Evolving Model of Weighted Networks

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Abstract. In this paper, in order to search the reason of the phenomena of power-law in the weighted networks, we present a general model for the growth of weighted networks that couples of new edges and vertices and the weights' and intrinsic strengths' dynamical evolution. This model is based on a simple weight and intrinsic strength driven dynamics and generates networks exhibiting the statistical properties observed in several real-world systems. Within this model we not only yields the scale-free behavior for the weight, strength and degree distributions, but also we give the analytical computation of the distributions of the weight, the strength and the degree .Simultaneity, by way of contrasting our results with those of the random model, we found the preferential attachment is necessary to the phenomena of scale-free of the strength and degree distributions. Finally, we found the analytical results are good consistent with those of numerical simulation. The conclusion from this model is helpful to the investigation of the topological role of weight and strength.

Keywords: weighted network, scale-free network, degree distribution, intrinsic strength.

1 Introduction

In the last few years, considerable efforts have been made to understand and characterize complex networks in the real world including the Internet [1], the World Wide Web [2], the scientific collaboration networks [3,4], and the world-wide airport networks (WAN) [5][6]. The empirical results show that these networks generally exhibit complex topological properties such as small-world phenomenon [7] and scale-free property [8]. In order to explore the mechanism responsible for these topological features, a wide array of models aimed at capturing various properties of real networks have been formulated [9]. But these researches on networks have mainly focused on unweighted networks. There is a need for a modeling approach to complex networks that evolving not only by the topology growth. Barrat, Barthélemy, and Vespignani present a weighted-driven model (BBV model) [6], the novel feature in the

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model is the weight dynamical evolution occurring when new vertices and edges are introduced in the system. BBV model yields scale-free properties of the degree, weight, and strength distributions, controlled by a physical parameter δ. But the weight dynamical evolution in BBV model is triggered only by the introduction of new vertices, and the empirical results [10, 11] show that the inner interaction plays an important role in the networks' evolution. In addition, in the company the executives usually adjust their funds' assignment in the several departments according to their achievements. Thirdly, in the scientific collaboration networks, the scientists' reputation will varies according to the grown of the networks. Fourthly, in the wide air networks, the number of the passengers varies according to the different period of time according the above matter; we firstly introduce the concept of intrinsic strength, which is the description of vertices' intrinsic property. Based on this concept, we present a general model depended on five dynamical evolutions: node addition, weight's growth and decrease, node intrinsic strength's growth and decrease. We find network generated by this model exhibits some statistical properties observed in several real-world systems. Through analytical way, we yield the analytical computations of the distributions of the weight, strength and degree. They all exhibit the behaviors of scale-free. At the same time, by way of contrasting our results with those of the random model, we found the preferential attachment is necessary to the phenomena of scale-free. Finally, we contrast the results deduced in theory with the results of practical data analysis, and we find the both results are good consistent. So we believe that through implementing some of the suggested modifications, the model could be well-suited to other types of networks.

The rest of the papers are organized as follows: In section 2, we firstly introduce the concept of intrinsic strength, which is the description of vertices' intrinsic property. Then we define a general simple model for weighted networks that consider the basic evolution of the system as driven by the weight properties of edges and vertices. In addition, differently from early models proposed in the past [6, 12], we allow the dynamical evolution of weights and intrinsic strength during the growth of the system. This mimics the evolution and reinforcements of interactions in natural and infrastructure networks. In Sec.3, we introduce our algorithm and get the analytical calculations for the weight, strength and degree distributions with continuum theory and rate equations method and we yield this model is scale free networks, .Simultaneity, we contrast our results with those of random model and we found the preferential attachment is necessary to the phenomena of scale-free. Finally, some numerical simulations are given in Sec.4.

2 Modeling Weighted Networks

2.1 The Definition of the Intrinsic Strength of the Node

From the [5, 12], we define the strength s_i of the vertex i as:

$$s_i = \sum_{j \in \Gamma(i)} w_{ij} \tag{2.1}$$

Where the sum runs over the set Γ(i) of the neighbors of i. Under this definition, the strength of a node just integrates the information of its connectivity and the weights of its links, but we can't see the difference of intrinsic characterization between vertices.

In order to proceed to general analysis of weighted networks, we introduce the concept of the intrinsic strength, and we use s_i' represent the intrinsic strength of the node i. For example, in the citation networks [13], the intrinsic strength s_i' represents the impact factor of the journal in which the paper i published. In our model, the strength s_i' of the vertex i defined as:

$$s_i = s'_i + \sum_{j \in \Gamma(i)} w_{ij} \tag{2.2}$$

Where the sum runs over the set Γ(i) of neighbors of i, the strength of a vertex not only integrates the information about its connectivity and the weights of its links, but also reflects the vertices' intrinsic property.

2.2 Modeling the Evolution of Weighted Networks

Let l_{ij} denote the edge between the node i and j, w_{ij} denote the weights of the line l_{ij}. The model starts from an initial configuration of N_0 isolated vertices whose intrinsic strengths are fixed to the value s_0. Each time step, we only perform one of the following five operations:

(1) We add a new node n with m_0 ($m_0 \le N_0$) edges to the network: the new node n is connected to node i already presented in the system with the preferential probability:

$$\Pi_{n \to i} = \frac{s_i}{\sum_j s_j}$$

And the weight of each new edge is fixed to w_0 and the intrinsic strength of the new node is fixed to s_0.

We perform this operation with probability λ.

(2) Weights' growth : Select a link l_{ij} existing from the system; where the vertex i is selected according to probability $\frac{s_i}{\sum_n s_n}$ and the vertex $j (j \ne i)$ is selected randomly. Next we update weight w_{ij} according to the mechanism: $w_{ij} \to w_{ij} + w_0$. This operation is repeated l times.

We perform this operation with probability p_1.

(3) Weights' decrease: Select a link l_{ij} existing from the system; where the vertex i is selected according to the anti-preferential probability $(N(t)-1)^{-1}(1-\frac{s_i}{\sum_n s_n})$ and the vertex

j ($j \neq i$) is selected randomly. Next we update weight w_{ij} according to the mechanism: $w_{ij} \rightarrow w_{ij} - w_0$. This operation is repeated q times. Where N(t) is the size of the network at the time t. In addition, we hold the line l_{ij} if the weight of it is decreased to w_0, which can keep the grown of this network.

We perform this operation with probability p_2.

(4) Intrinsic strengths' growth: select m_1 ($m_1 \leq N_0$) vertices from the network, where every vertex i is selected according to the probability $\frac{s_i}{\sum_n s_n}$. Each selected vertex is supposed to update its intrinsic strength according to the mechanism: $s_i' \rightarrow s_i' + s_{i_0}$.

We perform this operation with probability p_3.

(5) Intrinsic strengths' decrease: select m_2 ($m_2 \leq N_0$) vertices from the network, where every vertex j is selected according to the anti-preferential probability $(N(t)-1)^{-1}(1-\frac{s_j}{\sum_n s_n})$. Each selected vertex is supposed to update its intrinsic strength according to the mechanism: $s_i' \rightarrow s_i' - s_{j_0}$.

We perform this operation with probability p_4.

3 Analytical Results

We denote by $k_i(t)$ the number of links node i at time t; N(t) denotes the total number of nodes at time t and t_i denote the time of adding of the node i. For simplicity, we set q=1, $s_{i_0} = s_{j_0} = s_0$, i, j=1,2,…,t. For $t \rightarrow \infty$, we can get:

$$N(t) = N_0 + \lambda t \approx \lambda t \; ; \; \lambda + p_1 + p_2 + p_3 + p_4 = 1; \; w_{ij}(t_i) = w_0 \; ;$$

$$s_i(t_i) = s'_i + \lambda m_0 w_0 = s_0 + m_0 w_0 \; ;$$

$$\sum_j s'_j = N(t) s_0 + p_3 m_1 s_0 t - p_4 m_2 s_0 t \; ;$$

$$\sum_j s_j(t) = [2\lambda m_0 w_0 + 2 p_1 l w_0 - 2 p_2 w_0 + \lambda s_0 + s_0(p_3 m_1 - p_4 m_2)] \cdot t$$

$k_i(t_i) = m_0$;

3.1 Distribution of the Strength

We assume that the strength s_i, the degree k_i, and the weight w_{ij} change continuously and we can get:

$$\frac{ds_i}{dt} = \lambda m_0 \left(\frac{s_i}{\sum_j s_j} \cdot \right) + p_1 l \cdot \left(\frac{1}{N(t)} + (1 - \frac{1}{N(t)}) \cdot \frac{s_i}{\sum_j s_j} \right) w_0$$

$$+ p_2 \left(\frac{1}{N(t)} + (1 - \frac{1}{N(t)}) \frac{1}{N(t)-1} \cdot (1 - \frac{s_i}{\sum_j s_j}) \right) (-w_0) + m_1 p_3 s_0 (\frac{s_i}{\sum_j s_j}) \qquad (3.1)$$

$$+ m_2 p_4 (-s_0) \frac{1}{N(t)-1} \cdot (1 - \frac{s_i}{\sum_j s_j})$$

Indeed, the quantity w_0 sets the scale of the weights for intrinsic edge, and s_0 sets the scale of the intrinsic strength, without loss generality, we can let w_0=1, s_0=1, and then we get the initial condition:

$$s_i(t_i) = s_0 + m_0 w_0 = 1 + m_0 \qquad (3.2)$$

We let $\sum_j s_j$ =At, where A=$2\lambda m_0 + 2p_1 l - 2p_2 + \lambda + p_3 m_1 - p_4 m_2$, then we rewrite the equation (3.1):

$$\frac{ds_i}{dt} = \frac{\lambda m_0 + m_1 p_3 + p_1 l}{A} \cdot \frac{s_i}{t} + \frac{p_1 l - 2p_2 - m_2 p_4}{\lambda} \cdot \frac{1}{t} + \frac{p_2 + m_2 p_4 - p_1 l}{A\lambda} \cdot \frac{s_i}{t^2} \qquad (3.3)$$

For enough large t; we can get the following expression:

$$\frac{ds_i}{dt} \approx a_1 \frac{s_i}{t} + a_2 \frac{1}{t} \qquad (3.4)$$

Where $a_1 = A^{-1}(\lambda m_0 + p_3 m_1 + p_1 l)$, $a_2 = \lambda^{-1}(p_1 l - 2p_2 - p_4 m_2)$;

The differential equation (3.4) has the following solution:

$$s_i(t) = (\lambda + \lambda m_0 + \frac{a_2}{a_1})(\frac{t}{t_i})^{a_1} - \frac{a_2}{a_1} \; ; \qquad (3.5)$$

We may get:

$$P(s_i(t) < s) = P(t_i > (\frac{\lambda + \lambda m_0 + \frac{a_2}{a_1}}{s + \frac{a_2}{a_1}})^{\frac{1}{a_1}} \cdot t) \tag{3.6}$$

Assuming that we add the nodes at equal time intervals to the system, i.e., t_i denote the time adding of node *i* and follow the uniform distribution over interval (0, λt). Hence, when s> $\lambda + \lambda m_0$,

$$P(s_i(t)<s)=1-(\frac{\lambda + \lambda m_0 + \frac{a_2}{a_1}}{s + \frac{a_2}{a_1}})^{\frac{1}{a_1}} \cdot \frac{1}{\lambda} \tag{3.7}$$

So

$$P(s)=\frac{dP(s_i(t)<s)}{ds}=(\lambda a_1)^{-1}(\lambda+\lambda m_0+\frac{a_2}{a_1})^{\frac{1}{a_1}}(s+\frac{a_2}{a_1})^{-\gamma} \tag{3.8}$$

Where

$$\gamma = 1 + \frac{1}{a_1} = 3 + \frac{\lambda - p_3 m_1 - p_4 m_2 - 2p_2}{\lambda m_0 + m_1 p_3 + p_1 l};$$

This indicates that the probability distribution P(s) has behavior $P(s) \sim s^{-\gamma}$.

Additional, we consider the random model, that is, the five operations, node addition, weight's growth and decrease, node intrinsic strength's growth and decrease are all uniformly at random. For simplicity, we call the above model random model. In random model, we get

$$\frac{ds_i}{dt} = \lambda m_0 \frac{1}{N(t)} + p_1 l \frac{2}{N(t)} w_0 + p_2 \frac{2}{N(t)}(-w_0) + m_1 p_3 s_0 \frac{1}{N(t)} + m_2 p_4 (-s_0) \frac{1}{N(t)} \tag{3.9}$$

According to $N(t)=N_0+\lambda t \approx \lambda t$, we set

$$M = \frac{\lambda m_0 + 2(p_1 - p_2) + m_1 p_3 - m_2 p_4}{\lambda},$$

then

$$\frac{ds_i}{dt} = \frac{M}{t} \tag{3.10}$$

The differential equation (3.10) has the following solution:

$$s_i(t) = (1+m_0)\frac{\ln t}{\ln t_i} \tag{3.11}$$

We may get:

$$P(s_i(t) < s) = P(t_i > t^{\frac{1+m_0}{s}})$$

Assuming that we add the nodes at equal time intervals to the system, i.e., t_i denote the time adding of node i and follow the uniform distribution over interval (0, λ t). Hence

$$P(s_i(t)<s)=1-\frac{t^{\frac{m_0+1}{s}-1}}{\lambda}$$

So

$$P(s)=\frac{dP(s_i(t)<s)}{ds}=\frac{1+m_0}{\lambda t s^2 \ln t} t^{\frac{1+m_0}{s}} \tag{3.12}$$

The above expression indicates that P(s) isn't power-law. That is the preferential attachment is necessary to the phenomena of scale-free of the strength distribution.

3.2 The Distribution of the Node's Degree

Similarly, the rate equation for the degree evolution reads:

$$\frac{dk_i}{dt} = \lambda m_0 \frac{s_i(t)}{\sum_j s_j(t)} \tag{3.13}$$

According to the condition: $s_i(t)=(\lambda+\lambda m_0+\frac{a_2}{a_1})(\frac{t}{t_i})^{a_1}-\frac{a_2}{a_1}$, $\sum_j s_j(t)=At$; We may get:

$$\frac{dk_i}{dt} = \frac{\lambda m_0}{A} \cdot (\lambda + \lambda m_0 + \frac{a_2}{a_1}) \cdot \frac{t^{a_1 - 1}}{t_i^{a_1}} - \frac{\lambda m_0}{A} \cdot \frac{a_2}{a_1} \cdot \frac{1}{t} \tag{3.14}$$

For enough large t, we can get:

$$\frac{dk_i}{dt} \approx \frac{\lambda m_0}{A} \cdot (\lambda + \lambda m_0 + \frac{a_2}{a_1}) \cdot \frac{t^{a_1 - 1}}{t_i^{a_1}} \tag{3.15}$$

Where we seemly choose the parameter to make $\lambda (m_0+1)+p_1 l< 2p_2+p_4 m_2$. And we using the initial condition $k_i(t_i)=m_0$, then the equation (3.15) has the following solution:

$$k_i(t) = \frac{\lambda^2 m_0 + \lambda m_0 (\lambda m_0 + \frac{a_2}{a_1})}{A a_1} \left(\frac{t}{t_i} \right)^{a_1} + m_0 - \frac{\lambda^2 m_0 + \lambda m_0 (\lambda m_0 + \frac{a_2}{a_1})}{A a_1}$$

We let $B= \lambda m_0 - \frac{\lambda^2 m_0 + \lambda m_0 (\lambda m_0 + \frac{a_2}{a_1})}{A a_1}$; then we can get:

$$k_i(t) = (\lambda m_0 - B) \left(\frac{t}{t_i} \right)^{a_1} + B \tag{3.16}$$

We may get:

$$P(k_i(t) < k) = P(t_i > \left(\frac{\lambda m_0 - B}{k - B} \right)^{\frac{1}{a_1}} \cdot t)$$

Assuming that we add the nodes at equal time intervals to the system, i.e., t_i denote the time adding of node *i* follow the uniform distribution over interval (0, λ t). So, when $k> \lambda m_0$,

$$P(k_i(t) < k) = 1 - \left(\frac{\lambda m_0 - B}{k - B} \right)^{\frac{1}{a_1}} \cdot \frac{1}{\lambda}$$

We choose parameters properly to make $0<(\frac{\lambda m_0 - B}{k-B})^{\frac{1}{a_1}}<1$; So when $k> \lambda m_0$,

$$P(k) = \frac{dP(k_i < k)}{dk} = (\lambda m_0 - B)^{\frac{1}{a_1}} \cdot \frac{1}{\lambda a_1} \cdot (k - B)^{\gamma}$$

Where $\gamma=1+a_1^{-1}$. This result indicates that the model self-organizes into a scale-free network with scaling exponent γ.

Additional, we consider the random model, we get

$$\frac{dk_i}{dt} = \lambda m_0 \frac{1}{N(t)}$$

According to $N(t)=N_0+\lambda t \approx \lambda t$; we may get

$$k_i(t) = m_0 \frac{\ln t}{\ln t_i} \tag{3.17}$$

Hence, we may get

$$P(k_i(t) < k) = P(t_i > t^{\frac{m_0}{k}})$$

Assuming that we add the nodes at equal time intervals to the system, i.e., t_i denote the time adding of node i follow the uniform distribution over interval (0, λ t). So, when $k>m_0$

$$P(k_i(t) < k) = 1 - t^{\frac{m_0}{k}-1} \cdot \frac{1}{\lambda}$$

So we can get

$$P(k) = \frac{dP(k_i < k)}{dk} = \cdot \frac{1}{k^2 \lambda t \ln t} \cdot t^{\frac{m_0}{k}}$$

From the above result, we know that P(k) is not power-law. That is the preferential attachment is necessary to the phenomena of scale-free of the degree distribution.

3.3 The Distribution of the Weight

Similarly, the rate equation for the weights evolution reads

$$\frac{dw_{ij}}{dt} = 2p_1 l \frac{s_{ij}}{\sum_a s_a} \cdot \frac{1}{N(t)} - 2p_2 \frac{1}{N(t)-1} \cdot (1 - \frac{s_{ij}}{\sum_j s_j}) \cdot \frac{1}{N(t)} \tag{3.18}$$

Where $s_{ij} = s_{\max(i,j)}$, then we rewrite the above expression:

$$\frac{dw_{ij}}{dt} = 2p_1 \frac{s_{ij}}{At} \cdot \frac{l}{\lambda t} - \frac{2p_2}{\lambda^2 t^2} + \frac{2p_2 s_{ij}}{A\lambda^2 t^3} \tag{3.19}$$

For enough large t, we may get the following expression:

$$\frac{dw_{ij}}{dt} \approx \frac{2p_1 l(\lambda+\lambda m_0+\frac{a_2}{a_1})}{A\lambda t_{ij}^{a_1}} t^{a_1-2} - \frac{2p_1 l\frac{a_2}{a_1}\lambda+2p_2 A}{A\lambda^2}\frac{1}{t^2} \tag{3.20}$$

Where $t_{ij}=t_{\max(i,j)}$, the equation (3.20) has the following solution:

$$w_{ij}(t)=\tilde{C}[\frac{2p_1 l(\lambda+\lambda m_0+\frac{a_2}{a_1})}{A\lambda(a_1-1)t_{ij}^{a_1}} t^{a_1-1} + \frac{2p_1 l\frac{a_2}{a_1}\lambda+2p_2 A}{A\lambda^2}\frac{1}{t}];$$

For the initial condition: $w_{ij}(t_{ij})=1$, we can get

$$\tilde{C}=Mt_{ij}$$

Where $M=\frac{A\lambda^2(a_1-1)}{2p_1 l\lambda(\lambda+\lambda m_0+a_2)+2p_2 A(a_1-1)}$. Then

$$w_{ij}(t)=\frac{2p_1 l\lambda(\lambda+\lambda m_0+\frac{a_2}{a_1})}{2p_1 l\lambda(\lambda+\lambda m_0+a_2)+2p_2 A(a_1-1)}(\frac{t}{t_{ij}})^{a_1-1}$$

$$+\frac{(2p_1 l\frac{a_2}{a_1}\lambda+2p_2 A)(a_1-1)}{2p_1 l\lambda(\lambda+\lambda m_0+a_2)+2p_2 A(a_1-1)}\frac{t_{ij}}{t}$$

For enough large t, we may get:

$$P(w_{ij}<w)\approx P(S(\frac{t}{t_{ij}})^{a_1-1}<w)=P(t_{ij}>(\frac{S}{w})^{\frac{1}{a_1-1}}\cdot t)$$

Where $S=\frac{2p_1 l\lambda(\lambda+\lambda m_0+\frac{a_2}{a_1})}{2p_1 l\lambda(\lambda+\lambda m_0+a_2)+2p_2 A(a_1-1)}$.

Assuming that t_{ij} follows the uniform distribution over interval $(0, \lambda t)$ and $w > S\lambda^{1-a_1} + \lambda$. Hence,

$$P(w_{ij} < w) = 1 - (\frac{S}{w})^{\frac{1}{a_1 - 1}} \cdot \frac{1}{\lambda} \tag{3.21}$$

$$P(w) = \frac{dP(w_{ij} < w)}{dw} = \frac{1}{\lambda(a_1 - 1)} \cdot S^{\frac{1}{a_1 - 1}} \cdot w^{-(1+\frac{1}{a_1 - 1})} \tag{3.22}$$

That is P(w)~ $w^{-\theta}$. Where $\theta = 1 + \frac{1}{a_1 - 1}$.

Additional, we consider the random model, we get

$$\frac{dw_{ij}}{dt} = 2p_1 l \frac{1}{N(t)} \cdot \frac{1}{N(t)} - 2p_2 \frac{1}{N(t)} \cdot \frac{1}{N(t)} \tag{3.23}$$

According to N(t)= λ t, we rewrite the equation (3.23)

$$\frac{dw_{ij}}{dt} = \frac{2p_1 l - 2p_2}{\lambda^2 t^2} \tag{3.24}$$

For simplicity, we set $N = -\frac{2p_1 l - 2p_2}{\lambda^2}$. For the initial condition $w_{ij}(t_{ij})=1$, we can get $w_{ij}(t) = \frac{t_{ij}}{t}$, where $t_{ij}=t_{\max(i,j)}$. So we may get

$$P(w_{ij} < w) = P(t_{ij} < \omega t)$$

Assuming that t_{ij} follows the uniform distribution over interval $(0, \lambda t)$. Hence

$$P(w_{ij} < w) = \frac{\omega}{\lambda}$$

$$P(w) = \frac{dP(w_{ij} < w)}{dw} = \frac{1}{\lambda};$$

This indicate P(w) is a constant and is independent of w.

4 Numerical Simulation

4.1 Comparison the Distribution of the Strengths between the Exact Analytical Solution and the Numerical Simulation

Figure1 display the fact that the probability distribution of the strengths obtained from the analytical solution well-suited to that of the numerical simulation. We start with an initial network N_0=50, and we let m_0=10, m_1=10, m_2=1, l=10, λ=0.3, p_1=0.3, p_2=0.05, p_3=0.3, p_4=0.05, t=50000. From the analytical expression (3.8), we get the scale exponentγ=2.6916. With the compute simulation, we get the probability distribution of the degree exhibit the behavior of scale-free. That is $P(s) \sim s^{-\gamma}$, and we get the scale exponentγ=2.6848 by linear data fitting.

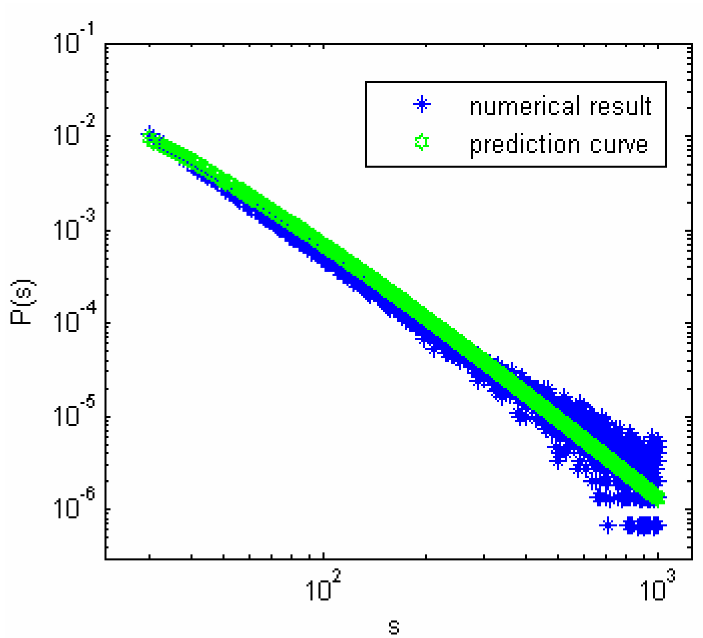

Fig. 1. Probability distribution of the strengths *P*(s)

Remark 1. Probability distribution of the strengths $P(s) \sim s^{-\gamma}$. The data are averaged over 100 networks with parameters N_0=50, m_0=10, m_1=10, m_2=1, l=10, λ=0.3, p_1=0.3, p_2=0.05, p_3=0.3, p_4=0.05, t=5000.

Remark 2. We found the existing of the phenomenon of the scale-free about the probability distribution of the strength P(s) when we only vary the parameters p_3 and p_4. In the figure 2, the blue data is obtained when p_3= p_4=0.03, and the green data is obtained when p_3= p_4=0.05, and the red data is gained when p_3= p_4=0.01. We found the

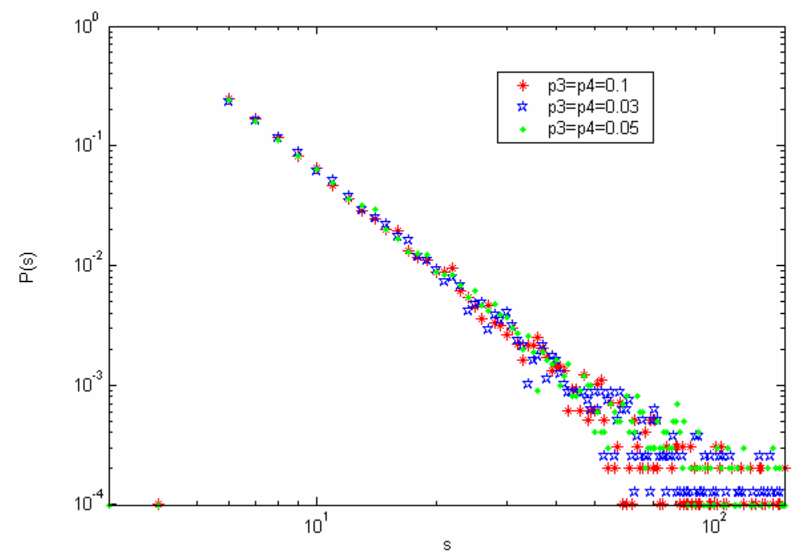

Fig. 2. Probability distribution of the strengths *P*(s) when we only change the Parameters p_3 and p_4.

changes of parameters p_3 and p_4 are very little to the probability distribution of the degree P(s).

4.2 Comparison the Distribution of the Degree between the Exact Analytical Solution and the Numerical Simulation

Figure3 display the fact the probability distribution of the degree obtained from the extra analytical solution well-suited to that of the numerical simulation. We start with an initial network of $N_0=50$, and we let $m_0=10$, $m_1=10$, $m_2=1$, $l=10$, $\lambda=0.3$, $p_1=0.3$, $p_2=0.05$, $p_3=0.3$, $p_4=0.05$, t=50000. From the analytical expression $P(k) \sim k^{-(1+\frac{1}{a_1})}$, we get the scale exponent$\gamma=2.6916$. Then we get he probability distribution of the degree exhibit the behavior of scale-free. That is $P(k) \sim k^{-\gamma}$, and we get the scale exponent $\gamma=2.6818$ by linear data fitting.

Additional, we found the existing of the phenomenon of the scale-free about the probability distribution of the degree P(k) when we only varies the parameters p_3 and p_4. In the following figure, the blue data is obtained when $p_3= p_4=0.05$ and the red data is gained when $p_3=0.1$, $p_4=0.05$. We found the changes of parameters p_3 and p_4 are very little to the probability distribution of the degree P(k).

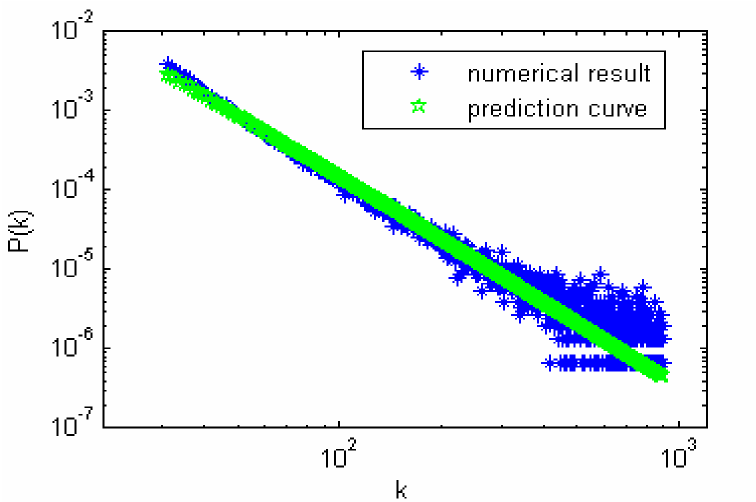

Fig. 3. Probability distribution of the degree P(k)$\sim k^{-\gamma}$. The data are averaged over 100 networks with parameters N_0=50, m_0=10, m_1=10, m_2=1, l=10, p_1=0.3, p_2=0.05, p_3=0.3, p_4=0.05, t=5000.

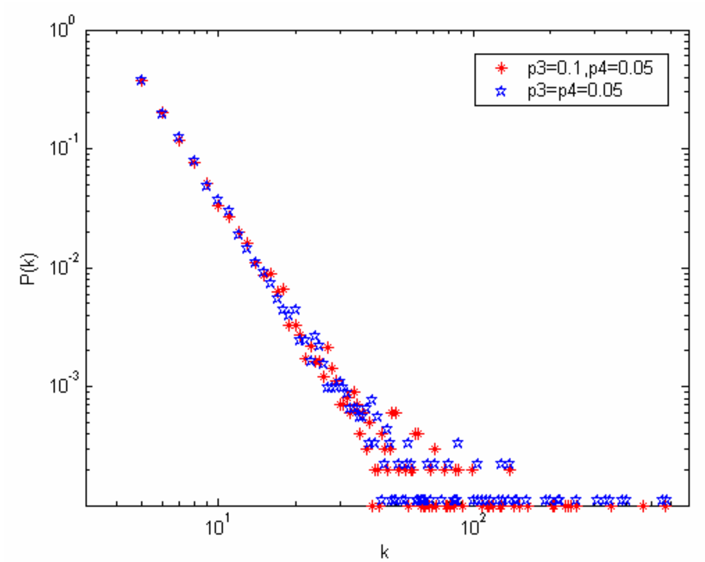

Fig. 4. Probability distribution of the strengths P(k) when we only change the parameters p_3 and p_4

5 Summary

The previous models of complex networks always incorporate the growth and the preferential attachment mechanism, which may result in scale-free properties. And the growths of the node and weigh are only being considered in the previous models. But the decrease of weight of a line and the strength of a node are usual in the real networks actually. In order to proceed to the general exact analysis of real networks, in this paper, we firstly introduce a concept of node intrinsic strength. Based on this concept, we present a general model depended on five dynamical evolutions: node addition, weight's growth and decrease, node intrinsic strength's growth and decrease. Through analysis and numerical simulation, we find network generated by this model exhibits some statistical properties observed in several real-world systems. Figure1 and figure2 show that the distributions of the weight, strength and degree from the analytical solution are well suited to those of the numerical simulation. Furthermore, we give the analytical computation of the distributions of the weight, the strength and the degree .Simultaneity, by way of contrasting our results with those of the random model; we found the preferential attachment is necessary to the phenomena of scale-free of the strength and degree distributions. Finally, we found the analytical results are good consistent with those of numerical simulation. they display the distributions of the weight, strength and degree all exhibit the behavior of scale-free., which further proved growth and preferential attachment existed not only in the unweighted networks but in the variations of the strength of the networks. We hope the conclusion from this general simple model is helpful to the further investigation of the topological role of weight and strength.

References

1. Pastor-Satorras, R., Vespignani, A.: Evolution and Structure of the Internet: A Statistical Physics Approach. Cambridge University Press, Cambridge (2004)
2. Albert, R., Jeong, H., Barabási, A.L.: The Scale-free Networks. Nature 401, 130–134 (1999)
3. Newman, M.E.J.: Clustering and preferential attachment in growing networks. Phys. Rev. E 64, 016131–016134 (2001)
4. Barabási, A.-L., Jeong, H., Néda, Z., Ravasz, E., Schubert, A., Vicsek, T.: Evolution of the social network of scientific collaborations. Physica A 50, 590–596 (2002)
5. Barrat, A., Barthélemy, M., Pastor-Satorras, R., Vespignani, A.: The architecture of complex weighted networks. Proc. Natl. Sci. U.S.A. 101, 3747–3752 (2004)
6. Barrat, A., Barthélemy, M., Vespignani, A.: Effects of Weight on Structure and Dynamics in Complex Networks. Phys. Rev. Lett. 92, 28701–28706 (2004)
7. Barabási, A.-L., Albert, R.: Emergence of Scaling in Random Networks. Science 286, 509 (1999)
8. Albert, R., Barabási, A.-L.: Mean-field theory for scale-free random networks. Rev. Mod. Phys. 74, 47–54 (2002)
9. Geng, X., Li, Q.: Random Models of Scale-free Networks. Physica A: Statistical Mechanics and it's Applications 365, 554–562 (2005)

10. Krause, A.E., Frank, K.A., Mason, D.M., Ulanowicz, R.E., Taylor, W.W.: Weighted Evolving Networks: Coupling Topology and Weight Dynamics. Nature 426, 282–286 (2003)
11. Almaas, E., Kovács, B., Viscek, T., Oltval, Z.N., Barabási, A.L.: Modeling the evolution of weighted networks. Nature 427, 839–843 (2004)
12. Yook, S.H., Jeong, H., Barabási, A.L., Tu, Y.: Phys. Weighted Evolving Networks. Rev. Lett. 86, 5835–5840 (2001)
13. Hajra, K.B., Sen, P.: Aging in citation networks. Physica A 346, 44–48 (2005)