# Epidemic Self-synchronization in Complex Networks

Ingo Scholtes[1], Jean Botev[1], Markus Esch[2], and Peter Sturm[1]

[1] Systemsoftware and Distributed Systems
University of Trier, D-54286 Trier, Germany
scholtes@syssoft.uni-trier.de
http://syssoft.uni-trier.de

[2] Faculty of Sciences, Technology and Communication, University of Luxembourg,
L-1359 Luxembourg-Kirchberg, Luxembourg
markus.esch@uni.lu

**Abstract.** In this article we present and evaluate an epidemic algorithm for the synchronization of coupled Kuramoto oscillators in complex network topologies. The algorithm addresses the problem of providing a global, synchronous notion of time in complex, dynamic Peer-to-Peer topologies. For this it requires a periodic coupling of nodes to a single random one-hop-neighbor. The strength of the nodes' couplings is given as a function of the degrees of both coupling partners. We study the emergence of self-synchronization and the resilience against node failures for different coupling strength functions and network topologies. For Watts/Strogatz networks, we observe critical behavior suggesting that small-world properties of the underlying topology are crucial for self-synchronization to occur. From simulations on networks under the effect of churn, we draw the conclusion that special coupling functions can be used to enhance synchronization resilience in power-law Peer-to-Peer topologies.

**Keywords:** Self-Synchronization, Networks, Coupled Oscillators, Kuramoto Model, Peer-to-Peer.

## 1 Introduction

Self-synchronization phenomena have been observed in various branches of science, ranging from biology, physics, astronomy or chemistry to computer science. During the last decades, several models have been proposed which are suitable to explain the self-synchronizing behavior of multi-agent systems like interacting cells, celestial bodies, insects or neurons. Among these, the Kuramoto model [13] for coupled oscillators is among the most interesting ones since it has proven to be exactly solvable despite considering a large set of non-linearly coupled entities.

Following the discovery and analysis of such models, an important question is if and how these can be utilized in engineered systems. Since they address a large number of constituents with no centralized control, complex Peer-to-Peer topologies are an interesting domain for the application of self-organization

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principles. In such systems, self-synchronizing behavior can be used to provide a network-wide synchronous chronology of states without requiring central coordination. Such a synchrony facilitates distributed algorithms requiring a number of peers to synchronously perform concerted actions and can e.g. be utilized by round-based protocols [2]. However, when applying self-synchronization models to real-world Peer-to-Peer systems, special requirements need to be taken into account. First of all, their decentralized character prohibits the usage of models that involve an all-to-all coupling as e.g. adopted in the original Kuramoto model. Populations of locally interacting nodes need to be considered instead. Considering topologies with highly diverse vertex degrees (like e.g. scale-free networks), even the applicability of models that consider a coupling to all nearest neighbors appears questionable as it would seriously overburden network hubs. Furthermore, other than those considered in common synchronization models, Peer-to-Peer topologies are characterized by high rates of node fluctuations and constant topology changes. In order to be applicable in real-world scenarios, it is therefore important to assess the resilience of synchronization mechanisms against this kind of dynamics.

In this article we investigate an epidemic algorithm for synchronizing complex networks of coupled Kuramoto oscillators. For this, we consider populations of nodes with Gaussian distributed frequencies and equally distributed phase skews. The synchronization algorithm involves nodes to exchange their current oscillator phase with a single random neighbor in regular intervals. Different coupling strengths based on the degrees of both coupling partners are considered. Having presented some background of research on self-synchronization in section 2, a more detailed description of our algorithm and some basic assumptions will be given in section 3. While not giving formal arguments for the emergence of self-synchronization, we argue that the algorithm is informally justified by the application of mean-field arguments in Kuramoto's original model and the convergence of epidemic aggregation in complex networks [11]. Furthermore, in section 4 we present simulation results which indicate that the algorithm provides for a self-organized synchronization in small-world network topologies.

## 2 Background

The phenomenon of self-synchronization has been extensively studied especially in the field of biology where it appears to be almost pervasive. Its occurrences range from neuronal activation patterns, the synchronization of cardiac cells to the flashing of fireflies [23]. One of the earliest models to explain self-synchronizing phenomena in biology is based on the concept of frequency pulling, i.e. oscillators pulling on each other's intrinsic frequencies. This model has e.g. been investigated in [25] in the context of neuronal cells. Considering oscillator interaction as phase-dependent influence and sensitivity functions, the *generalized relaxation oscillator* model introduced in [26] has opened the problem to formal analysis. In [13], a variant with sinusoidal all-to-all coupling has been solved exactly despite its non-linear nature. Formal analysis of this so-called Kuramoto model is facilitated by the application of ideas from mean-field theory, i.e. a single oscillator can be looked at as being influenced by the average phase of all other oscillators. In the subsequent years, chains and rings of oscillators with a nearest-neighbor-coupling have been studied as well [8] [6].

More recently, nearest-neighbor couplings have been investigated in complex network topologies. In [10], a graph theoretical reformulation of Kuramoto's model is studied and the critical coupling constant is found to depend on a graph's Laplacian matrix. In [5], it has been shown that synchronizability benefits from disassortative networks in which high degree nodes are connected to low degree ones. Most recently, asymmetric couplings that are based on a node's degree have been studied for complex network topologies [18] [19] [15].

A special class of models inspired by biological systems is called "Integrate-and-Fire". Here a state variable - e.g. a voltage - builds up until a threshold value is exceeded which leads to a sudden discharge. This discharge advances the state variables of neighbor nodes. Such a model for self-synchronization has e.g. been proposed by [20] to explain synchrony in cardiac pacemaker cells. In [17] this model has been generalized, showing that systems with a monotonic and concave down evolution of local state variables will eventually synchronize. An adaptive variation of this model has been studied in [7].

These discoveries have paved the way for the application of self-synchronization mechanisms in engineered networks. For small-scale sensor network topologies, the self-synchronizing behavior of pulse-coupled oscillators has e.g. been investigated by [16]. A self-organized synchronization of heartbeat events in the Peer-to-Peer topology Newscast [12] has been proposed in [2]. Closest in scope to our work, this algorithm is based on the adaptive Ermentrout model [7] and assumes a coupling of nodes to a continuously updated random sample of nodes from the entire network.

Generally, a network's topology has great influence on the synchronization behavior of nearest-neighbor-coupled oscillators. In [24] a model for graphs of varying randomness and small-world characteristics is introduced: The authors mention that self-synchronization in small-world networks of coupled oscillators emerges about as fast as in Kuramoto's mean-field model although these networks have significantly fewer edges. This result has been substantiated by a formal analysis of $n$ Rössler systems performed in [4], showing that the number of required edges for self-synchronization to occur is especially low for small-world graphs.

Finally, our work has been inspired by the concept of epidemic aggregation. In [11], it has been shown that a set of aggregate functions can be efficiently calculated in large dynamic networks based on periodic information exchanges with random nodes. A study of this scheme in different networks has shown that it works efficiently in small world topologies.

## 3 Epidemic Self-synchronization in Complex Networks

Based on the large body of existing work on coupled oscillators, in this article we seek to find a robust synchronization mechanism for large-scale complex Peer-to-Peer topologies. Since our algorithm is basically an epidemic variation of the

Kuramoto model with weighted coupling strengths, we briefly discuss the basic Kuramoto model in its most common notation prior to describing our algorithm in more detail.

### 3.1 Kuramoto Model Primer

In its simplest mean-field form, the Kuramoto model consists of $N$ all-to-all coupled oscillators with frequencies $\omega_i$ and a coupling constant $K$ [1]. The advance of an oscillator's phase $\Theta_i$ is coupled to the current phase of all other oscillators in the following way:

$$\frac{\partial \Theta_i}{\partial t} = \omega_i + K \cdot \sum_{j=1}^{N} sin(\Theta_j - \Theta_i)$$

Hence, for $K = 0$, a node $i$'s phase $\Theta_i$ advances with constant speed at its intrinsic frequency $\omega_i$. Assuming $\Theta_k \in [0, 2\pi)$ and interpreting $\Theta_k$ as a node $k$'s position on a limit-cycle, for $K > 0$ the advance of an oscillator's phase at any given time $t$ depends on the angle between its local phase and the phases of all other nodes. Positive angles speed up, negative angles slow down an oscillator's phase advance. Although this model is fully nonlinear, it has been shown that frequencies and phases of all oscillators eventually self-synchronize if (a) the coupling constant $K$ exceeds a certain critical value $K_c$ and (b) oscillators are identical or near-identical [13]. For a detailed survey of the Kuramoto model and several dependent variations, we refer to [1]. A review of a large body of research based on the Kuramoto model can be found in [22].

### 3.2 Epidemic Degree-Weighted Self-synchronization

In this section we introduce an epidemic synchronization algorithm based on the Kuramoto model. Rather than being continuously coupled to all nodes, the algorithm assumes that nodes are coupled to a single random one-hop-neighbor in periodic intervals. We further assume that the coupling strength is given as a function of the degree of both coupling partners, i.e. depending on the chosen coupling partner, the coupling strength may differ across subsequent coupling iterations. A detailed and formal description of the algorithm is given in the remainder of this section.

Let each node $v \in V$ in an undirected graph $G = (V, E)$ possess a randomly skewed local clock $t_v$, a Gaussian distributed frequency $\omega_v$ as well as an oscillating signal $\gamma_v(t_v)$. Defining its period $T_v$ as $T_v = \frac{1}{\omega_v}$, a node's current position within this period can be written as

$$\Theta_v = \Theta_v(t_v) = \frac{(t_v \, mod \, T_v) \cdot 2\pi}{T_v}.$$

$\Theta_v$ represents a node $v$'s current phase in its oscillator cycle. With this, we define a node's oscillating signal $\gamma_v(t_v)$ as

$$\gamma_v = \gamma_v(t_v) = sin\left(\Theta_v\right).$$

For self-synchronization to occur, we assume that in regular intervals a node $v$ selects a random neighbor $w$ and that both nodes exchange their current phases $\Theta_v$ and $\Theta_w$. With $deg(v)$ denoting the number of $v$'s neighbors, we further assume that the nodes $v$ and $w$ update their periods (and thus frequencies) in the following way:

$$T_v = T_v + f(deg(v), deg(w)) \cdot sin(\Theta_v - \Theta_w)$$
$$T_w = T_w + f(deg(w), deg(v)) \cdot sin(\Theta_w - \Theta_v)$$

In this definition, $f$ denotes a coupling strength function, whose value depends on the vertex degrees of both coupling partners. In the following we give three intuitive examples for such coupling functions. We start by using the constant, mean-field coupling strength of the original Kuramoto model. For some real value $C$ it is defined as

$$f_1(deg(v), deg(w)) = \frac{C}{|V|}$$

Speaking informally, the intention of using this coupling strength in the original Kuramoto model was to scale the influence exerted on a certain oscillator to $C$ times the mean of all phase differences. With this in mind, it is questionable whether this coupling can produce self-synchronizing behavior for locally coupled oscillators. Considering the heterogeneity of vertex degrees it would be surprising if there existed a uniform critical value $C$ for different complex topologies. Although a formal analysis of critical coupling strength for arbitrary network topologies with nearest-neighbor couplings is still an issue of ongoing research, recent numerical simulations suggest that degree-based weighting provides uniform criticality in complex networks of oscillators coupled to all neighbors [14]. Although so far not substantiated by formal analysis, it seems intuitive when considering the initial intention of letting oscillators be influenced by a "mean phase difference". Thus - justified by the accumulation of influences across subsequent couplings to different neighbors - for our epidemic algorithm it appears reasonable to investigate the following coupling function:

$$f_2(deg(v), deg(w)) = \frac{C}{deg(v)}$$

With both coupling functions $f_1$ and $f_2$, a node does not differentiate between the influence exerted by different neighbors and thus inherently implies equal credibility. In real-world networks occurring in nature or engineering this assumption is often not justified. Most of these networks exhibit highly heterogeneous vertex degrees and are disassortative, i.e. high degree nodes are often coupled to many small degree nodes. Being influenced by more nodes, the synchronization state of such "hubs" is likely to be more reliable. Power-law networks emerging from preferential attachment are an important example for disassortative networks. The fact that the attractiveness of hubs in real-world networks often stems from higher stability or better capabilities can be assessed as another argument for a degree-based differentiation of a neighbor's importance. Based on these considerations, it seems justified to privilege hubs by selectively elevating the coupling strength to high degree nodes. Similar ideas have been considered e.g. in [9]. While maintaining the degree-based weighting in $f_2$, we define $f_3$ as follows:

$$f_3(deg(v), deg(w)) = \frac{g(deg(w))}{deg(v)}$$

In this definition, $g$ denotes some real-valued, monotonically increasing function. We further assume that for all vertex degrees $x$ of a network, $g(x)$ is within the critical range for $K$ that guarantees self-synchronization in the original Kuramoto model. This can be achieved by estimating network size and thus maximum degree. For the simulations presented in the following section, a function of the form

$$g(x) = C + \frac{x}{S}$$

has been used, with $C$ denoting a certain lower threshold of critical coupling strength and $S$ being a reasonably chosen scaling constant.

In the following sections, simulation results for different topologies and coupling strength functions will be presented. Particular emphasis is placed on the question which functions result in the emergence of self-synchronization and how they relate to the resilience of synchrony against failing nodes.

## 4 Experimental Results

In this section we study the emergence of self-synchronization in experiments based on the epidemic synchronization scheme described above. In order to evaluate the evolution of synchrony, we define the normalized macroscopic signal $\Gamma$ of a graph $G = (V, E)$ as the superposition of microscopic node oscillations with reference to a global time frame, i.e.

$$\Gamma(t) = \frac{\sum_{v \in V} \gamma_v(t)}{|V|}.$$

Another way of looking at $\Gamma$ is as a "sound" being composed of "tones" $\gamma_v$ that are generated by the oscillations of individual nodes. In the terminology of acoustics, the degree of synchronization of individual tones can be assessed by looking at the loudness and harmonics of the global sound. In analogy, one can intuitively attribute the degree of synchronization to certain properties of $\Gamma$: Phases and frequencies of a population of sinusoidal oscillators are synchronized if $\Gamma$ is pure-sinusoidal with a peak-to-peak amplitude of 2 and if the spectrum of $\Gamma$ exhibits only a single frequency. Obviously $\Gamma$ is a rather illustrative and informal metric which will be used to monitor the evolution of synchrony. In order to reliably compare different states of synchrony, a more objective metric is required. $I(G)$ denoting the maximum intensity in the Fourier spectrum of a network $G$ and $I_{Max}$ denoting the maximum intensity in a reference graph with synthetically synchronized identical oscillators, we define

$$\sigma(G) = \frac{I(G)}{I_{Max}}$$

for this purpose. It is clear, that $\sigma(G) = 1$ holds true for a population of perfectly synchronized oscillators. Generally, values of $\sigma$ that are closer to 1 indicate that more oscillators exhibit the same frequency.

In order to assess the synchronization behavior of our algorithm in different network topologies, simulations have been performed using TopGen [21], an open-source, event-based network simulator that has been developed within our working group. TopGen can generate and analyze various network topologies and is extensible by custom modules. Experiments can be assembled much like combining building blocks by using several modules at once. Hands-on and visual experimenting with coupled oscillators is facilitated by special support like e.g. the visualization of oscillator populations, oscilloscopic real-time analysis of definable global characteristics (like e.g. $\Gamma$) or integrated Fast Fourier Transform on streams of configurable measurands. TopGen source code and binaries as well as several video captures of coupled oscillator experiments are available online from the author's website[1].

### 4.1 Static Topologies

In the following, we study oscillograms of the signal $\Gamma$ for different topologies and coupling functions. We also study Fourier spectra of $\Gamma$, which have been obtained using a discrete 8192-sample Fourier transformation. All results presented in this section have been obtained by simulating 20000 iterations of the epidemic algorithm described above. For this, different random graph topologies with 10000 nodes and roughly 50000 edges were used. All of them were checked to be connected prior to performing simulations. The natural periods of oscillating nodes were Gaussian-distributed with $\mu = 100$ and $\sigma = 25$ iterations, initially phase skews of individual nodes were uniformly distributed within a node's natural period. For the simulations presented in this section, we did not consider any potential message exchange latencies that might occur in reality. Figure 1 shows oscillogram and frequency spectrum of the macroscopic signal

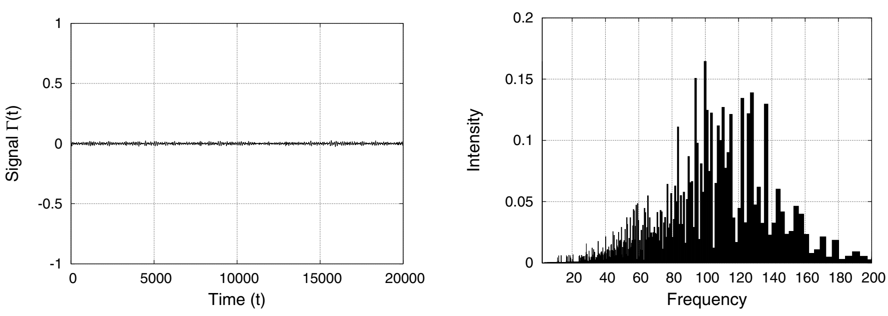



**Fig. 1.** Oscillogram and Frequency Spectrum of $\Gamma$ in a $10^4$ Node Reference Graph (No Synchronization)

[1] http://syssoft.uni-trier.de/˜scholtes

$\Gamma$ in a reference graph with 10000 nodes and with no self-synchronization involved. As one would expect from the superposition of unsynchronized Gaussian oscillations, the oscillogram exhibits no pronounced dynamics and the frequency spectrum is Gaussian shaped with a small maximum intensity around the mean frequency $\mu = 100$. Our $\sigma$-metric yields a value of $2 \cdot 10^{-3}$ for this reference network.

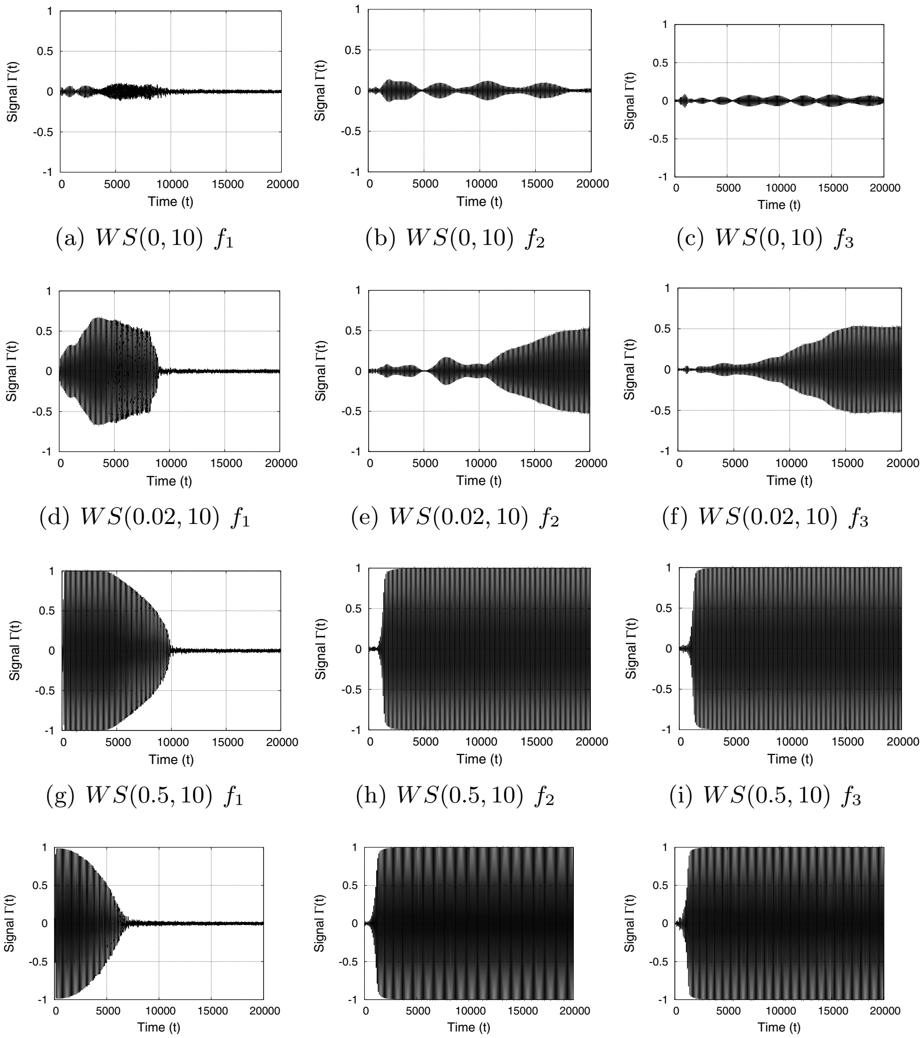
We begin our analysis by considering graphs generated by the Watts/Strogatz [24] model. The main advantage of this model is that it allows to study the interrelation between a network topology's small-world characteristics and the emergence of self-synchronization. For this, two model parameters $k$ (number of nearest-neighbors in initial ring lattice) and $p$ (probability to reconnect a node to a random node) can be be tuned to produce graphs of increasing randomness and with increasingly pronounced small-world characteristics. The resulting graph can range between a regular ring lattice ($p = 0$) or a completely random graph ($p = 1$). Based on these parameters, we will denote Watts/Strogatz graphs with $WS(p, k)$. Because a $WS(p, k)$ graph with $N$ nodes has $\frac{N \cdot k}{2}$ edges and we intended to perform all simulations on equally-sized graphs with 10000 nodes and roughly 50000 edges, the parameter $k$ has been set to 10 for all subsequent experiments.

In Figure 2, the evolution of the macroscopic signal $\Gamma$ is depicted for different topologies and coupling strength functions over 20000 simulation steps. Since an averaging over several simulation runs would impair $\Gamma$'s significance as an illustration of the process of self-synchronization, results of a single but representative simulation are shown instead. The results suggest that self-synchronization did not emerge for $f_1$, while for $WS(p, 10)$ graphs the degree of synchronization with couplings $f_2$ and $f_3$ seems to depend on the parameter $p$. Complementing these rather illustrative results, Figure 3(a) shows the averaged degree of synchronization $\sigma$ for several simulation runs and different network topologies. These results suggest that for our epidemic synchronization scheme, the synchronizability of a $WS(p, 10)$ graph increases with higher parameters $p$.

Above results for $WS$-model graphs incite to explore several more points in its parameter space. For this, the average synchronization degree $\sigma(G)$ has again been measured for several simulations of our epidemic algorithm in a $WS(p, 10)$ graph with coupling function $f_3$ and different values $p$. The results that can be seen in Figure 3(b)[2], show that the onset of self-synchronization in Watts/Strogatz graphs occurs suddenly at roughly the same critical point $p_c = 0.01$ that has been identified for the emergence of small-world properties [24].

Another important class of complex networks are those whose vertex degree distributions obey a power law, i.e. a node's probability to have exactly $k$ links is proportional to $k^{-\gamma}$ for some real-valued $\gamma$. A simple and convenient model for power-law graphs which exhibit small-world characteristics is the Barabási/Albert ($BA$)[3] model. It is based on preferential attachment, i.e. the probability of incrementally added nodes to establish links to existing nodes is proportional to the target nodes' degree, higher degree nodes being more likely

[2] Please note the log-scale x-axis.




(a) $WS(0, 10)$ $f_1$ (b) $WS(0, 10)$ $f_2$ (c) $WS(0, 10)$ $f_3$

(d) $WS(0.02, 10)$ $f_1$ (e) $WS(0.02, 10)$ $f_2$ (f) $WS(0.02, 10)$ $f_3$

(g) $WS(0.5, 10)$ $f_1$ (h) $WS(0.5, 10)$ $f_2$ (i) $WS(0.5, 10)$ $f_3$

(j) BA $f_1$ (k) BA $f_2$ (l) BA $f_3$

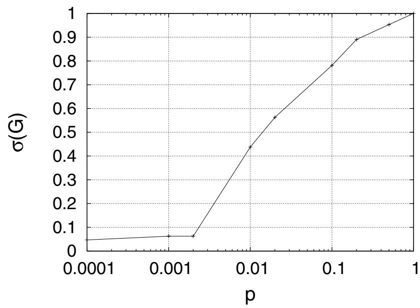
**Fig. 2.** Synchronization in Static Watts/Strogatz and Barabási/Albert Graphs

to receive even more links. For the simulations the following results are based on, the number of links $l$ that each newly added node established has been set to $l = 5$. Figures 2(j) - 2(l) and 3(a) show the results of experiments with $BA$ graphs with 10000 nodes and roughly 50000 edges. Similar to the results for $WS(p, 10)$ graphs with $p > 0.5$, one observes a fast convergence towards a synchronized state for the degree-weighted coupling strength functions $f_2$ and $f_3$, while $f_1$ did not result in a stable synchronization state.

Based on above findings, it is interesting to study the coupling strength function $f_1$ in more detail. In Figures 2(d), 2(g) and 2(j) one observes an initial

| Topology | $f_1$ | $f_2$ | $f_3$ |
|---|---|---|---|
| WS(0,10) | 0 | 0.03 | 0.08 |
| WS(0.02,10) | 0 | 0.44 | 0.59 |
| WS(0.5,10) | 0 | 0.97 | 0.94 |
| WS(1,10) | 0 | 0.79 | 1 |
| BA | 0 | 0.95 | 0.95 |

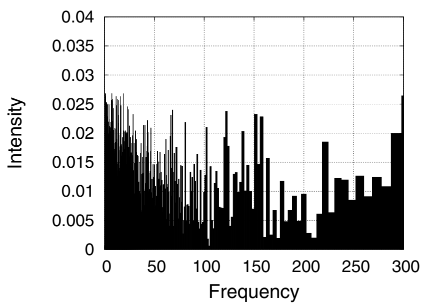
(a) Avg. $\sigma$ for Different Couplings



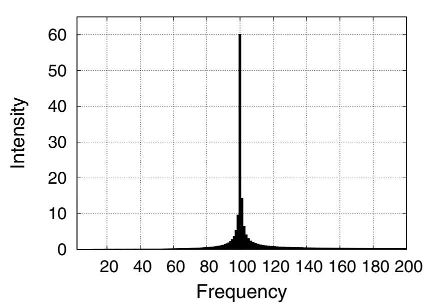

(b) $\sigma$ in WS graph for $f_3$ (Lines are drawn to guide the eye)

**Fig. 3.** Synchronization Degree $\sigma$ in Different Topologies

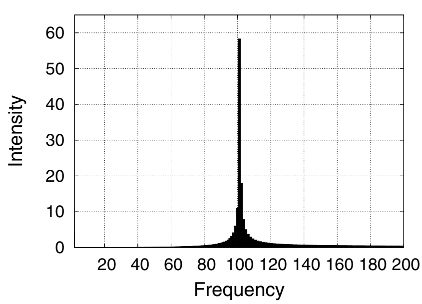
convergence towards a (partially) synchronized state. After having quickly reached a certain maximum amplitude, it gradually fades and the amplitude stabilizes at a low level. At first glance it seems strange that the system eventually stabilizes in an unsynchronized state although it initially started to synchronize from an unsynchronized state as well. We can clarify this by looking at the frequency spectra of $\Gamma$ after having performed 20000 iterations of our




(a) Spectrum $f_1$




(b) Spectrum $f_2$




(c) Spectrum $f_3$

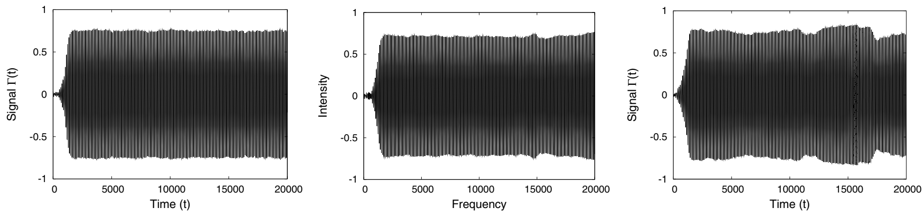
**Fig. 4.** Frequency Spectra of BA Graphs after 20000 Iterations

algorithm in a $BA$ graph. In the spectra of coupling functions $f_2$ and $f_3$ which are shown in Figure 4, one observes sharp peaks with maximum intensity at the global mean frequency, i.e. the spectra one would expect from a population of synchronized oscillators. In contrast, the spectrum for $f_1$ is highly heterogeneous and exhibits no dominant frequency. With this we can explain the stabilization in an unsynchronized state, since for the Kuramoto model it has been shown that self-synchronization can only occur if oscillator frequencies are homogeneous enough (as e.g. the case for the Gaussian distributed initial state shown in Figure 1). For this reason, states of total desynchronization with non-Gaussian distributed frequencies are irreversible and the macroscopic signal $\Gamma$ stabilizes at a low amplitude.

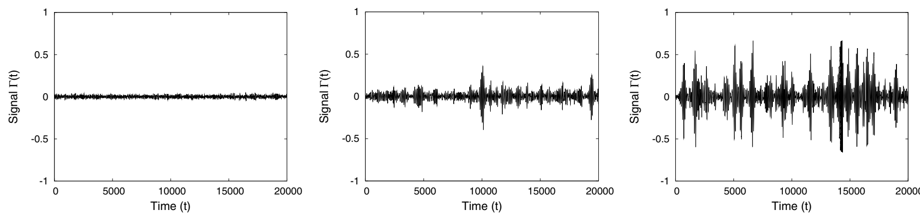
### 4.2 Dynamic Topologies

A major hurdle that complicates the application of distributed technologies in large-scale distributed computing systems is the impact of the dynamics induced by leaving or joining network nodes. Thus, an important task when designing distributed algorithms is to assess their susceptibility to this kind of dynamics, so-called *churn*. In this section, we present simulation results that have been acquired in order to evaluate the resilience of our epidemic synchronization scheme against dynamic nodes. Based on the results from the previous section, simulations have been performed using $WS(1, 10)$ and $BA$ graphs. In the remainder of this section we only consider the coupling functions $f_2$ and $f_3$, since only these promise the emergence of self-synchronization.

In a first series of experiments that have again been performed using the simulation environment TopGen, the effect of randomly failing and joining nodes has been investigated. For this, in each simulation step a randomly chosen fraction of 0.1% respectively 1% of all nodes was removed from random graph topologies consisting of 10000 nodes. The same number of nodes was added again in the same simulation step in a way that did not alter the graph's vertex degree distribution. Hence the graphs remained in a state of equilibrium, newly added nodes being initialized with random frequencies and phases using the same parameters as in the original graph initialization (see section 4.1). Simulation results for different coupling functions, graph topologies and rates of random churn are shown in Figure 5 and 6. While a churn rate of 0.1% resulted in a state of at least partial synchronization both for $WS(1, 10)$ and $BA$ graphs, synchronization largely broke down when the churn rate was set to 1%. As can be seen in Figure 5, there are notable differences in the signal evolution for $BA$ and $WS(1, 10)$ topologies in terms of amplitude fluctuations: These fluctuations are much stronger in $BA$ graphs than in $WS(1, 10)$ topologies. Although this finding requires further evaluation, it can be intuitively attributed to situations where hubs were removed from the network based on the random churn model. Comparing the performance of the different coupling strength functions $f_2$ and $f_3$, one recognizes that using $f_3$ seems to intensify amplitude fluctuations in $BA$ model graphs while increasing the average degree of synchronization (see Figure 6).




(a) $WS(1, 10)$, $f_2$, 0.1% (b) $BA$, $f_2$, 0.1% (c) $BA$, $f_3$, 0.1%




(d) $WS(1, 10)$, $f_2$, 1% (e) $BA$, $f_2$, 1% (f) $BA$, $f_3$, 1%

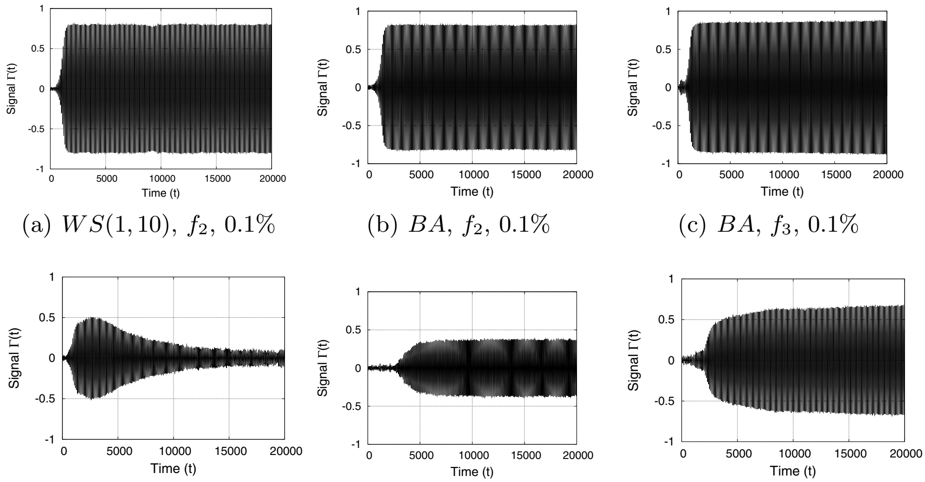
**Fig. 5.** Oscillograms of $\Gamma$ in $WS$ and $BA$ Graphs with Different Rates of Random Churn

| Churn Rate | WS(1,10), $f_2$ | WS(1,10), $f_3$ | BA, $f_2$ | BA, $f_3$ |
|---|---|---|---|---|
| 0.1 % | 0.48 | 0.52 | 0.54 | 0.62 |
| 1 % | 0 | 0 | 0.03 | 0.09 |

**Fig. 6.** Avg. Synchronization Degree $\sigma$ under the Effect of Random Churn

Based on the considerations leading to the introduction of the coupling strength function $f_3$ in section 3, for many real-world network topologies with highly diverse vertex degrees it is justified to relate a node's degree to its reliability: Highly connected Internet routers are probably - or at least hopefully - better secured against failures and attacks, Websites with many incoming links would probably not have become popular if they were not continuously accessible and hubs in prevalent Peer-to-Peer overlay topologies usually emerge because their past behavior has shown superior reliability. Thus, it seems reasonable to consider a degree-coupled churn model in which a node's probability to fail is reciprocally proportional to its degree. Figures 7 and 8 show results of a series of simulations based on such a *degree-coupled churn* model, again using churn rates of 0.1% respectively 1% per simulation step and different coupling strength functions. As in the previous random churn experiments, all exitting nodes were immediately replaced with new, randomly initialized ones in a way that did not change the graph's vertex degree distribution.

The results in Figure 7 show that under the effect of degree-coupled churn, synchrony in Watts/Strogatz networks eventually fades for a churn rate of 1% per iteration. For Barabási/Albert graphs, one observes better resilience against degree-coupled churn both for the coupling strength functions $f_2$ and $f_3$. We




(a) $WS(1,10)$, $f_2$, 0.1%  (b) $BA$, $f_2$, 0.1%  (c) $BA$, $f_3$, 0.1%

(d) $WS(1,10)$, $f_2$, 1%  (e) $BA$, $f_2$, 1%  (f) $BA$, $f_3$, 1%

**Fig. 7.** Oscillograms of $\Gamma$ in $WS$ and $BA$ Graphs with Different Rates of Degree-Coupled Churn

| Churn Rate | WS(1,10), $f_2$ | WS(1,10), $f_3$ | BA, $f_2$ | BA, $f_3$ |
|---|---|---|---|---|
| 0.1 % | 0.6 | 0.63 | 0.81 | 0.67 |
| 1 % | 0.08 | 0.01 | 0.38 | 0.66 |

**Fig. 8.** Avg. Synchronization Degree $\sigma$ under the Effect of Degree-Coupled Churn

attribute this to the stabilizing effect exerted by hubs in $BA$ model topologies. Since the degree-coupled churn model assumes that hubs fail with smaller probability, there likely remains a stable "core" that is highly connected to the residual network. A surprising outcome of our simulations is that, compared to $f_2$, for $BA$ networks under 0.1% churn the coupling function $f_3$ leads to a 17% decrease of $\sigma$, while resulting in a 70% increase for a 1% churn rate (see Figure 8). Further simulations are required in order to reveal how different scaling parameters for $f_3$ and different vertex degreee exponents influence this result.

## 5 Conclusion

In this article, we introduced and evaluated an epidemic self-synchronization mechanism for complex networks of Kuramoto-based oscillators. The algorithm is based on a periodic exchange of the oscillator's phase with a single random neighbor. Simulations suggest that the self-synchronizing behavior depends on small-world characteristics as well as on a degree-weighted coupling strength. Intuitively, these findings can be attributed to the mean-field coupling in the Kuramoto model as well as to the efficiency of epidemic aggregation in small-world topologies. In accordance to [24], our simulations revealed critical behavior with respect to the parameter $p$ in the Watts/Strogatz model. Furthermore, it has been argued that selectively boosting coupling strengths to high degree vertices provides a basis for improving synchronization resilience of power-law networks in situations of massive churn. From these findings we draw the conclusion that the proposed scheme is a promising approach for the self-organized provision of a global concept of time in complex and dynamic Peer-to-Peer network topologies.

Another interesting question when applying the scheme to real-world Peer-to-Peer network topologies is the controllability of the network's global frequency and phase. Although rigid results could not yet be included in this article, first experiments have been performed suggesting that using the coupling strength function $f_3$, the synchronization state (frequency and phase) in power-law networks can be efficiently adjusted by instrumenting a small number of highest-degree nodes[3]. Epidemic self-synchronization could thus be utilized to efficiently provide an adjustable, network-wide, synchronous chronology of states. By this, it can serve as a foundation for adaptable, cycle-based distributed algorithms operating on large-scale dynamic Peer-to-Peer power-law topologies. Representing a first application of our mechanism, a probabilistic mutual exclusion algorithm for dynamic complex networks has been implemented. Here subsequent oscillator cycles are used to elect privileged nodes and access critical sections. The algorithm will be presented in more detail in a subsequent publication. Apart from further substantiating our claims with more detailed and comprehensive simulations, investigating further applications of our epidemic self-synchronization scheme will be an interesting future task.

We conclude this article with a discussion of its main weaknesses and open issues. A major - and probably the foremost - deficiency of our work is the lack of a formal analysis of the algorithm's synchronization behavior. Such an analysis is complicated by the fact that it involves randomized neighbor couplings with nonlinear and asymmetric coupling strengths. Although apparently being a simpler problem, the analysis of topologies of all-neighbor coupled oscillators with symmetric coupling strengths has proven to be hard and is subject of ongoing research. Notwithstanding the fact that they can not replace formal analysis, we argue that at this borderline simulations are to date the best available tool to assess an algorithm's behavior. Another important aspect that has not yet been considered is the effect of communication latency on the performance of our synchronization scheme. Intuitively one would expect message latency to limit the maximum frequency at which oscillators can be synchronized. Extensive simulations based on realistic communication latencies are required to investigate this issue in more detail. Conveniently, the TopGen simulation environment contains facilities to generate and respect latency data based on realistic router-level Internet graphs. Another issue that has not yet been considered is a further evaluation of the epidemic algorithm's parameter space. It will be interesting to investigate the interrelations between parameters like e.g. frequency of inter-node couplings, coupling strength parameters, network size, power law exponent, initial frequency distribution and synchronization behavior. For the experiments

[3] Videos of simulations available at http://syssoft.uni-trier.de/˜scholtes

presented in this article, some intuitively chosen, reasonable values have been used and more comprehensive evaluations must be considered future work. An important issue of using epidemic algorithms in graph topologies with highly diverse vertex degrees is the potential burden put on the highest degree vertices since those can be potentially chosen as random coupling partner by a large number of neighbors. An investigation how this problem can be mitigated for our epidemic synchronization scheme is an important issue and must be considered future work.

Finally, during our simulation experiments with different network topologies, another interesting question emerged that could not yet be addressed: One might ask whether the effects of topology on a network's synchronization behavior can be utilized to provide an efficient and decentralized monitoring of global network characteristics. For this, it will be important to investigate whether global properties can be related to the behavior of local oscillator signals. Such a relation might empower individual nodes to efficiently infer valuable global knowledge of network characteristics that can otherwise be hard to extract.

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