Evolutionary Prisoner's Dilemma Game in Flocks

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Abstract. We investigate the effect of mobility on the evolution of cooperation in a flock model, where each player moves on the two-dimensional plane with the same absolute velocity. At each time step every player plays the prisoner's dilemma game and aligns moving direction with its neighbors, who are chosen according to distances between them in the two-dimensional space. Experimental results have shown that with unconditional cooperation or defection, cooperation can be maintained in mobile players even for high velocities, as local interactions among players are enhanced by the expansion of neighborhood. However, the movement of players can only be offset within a certain range of temptation b, while outside this range a rapid decrease of cooperators will appear in the population because too many neighbors are involved.

Keywords: Cooperation, Flocks, Evolutionary games, Prisoner's Dilemma.

1 Introduction

Cooperation is commonly observed in genomes, cells, multi-cellular organisms, social insects, and human society, but Darwin's Theory of Evolution implies fierce competition for existence among selfish and unrelated individuals. In past decades, many efforts have been devoted to understanding the mechanisms behind the emergence and maintenance of cooperation, in which the prisoner's dilemma game is a widely used model to illustrate the conflict between selfish and cooperative behavior in the context of evolutionary game theory.

Recently the evolution of cooperation in structured populations has attracted much attention. To study the spatial effects upon strategy frequencies in the population, Nowak and May have introduced the spatial PD game [1]. In their model, players are located on the vertices of a two-dimensional lattice and each player only interacts with his neighbors. Without any strategic complexity, the stable coexistence of cooperators and defectors can be achieved. And to characterize the effect of noise, Szabo and Toke [2] have presented a stochastic update rule that permitting irrational choice. Besides, Perc and Szolnoki [3] account for social diversity by stochastic variables that determine the mapping of game

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payoffs to individual fitness. But as pointed in [4], the survival of cooperators dose not always benefit from spatial structure.

In spatial games players are located on the nodes of networks and only those who are directly connected will interact with each. The underlying networks are often assumed with fixed topologies, and can also evolve with the game dynamics. However social networks are constructed through the motion of nodes [5], and cooperation in animals usually exist in migration and prey, where the mobility of individuals might play a certain role. So it is a natural interest to explore cooperative behavior in mobile individuals among the structured populations.

Mobility can be implicitly represented as travelling cost or change of group membership, and in agent-based models it can be involved explicitly as the movement of agents. By intuition the introduction of mobility would lead to the dominance of defection strategy because mobile defectors can escape retaliation of former partner by moving away. But previous investigations have shown that cooperation can be sustained in populations composed of mobile agents [6,7], and through contingent movement cooperators can avoid repeated interaction with defectors [8].

In biological systems, such as flocks of birds and schools of fish, highly aligned collective motion can be achieved only through local behavioral rules without centralized coordination. And individuals in flocks are tending to align their directions of travel with that of nearby neighbors. In the work below we will let agents move to simulate direction alignment process in flocks, and investigate the effects of mobility on the evolution of cooperative behaviors in the population.

2 The Model

To study the effect of mobility in the evolution of cooperation, we consider a system of self-propelled agents, which contains $N = 500$ mobile agents. At time $t = 0$ all agents are randomly distributed in a square-shaped cell of linear size L and each has the same absolute velocity v with different directions uniformly distributed in $[0, 2\pi)$. At the next time ($t \geq 0$) each agent simultaneously update its position according to

$$x_i(t+1) = x_i(t) + \overrightarrow{V_i}(t)\Delta t \tag{1}$$

where the velocity $\overrightarrow{V_i}$ of agent i contains an absolute value v and a direction given by the angle $\theta_i(t)$. To simulate direction alignment process in flocks, the angle $\theta_i(t)$ is updated as

$$\theta_i(t+1) = \frac{1}{1+n_i(t)}(\theta_i(t) + \sum_{j \in N_i(t)} \theta_j(t)) \tag{2}$$

where $N_i(t)$ denotes the set of agents that agent i aligns itself with, and $n_i(t)$ represents the size of $N_i(t)$. And $N_i(t)$ is defined as the set composed of the neighbors of agent i which can be written as

$$N_i(t) = argmin_k\{\sqrt{\|x_i(t) - x_j(t)\|}, j \in N, j \neq i\} \tag{3}$$

where the function $argmin_k\{\bullet\}$ means to find k smallest elements given in $\{\bullet\}$ and the function $\sqrt{\| \bullet \|}$ is used to calculate distances between each agent in the two-dimensional space.

Note that the model proposed in [9] defines the neighbors of agent i as agents within the circle of radius r centered at itself. But some agents may have more neighbors than that of the others. To exclude those effects coming from the fluctuations of the number of neighbors, we choose k nearest agents to agent i as its neighbors.

Initially an equal percentage of strategies, namely cooperators or defectors, are randomly distributed in the population. At each time step, every player plays the PD game with his neighbors defined as (3), and all the income is accumulated. Following the common practices, the payoff matrix of the prisoner's dilemma takes a rescaled form as

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \quad (4)$$

where $1 < b < 2$. Then each player compares its income with that of his neighbors, and at the next time the player will choose the strategy that gains the highest payoff among itself and its neighbors. This process is repeated until the system reaches equilibrium.

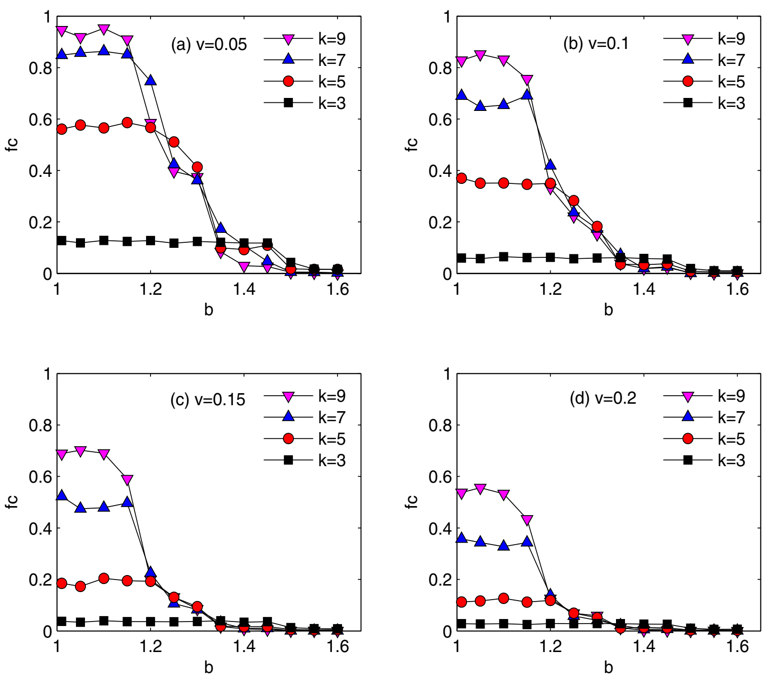

Fig. 1. The frequencies of cooperators fc vs. the temptation to defect b for $v = 0.05$, 0.1, 0.15 and 0.2 respectively, where the cases $k = 9, 7, 5, 3$ correspond to different sizes of neighborhood

3 Results and Discussions

In the model mentioned above we investigate the PD under different values of the temptation b, the absolute velocity v and the neighborhood size k. Here the cooperation level is characterized by the frequency of cooperators, which is defined as the percentage ratio of cooperators in the population. And equilibrium frequencies of cooperators are obtained by averaging over 100 generations after a transient time of 300 generations. Besides every data point in all figures below is acquired by averaging over 200 realizations of independent initial states.

Fig. 1(a) illustrates the dependence of the frequency of cooperators fc on the temptation b in the stationary state for different values of k with a fixed absolute velocity $v = 0.05$, while Fig. 1(b-d) correspond to the cases of $v = 0.1$, $v = 0.15$ and $v = 0.2$. Under a fixed v, we can find that when $k = 3$, defectors dominate the population significantly. If the velocity is increased, such as illustrated in Fig. 1(c) and Fig. 1(d), the cooperator is nearly extinct even the temptation b is small. Besides for any value of k, there exists a threshold $b(k)$. If $b < b(k)$, the frequency of cooperators, $fc(k)$, can be maintained in a relatively stable level with small fluctuations and for a fixed temptation b, the increase of k can enhance fc in the system. Furthermore, the threshold $b(k)$ decreases as the increase of the neighborhood size k, and if $b > b(k)$, fc decreases gradually as the increase

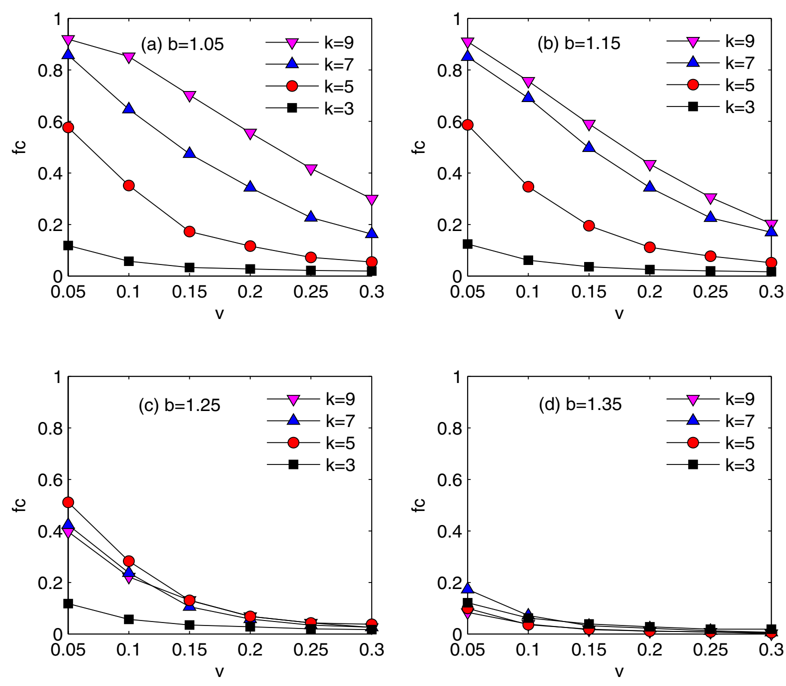

Fig. 2. The frequency of cooperators fc as a function of the absolute velocity v with various values of the temptation b, where the cases $k = 9$, 7, 5, 3 correspond to the neighborhood sizes

of b. In contrast with the case of $b < b(k)$, increasing the number of neighbors can not restrain the decrease of fc, and even makes fc decline a little faster than that with less neighbors.

Fig. 2(a) demonstrates the frequency of cooperators fc as a function of the absolute velocity v in the stationary state for different values of k with a fixed temptation $b = 1.05$, while Fig. 2(b)-(d) correspond to the cases of $b = 1.15$, $b = 1.25$ and $b = 1.35$. In our model the absolute velocity v can be used to measure mobility of players. As the increase of v, the interaction range of every player is enlarged and one player can expect to interact with more neighbors than before, which means the system is approximating to the mean field situation. We can find that in Fig. 2 defectors dominate the population as the velocity of players, v, is increased. But when players move with $v = 0.3$, there are still cooperators surviving in the system when $k = 7$ or $k = 9$ shown in Fig. 2(a) and Fig. 2(b). As suggested in [7], expanding the size of neighborhood prevents each player escape from its neighbors and keeps a continuous interaction between each player and local neighborhood [6]. In other words, the drop of fc caused by the movement of players can be offset by the enlargement of interaction range k. However, we have pointed out that for a fixed v, cooperative behavior can be maintained within a certain range as long as the temptation b does not exceed the threshold $b(k)$. This helps to explain why in Fig. 2(a) and Fig. 2(b) curves corresponding to the same k do not change a lot. And while for $b = 1.25$ and

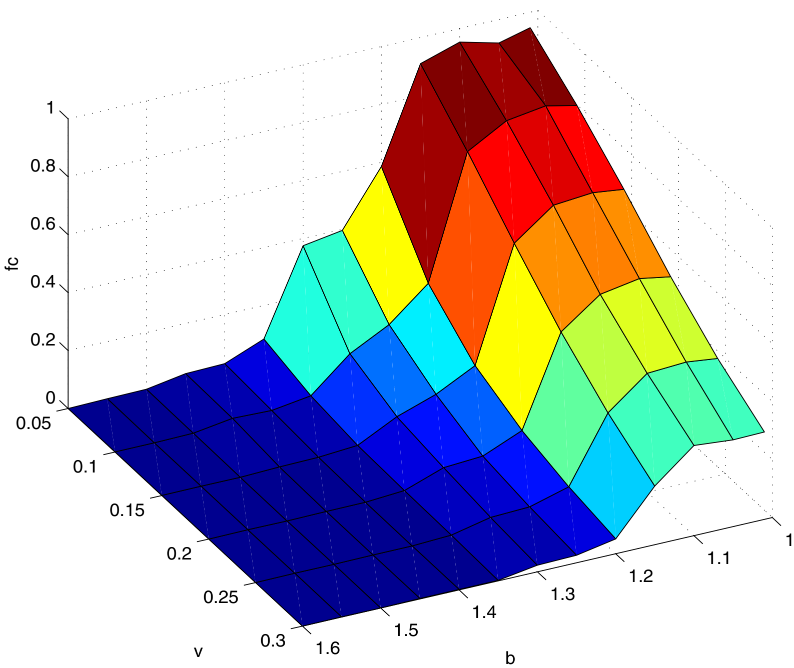

Fig. 3. The frequency of cooperators fc vs. the parameter space (v, b) for $k = 9$

$b = 1.35$ shown in Fig. 2(c) and Fig. 2(d), cooperators are completely extinct as the increase of v because of $b > b(k)$.

Fig. 3 shows the dependence of fc on b and v together when $k = 9$. We can find when $b < 1.15$, fc declines slowly as the increase of v and there are still a fraction of players sticking to cooperate when $v = 0.3$. When $b > 1.15$, there is a sudden drop of fc and even for $v = 0.05$, corresponding fc is below 0.6.

4 Conclusion

To summarize, we have investigated the effects of mobility on the evolution of cooperation in the direction alignment process of flocks and found that cooperation can be maintained in mobile players. Increasing the size of neighborhood offsets the effects caused by the movement of players within a certain range of b, while involving too many neighbors may leads a rapid decrease of cooperators outside this range. In future work the role of neighborhood in the evolution of cooperation, and the dependence of the threshold on the neighborhood size will be explored.

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