

## 5 Conclusion

The collective dynamics of intelligent multi-agents is not only the common phenomena in nature, but also is required in-depth investigation in engineering. Utilizing the topology structure of mobile network to enhance the convergence efficiency is rarely reported. In this paper, we introduce the weight based on the size of neighborhood. The simulation results show that this approach can accelerate consensus process and improve convergence efficiency. Furthermore, when the exponent is increasing, the contribution of the weight is larger, the self-propelled particle system is much easier to obtain consensus.

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