Exponential Synchronization of General Complex Delayed Dynamical Networks via Adaptive Feedback Control

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Abstract. This paper studies the problem of controlling complex delayed dynamical networks by applying adaptive linear feedback controllers to each node. By using Lyapunov functional method, we give the criteria of globally exponential synchronization of the controlled complex delayed dynamical networks. The obtained criteria are suitable to the network with general properties, such as, with hybrid coupling, with different time delays, with asymmetric coupling configuration matrices, and so on. It is useful for future practical engineering design for its universality. Some simulations are given to verify the effectiveness of our scheme.

Keywords: complex dynamical networks, exponential synchronization, adaptive feedback control, hybrid coupling, lyapunov function.

1 Introduction

Since the discovery of small-world effect by Watts and Strogatz [1] and scale-free property by Barab*á*si and Albert [2], the science of complex networks have attracted growing interest and attention from the various disciplines, including mathematical, physical, social science, and so on. Many studies have investigate the topological properties of various systems such as the Internet, metabolic networks, social networks, airline networks, power networks, etc.. Whereas research of collective behavior in arrays of coupled nonlinear systems is also an important issue for understanding the essence of complex networks [3]. Complex dynamical networks can exhibit many interesting and unexpected phenomena, such as, spatio-temporal chaos [4], outbreak of epidemic [5,6], synchronization of signals, etc.[7]. Among them, synchronization of complex systems has obtained an intensive attention from communication, engineering, physics, etc.,

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for its many applications. Recently, many researchers have studied the synchronization of complex networks, such as in Refs [7,8,9,10,11,12,13]. Time delay is quite ubiquitous in many physical systems due to the finite switching speed of amplifiers, finite signal propagation time in biological networks, memory effects, etc.. What's more, time delay in the interaction may modify drastically dynamic behavior of system [8]. In view of these reasons, the synchronization of complex delayed dynamical networks are also studied by many researchers [3,9,10,11,12,13].

Though many good results have been obtained for the synchronization of the complex dynamical networks, the models considered in previous work are too simple or too restricted. For instance, the coupling matrix is considered to be undirected and unweighted and symmetric. However, this simplification does not match the real-world system due to actual circumstances in far too many cases. So in this Letter, our models relax these unperfected restrictions to better simulate practical systems.

Since coupling styles between nodes may be hybrid in complex networks, such as in Ref [11], Cao et al considered the hybrid coupling on delayed neural networks. We introduce the hybrid coupling style in our models too.

In addition, sometimes the network may be chaotic or stochastic at the initial states, so it is an interesting and significant work to control the system to an desired state from the point of engineering. In Refs [11,12,13], authors proposed many effective schemes to control the system to synchronization state. Among these schemes, the adaptive linear feedback control scheme is better than others because it is simple and efficient, what's more, which can adapt the feedback strength with time.

Motivated by the aforementioned discussions, this paper aims to investigate the globally exponential synchronization of complex delayed dynamical networks by introducing adaptive linear feedback controllers to nodes. The models considered are very general, which can have the properties with asymmetric coupling matrix, with different time delays, with hybrid coupling and so on. Moreover, the synchronization of networks can be quackier achieved than other schemes because of the globally exponential synchronization. By using Lyapunov functional method, some novel and useful criteria are given in Letter.

The rest of this paper is organized as follows: In Section 2, some preliminaries are presented. In Section 3, two theorems are given for the two models. Then, simulations are showed to verify the effectiveness of our scheme in Section 4. Finally, conclusions are presented in Section 5.

2 Preliminaries

This section introduces a general complex delayed dynamical system model and give some preliminary definitions and assumptions.

We consider a general delayed dynamical network consisting of N diffusively coupled identical nodes, the state equations of such network are described as:

$$\dot{x_i}(t) = f(x_i(t)) + \sum_{j=1,j\neq i}^{N} b_{ij}(g(x_j(t)) - g(x_i(t))) + \sum_{j=1,j\neq i}^{N} c_{ij}(h(x_j(t-\tilde{\tau})) - h(x_i(t-\tilde{\tau}))) \quad (1)$$

where $x_i \in R^n, i = 1, 2, \cdots, N$ are the state variables of node i. $\tilde{\tau} \geq 0$ is time delay. $B = (b_{ij}) \in R^{N\times N}$ and $C = (c_{ij}) \in R^{N\times N}$ are the coupling configuration matrices representing the coupling strength and topology of the network, if there is a connection between node i and node j $(i \neq j)$, then $b_{ij} > 0, (c_{ij} > 0)$; otherwise, $b_{ij} = 0,\ (c_{ij} = 0)$.

Remark 1. The coupling matrix B and C are not necessary to be symmetric and the elements b_{ij} and c_{ij} can be arbitrary nonnegative numbers.

Let B and C are the diffusively matrices of network, that is,

$$b_{ii} = -\sum_{j=1,j\neq i}^{N} b_{ij}, \qquad c_{ii} = -\sum_{j=1,j\neq i}^{N} c_{ij}, \quad i = 1, 2, \ldots, N \quad (2)$$

Then Eq.(1) can be rewritten as

$$\dot{x_i}(t) = f(x_i(t)) + \sum_{j=1}^{N} b_{ij} g(x_j(t)) + \sum_{j=1}^{N} c_{ij} h(x_j(t-\tilde{\tau})) \quad (3)$$

Our task is to globally exponentially synchronize the states of the network (3) on the manifold defined (4) by introducing adaptive linear feedback controllers into each individual node.

$$x_1(t) = x_2(t) = ... = x_N(t) = s(t) \quad (4)$$

where $s(t) \in R^n$ is a solution of an isolated node

$$\dot{s}(t) = f(s(t)) \quad (5)$$

Before main results are given, the rigorous mathematical definition of globally exponential synchronization for dynamical network with time delay is introduced.

Definition 1. *Let* $X(t; t_0; \phi) = (x_1(t; t_0; \phi)^T, ..., x_N(t; t_0; \phi)^T)^T \in R^{nN}$ *be a solution of delayed dynamical network (3), where* $\phi = (\phi_1^T, \phi_2^T, ..., \phi_N^T)^T, \quad \phi_i = \phi_i(\theta) \in C([-\tilde{\tau}, 0], R^n)$ *are initial conditions of node* i*, and* $f, g, h:\ R \times \Omega \to R^n$ *are continuously differentiable on* $\Omega \subseteq R^n$*. If there exist constants* $\alpha > 0,\ \lambda > 0$

and a nonempty subset $\Lambda \subseteq \Omega$ *with* $\phi_i \in \Lambda,\ i = 1, 2, ..., N$*, such that* $X(t; t_0; \phi) \in \Lambda \times ... \times \Lambda$ *for all* $t \geq t_0$*, and*

$$\|X(t; t_0; \phi) - S(t; t_0; \phi)\| \leq \alpha e^{-\lambda t} \sup_{-\tilde{\tau} \leq \theta \leq 0} \|\phi(\theta) - S_0\| \tag{6}$$

where $\|X(t)\| = \sqrt{x_1^2 + ... + x_n^2}$ *,*$S(t; t_0; s_0) = (s(t; t_0; s_0)^T, ..., s(t; t_0; s_0)^T)^T \in R^{nN}$*, and* $S_0 = (s_0^T, ..., s_0^T)^T$*,* $s(t; t_0; s_0)$ *is a solution of (5) with the initial condition* $s_0 \in \Omega$*, then the dynamical network (3) with time delay is said to realize exponential synchronization such that* λ *is the exponential rate and* $\Lambda \times ... \times \Lambda$ *is called the region of synchrony of delayed network (3).*

Proposition 1. *Suppose there exist three positive* L_1*,* L_2*, and* L_3 *such that*

$$\|f(x_i(t)) - f(s(t))\| \leq L_1 \|e_i(t)\| \tag{7}$$

$$\|g(x_i(t)) - g(s(t))\| \leq L_2 \|e_i(t)\| \tag{8}$$

and

$$\|h(x_i(t - \tilde{\tau})) - h(s(t - \tilde{\tau}))\| \leq L_3 \|e_i(t - \tilde{\tau})\| \tag{9}$$

hold for $i = 1, 2, ..., N$*, where* $e_i(t) = x_i(t) - s(t)$*, and* $e_i(t - \tilde{\tau}) = x_i(t - \tilde{\tau}) - s(t - \tilde{\tau})$*.*

Proposition 2. *Suppose the inequalities (8) and (9) hold, and there exist a positive constant* L_4*, such that*

$$\|f(x_i(t), x_i(t - \tau)) - f(s(t), s(t - \tau))\| \leq L_4[\|e_i(t)\| + \|e_i(t - \tau)\|] \tag{10}$$

hold for $i = 1, 2, ..., N$*, where* $e_i(t - \tau) = x_i(t - \tau) - s(t - \tau)$*.*

3 Main Results

In this section, we will give several effectively adaptive feedback synchronization criteria for the complex dynamical network with time delay.

When (3) are given adaptive linear feedback controllers into each individual node, the controlled network can be described as:

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} b_{ij} g(x_j(t)) + \sum_{j=1}^{N} c_{ij} h(x_j(t - \tilde{\tau})) - u_i \tag{11}$$

where $u_i,\ i = 1, 2, ..., N$ are the feedback controllers, which forms are given as (13) and (14).

In order to achieve the objective on the manifold (5), subtracting (5) from (11) yields the error dynamical system

$$\begin{aligned} \dot{e}_i(t) = f(x_i(t)) - f(s(t)) + \sum_{j=1}^{N} b_{ij}(g(x_j(t)) - g(s(t))) \\ + \sum_{j=1}^{N} c_{ij} h(x_j(t - \tilde{\tau})) - h(s(t - \tilde{\tau})) - u_i \end{aligned} \tag{12}$$

Based on above preliminaries work, we have following theorem

Theorem 1. *Suppose proposition 1 holds, then the controlled network (11) is globally exponentially stable under controllers*

$$u_i = d_i e_i(t), \quad i = 1, 2, ..., N \tag{13}$$

where

$$\dot{d}_i = k_i \parallel e_i \parallel^2 exp(\mu t) \tag{14}$$

k_i, $i = 1, 2, ..., N$ and μ are positive constants. □

Proof. Select the following Lyapunov functional candidate:

$$V(t) = \frac{1}{2}\left[\sum_{i=1}^{N}\left(e_i^T(t)e_i(t)exp(\mu t) + cL_3\sum_{j=1}^{N}\int_{t-\tilde{\tau}}^{t} e_j^T(\theta)e_j(\theta)exp(\mu(\theta+\tilde{\tau}))d\theta\right)\right]$$
$$+\frac{1}{2}\sum_{i=1}^{N}\frac{(d_i-\hat{d}_i)^2}{k_i} \tag{15}$$

where $c = \max_{1\leq i,\ j\leq N} c_{ij}$.

Along (12)-(15), we have

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N}\Big\{e_i^T(t)\dot{e_i}(t)exp(\mu t) + \frac{\mu}{2}\|e_i(t)\|^2 exp(\mu t) + (d_i-\hat{d}_i)\|e_i(t)\|^2 exp(\mu t)\\
&\quad+\frac{cL_3}{2}\sum_{j=1}^{N}\Big[\|e_j(t)\|^2 exp(\mu\tilde{\tau}) - \|e_j(t-\tilde{\tau})\|^2\Big]exp(\mu t)\Big\}\\
&= \sum_{i=1}^{N}\Big\{e_i^T(t)\dot{e_i}(t) + \frac{\mu}{2}\|e_i(t)\|^2 + (d_i-\hat{d}_i)\|e_i(t)\|^2\\
&\quad+\frac{cL_3}{2}\sum_{j=1}^{N}\Big[\|e_j(t)\|^2 exp(\mu\tilde{\tau}) - \|e_j(t-\tilde{\tau})\|^2\Big]\Big\}exp(\mu t)\\
&= \sum_{i=1}^{N}\Big\{e_i^T(t)\Big[f(x_i(t)) - f(s(t)) + \sum_{j=1}^{N} b_{ij}\Big(g(x_j(t)) - g(s(t))\Big)\\
&\quad+\sum_{j=1}^{N} c_{ij}\Big(h(x_j(t-\tilde{\tau})) - h(s(t-\tilde{\tau}))\Big) - d_i(t)e_i(t)\Big]\\
&\quad+\frac{\mu}{2}\|e_i(t)\|^2 + (d_i-\hat{d}_i)\|e_i(t)\|^2 + \frac{cL_3}{2}\sum_{j=1}^{N}\left[\|e_j(t)\|^2 exp(\mu\tilde{\tau}) - \|e_j(t-\tilde{\tau})\|^2\right]\Big\}exp(\mu t)\\
&\leq \sum_{i=1}^{N}\Big\{\|e_i^T(t)\|\Big[L_1\|e_i(t)\| + bL_2\sum_{j=1}^{N}\|e_j(t)\| + cL_3\sum_{j=1}^{N}\|e_j(t-\tilde{\tau})\|\Big]\\
&\quad+\frac{\mu}{2}\|e_i(t)\|^2 - \hat{d}_i\|e_i(t)\|^2 + \frac{cL_3}{2}\sum_{j=1}^{N}\left[\|e_j(t)\|^2 exp(\mu\tilde{\tau}) - \|e_j(t-\tilde{\tau})\|^2\right]\Big\}\ exp(\mu t)
\end{aligned}$$

By using the inequality $x^T y \leq \|x^T y\| \leq \|x\|\|y\|$, we have

$$\dot{V}(t) \leq \sum_{i=1}^{N} \Big\{ L_1\|e_i(t)\|^2 + \frac{bL_2}{2}\sum_{j=1}^{N}(\|e_j(t)\|^2 + \|e_i(t)\|^2) + \frac{cL_3}{2}\sum_{j=1}^{N}(\|e_j(t-\tilde{\tau})\|^2 + \|e_i(t)\|^2)$$
$$+(\frac{\mu}{2} - \hat{d}_i)\|e_i(t)\|^2 + \frac{cL_3}{2}\sum_{j=1}^{N}\left[\|e_j(t)\|^2 exp(\mu\tilde{\tau}) - \|e_j(t-\tilde{\tau})\|^2\right] \Big\} exp(\mu t)$$
$$\leq \sum_{i=1}^{N}\left(L_1 + \frac{\mu}{2} + \frac{N}{2}(bL_2 + cL_3) + \frac{M_i}{2}(bL_2 + cL_3 exp(\mu\tilde{\tau})) - \hat{d}_i \right) \|e_i(t)\|^2 exp(\mu t)$$

where $M_i > 0, \quad i = 1, 2, ..., N$ are selected to satisfy $\sum_{j=1}^{N} \|e_j(t)\|^2 \leq M_i\|e_i(t)\|^2$ and $b = \max_{1\leq i,\ j\leq N} b_{ij}$. By choosing suitable constants $\hat{d}_i, \quad i = 1, 2, ..., N$ such that

$$L_1 + \frac{\mu}{2} + \frac{N}{2}(bL_2 + cL_3) + \frac{M_i}{2}(bL_2 + cL_3 exp(\mu\tilde{\tau})) - \hat{d}_i < 0$$

Then, we have $\dot{V}(t) < 0$. It means that $V(t) < V(0)$ for any $t > 0$.
From $V(t)$ in Eq.(15), we have

$$\frac{1}{2}\|e_i(t)\|^2 exp(\mu t) = \frac{1}{2}e_i^T(t)e_i(t)exp(\mu t) \leq V(t) \leq V(0) \tag{16}$$

That is,

$$\|e_i(t)\| \leq R\ exp(-\frac{\mu}{2}t) \quad with \quad R = \sqrt{2V(0)} > 0 \tag{17}$$

Thus, $\lim_{t\to\infty} \|e_i(t)\| = 0$, i.e., the controlled network (11) is globally exponentially stable.

In theorem 1 , we do not assume that the dynamics of each node is time delayed, however, each node's dynamical behavior can also be time delayed, for example, in Refs [13], authors considered the synchronization of complex dynamical network when each node's dynamical behavior is time delayed.

So we generalize (3) as follow:

$$\dot{x_i}(t) = f(x_i(t), x_i(t-\tau)) + \sum_{j=1, j\neq i}^{N} b_{ij}(g(x_j(t)) - g(x_i(t)))$$
$$+ \sum_{j=1, j\neq i}^{N} c_{ij}(h(x_j(t-\tilde{\tau})) - h(x_i(t-\tilde{\tau}))) \tag{18}$$

At this time, the solution of an isolated node $s(t)$ is determined by

$$\dot{s}(t) = f(s(t), s(t-\tau)) \tag{19}$$

By giving adaptive linear feedback controllers into each individual node, the controlled network can be described as:

$$\dot{x_i} = f(x_i(t), x_i(t-\tau)) + \sum_{j=1}^{N} c_{ij} g(x_j(t)) + \sum_{j=1}^{N} c_{ij} h(x_j(t-\tilde{\tau})) - u_i \tag{20}$$

In order to achieve the objective on the manifold (4), subtracting (19) from (20) yields the error dynamical system

$$\dot{e_i}(t) = f(x_i(t), x_i(t-\tau)) - f(s(t), s(t-\tau)) + \sum_{j=1}^{N} b_{ij}(g(x_j(t)) - g(s(t)))$$
$$+ \sum_{j=1}^{N} c_{ij} h(x_j(t-\tilde{\tau})) - h(s(t-\tilde{\tau})) - u_i \tag{21}$$

We have:

Theorem 2. Suppose proposition 2 holds, then the controlled network (20) is globally exponentially stable when the adaptive feedback controllers u_i, $i = 1, 2, ..., N$ are described as (13) and (14).

Proof. Construct a Lyapunov function

$$V(t) = \frac{1}{2}\left[\sum_{i=1}^{N}\left(e_i^T(t)e_i(t)exp(\mu t) + cL_3 \sum_{j=1}^{N} \int_{t-\tilde{\tau}}^{t} e_j^T(\theta)e_j(\theta)exp(\mu(\theta+\tilde{\tau}))d\theta\right.\right.$$
$$\left.\left.+L_4 \int_{t-\tau}^{t} e_i^T(\theta)e_i(\theta)exp(\mu(\theta+\tau))d\theta\right)\right] + \frac{1}{2}\sum_{i=1}^{N}\frac{(d_i - \hat{d}_i)^2}{k_i} \tag{22}$$

The following steps are similar to the proof in Theorem 1, so we omit it.

4 Simulations

To show the effectiveness of the proposed globally exponential synchronization criteria, in this section, we consider two general delayed dynamical networks with 10 coupled Lorenz systems [14] and delayed Lorenz systems [13,15], respectively.

A Lorenz system is described as:

$$\begin{cases} \dot{x_1}(t) = a(x_2(t) - x_1(t)) \\ \dot{x_2}(t) = cx_1(t) - x_3(t) - x_2(t) \\ \dot{x_3}(t) = x_1(t)x_2(t) - bx_3(t) \end{cases}$$

here $a = 10,\ b = 8/3,\ c = 28$.

And a Lorenz system with time delay is given as [13]:

$$\begin{cases} \dot{x_1}(t) = a(x_2(t) - x_1(t)) \\ \dot{x_2}(t) = cx_1(t) - x_3(t-\tau) - x_2(t) \\ \dot{x_3}(t) = x_1(t)x_2(t-\tau) - bx_3(t-\tau) \end{cases}$$

here $a = 10,\ b = 1.3,\ c = -28;\quad \tau = 1$.

Choose a asymmetric coupling matrix A as

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -5 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & -8 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -6 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

To demonstrate our models are general, we let

$$b_{ij} = a_{ij}/k_i^{0.5},\ c_{ij} = a_{ij}/k_i^{0.5}$$

k_i is the out-degree of node i.

By letting $g(x(t))$ and $h(x(t))$ as:

$$g(x(t)) = [x_1(t); x_2(t); x_3(t)]^T$$

and

$$h(x(t)) = [a(x_2(t) - x_1(t)); x_2(t); x_1(t)x_2(t) - bx_3(t)]^T$$

$\tilde{\tau} = 0.1,\ \mu = 0.02$ and $d_i = 0.1,\ i = 1, 2, ..., N$.

Fig .1 and Fig .2 show the globally exponential synchronization of models (11) and (20), respectively.

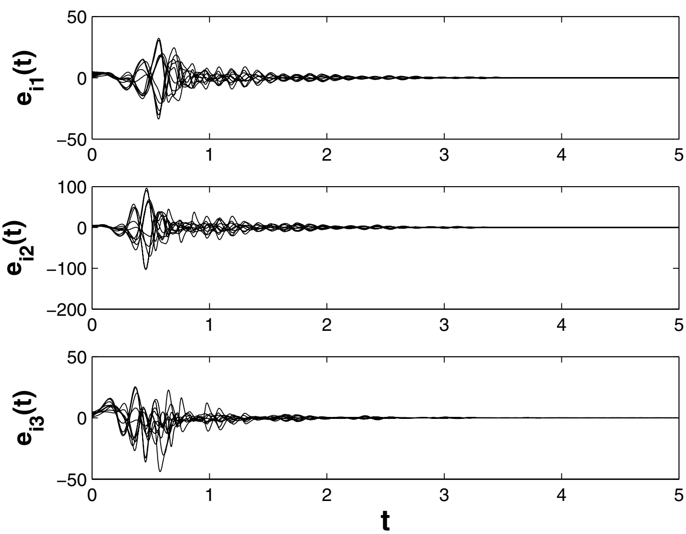

Fig. 1. Synchronization errors $e_{i1},\ e_{i2},\ e_{i3}$ $(i = 1, 2, ..., 10)$ of the controlled network (11)

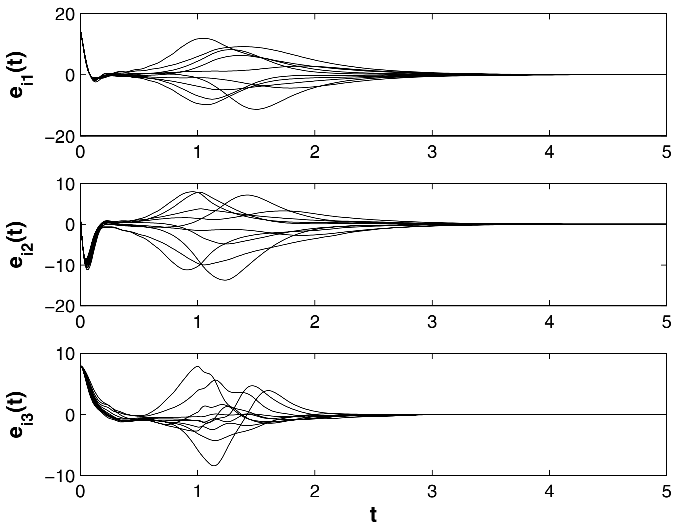

Fig. 2. Synchronization errors $e_{i1},\ e_{i2},\ e_{i3}$ $(i = 1, 2, ..., 10)$ of the controlled network (20)

5 Conclusion

We studied the globally exponential synchronization of controlled complex delayed dynamical networks by introducing adaptive linear feedback controllers to each node, and several criteria were obtained by using the Lyapunov functional method. The scheme is suitable to the network with general properties, such as, with hybrid coupling, with different delays, with asymmetric coupling configuration matrices, and so on. It is very useful for the future practical engineering design. Some simulations are given to verify the effectiveness of our scheme.

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