Adjustable Consensus of Mobile Agent Systems with Heterogeneous Delays

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Abstract. Consensus of mobile agent system is a question with practical significance in the complex dynamics system. In this paper, an adjustable multi-agent moving system with the heterogeneous communication delays is studied under the hypothesis of fixed, undirected and connected topology. The consensus of the multi-agent system is a weighted average consensus that can be adjusted by setting the parameters of the agents. Applying generalized Nyquist criterion, the multi-agent delayed algorithm is analyzed, and many sufficient condition is obtained to ensure the weighted average consensus of the moving system. Finally, many computer simulations are used to show the validity of the results.

Keywords: Mobile agent system, heterogeneous communication delays, adjustable, weighted average consensus.

1 Introduction

Over the past decade a considerable amount of attention has been focused on the problem of coordinated motion of multiple autonomous agents. Related problems have been studied in ecology and theoretical biology, in the context of animal aggregation and social cohesion in animal groups, statistical physics and complexity theory, non-equilibrium phenomena in many degree-of-freedom dynamical systems, as well as in distributed control of multiple vehicles and formation control. Researchers from many different communities have been trying to develop an understanding of how a group of moving agents can move in a formation only using local interactions and without a global supervisor.

In 1986 Craig Reynolds [1] developed a computer animation model for coordinated motion of groups of animals such as bird flocks and fish schools. Reynolds' model suggests that flocking is the combined result of three simple steering rules, which each agent independently follows:

- **Separation:** steer to avoid crowding local flockmates.
- **Alignment:** steer towards the average heading of local flockmates.
- **Cohesion:** steer to move toward the average position of local flockmates.

In Reynolds' model, each agent can access the whole scene's geometric description, but flocking requires that it reacts only to flockmates within a certain small neighborhood

J. Zhou (Ed.): Complex 2009, Part II, LNICST 5, pp. 1462 – 1471, 2009.

around itself. The superposition of these three rules results in all agents moving in a formation, while avoiding collision.

More recently, there has been a tremendous surge of interest among researchers from various disciplines of engineering and science in problems related to multi-agent networked systems with close ties to consensus problems. In 1995, an algorithm model was proposed by Vicsek *et al.* [2]. In Vicsek model, each agent heading is updated as the average of the headings of agent itself with its nearest neighbors plus some additive noise. Numerical simulations in [2] indicate the spontaneous development of coherent collective motion, resulting in the headings of all agents to converge to a common value.

Based on the study of the linearized Vicsek's Model, Jadbabaie et al. proved that all the agents converge to a common steady state provided that the graph formed by the agents is jointly connected, i.e., the agents are all "linked together" via their neighbors with sufficient frequency as the system evolves [3]. Further theoretical extensions of this work were presented in [4] and [5] with a look toward treatment of directed information flow in networks. A theoretical framework for posing and solving consensus problems for networked dynamic systems was introduced by Olfati-Saber and Murray in [6] and [7] building on the earlier work of Fax and Murray [8]. Based on these researches, the consensus for many subjects is studied from different fields [9-15].

In the multi-agent practical systems, the communication delays are important elements to affect the capacity of the systems. Moreover, the analysis of the stability or the convergence of the protocol is very difficult with non-negligible communication delays. Suppose the system is that with the same communication delay, the convergence of the protocol is studied by [6,9]. Based on the contraction theory and wave variable method, Wang and Slotine [10] studied the consensus problem for a simplified continuous-time model of schooling or flocking with time-delayed communications, and generalize recent results in the literature [9], where they proposed a simple consensus protocol with zero self-delay, which was robust to arbitrary communication delays.

In this paper, based on the multi-agent collaboration systems presented by Olfati-Saber[6], we present an auto-adjusted consensus algorithm and study the consensus of the system with heterogeneous feedback delays. The delayed collaboration algorithm in this paper is different from the delayed algorithm in [6], where the communication delays between the agents are homogeneous delays. In our protocol the self-delay is diverse communication delays, and the consensus value can be adjusted by setting the weighted parameters. With the help of the frequency-domain method, we analyze the effect of the heterogeneous communication delays on the convergence of the multi-agent system. Many sufficient conditions for the multi-agent system converging to a consensus are obtained.

This paper is organized as follows. In Section 2, we describe the interrelated problem of the multi-agent systems and the relevant background knowledge on graph theoretical results. An auto-adjusted consensus algorithm is presented, and the property and the consensus of the multi-agent system with diverse communication delays are analyzed in Section 3. In Section 4, many computer simulations are used to validate the result. Finally, a conclusion for the whole paper is given in Section 5.

2 Problem Descriptions

Consider a network of decision-making agents interested in reaching a consensus via local communication with their neighbors on a graph $G = \{V, E, A\}$, where the set of nodes as $V = \{v_1, \cdots, v_n\}$ and the set of edges $E \subseteq V \times V$. Let the adjacency matrix $A = [a_{ij}]$, where $a_{ij} \geq 0$ is the weighted value between agent *i* and agent *j*. If they change the information, $a_{ij} > 0$, otherwise, $a_{ij} = 0$. The set of neighbors of an agent *i* is defined as N_i by

$$N_i = \{ j \in V : a_{ij} \neq 0\}.$$

In topology graph *G*, if the elements of the adjacency matrix *A* satisfy $a_{ij} = a_{ji}$ for any $a_{ij} > 0$, then the topology *G* is an undirected connected graph, otherwise, it is called as a directed connected graph. For two agents *i* and *j*, if there exists a set of the index $\{k_1, \cdots, k_l\}$, satisfying $a_{ik_1} > 0$, $a_{k_1k_2} > 0$, …, $a_{k_l j} > 0$, we call that a directed connected path existing between this two agents. In the topology graph G, if there is a directed connected path for any two agents, the graph G is called as the strong connected graph.

Let the degree matrix $D = diag\{d_i, i = 1, \cdots, n\}$, where $d_i = \sum_{j=1}^{n} a_{ij}$, the sum of the element in the *i*th row of matrix *A*, is called as out-degree of the node *i*. Matrix $L = D - A$ is the Laplacian matrix of the graph *G*. In a strong connected graph, if the out-degree is equal to the in-degree for any node *i*, the graph G is called as an balanced graph.

The most common continuous consensus algorithm is given by Olfati & Murray[6],

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)), \tag{1}$$

where a_{ij} is the element of the adjacency matrix *A* of the network topology for multi-agent systems, if $j \in N_i, a_{ij} > 0$, otherwise, $a_{ij} = 0$. $x_i(t)$ is the information state of the *i*th agent. A consequence of (1) is that the information state $x_i(t)$ of agent *i* is driven toward the information states of its neighbors. If the graph *G* is strong connected, the consensus of the system is achieved [6].

When the information is exchanged among the vehicles, it must be considered for the communication delays in the process of the message transmission. Let τ_{ij} denote the time delay for information communicated from vehicle *j* to vehicle *i*. If we consider the time-delays, the system (1) is modified as

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij}[x_j(t-\tau_{ij}) - x_i(t-\tau_{ij})]. \tag{2}$$

In reference [6], Olfati-Saber & Murray studied a simpler case for the homologous delay $\tau_{ij} = \tau$. When the communication topology is fixed, undirected, and connected, the average consensus is achieved, if $0 \le \tau < \frac{\pi}{2\lambda_{\max}}$, where $\lambda_{\max} = \lambda_{\max}(D)$, and D is the Laplacian of the communication graph.

Alternatively, the references [9,10] consider the case in which the time delay affects only the information state that is being transmitted so that (2) is modified as

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij}[x_j(t-\tau_{ij}) - x_i(t)], \tag{3}$$

where $\tau_{ij} = \tau$ and the communication topology is directed and switching. The consensus result for switching topologies remains valid for arbitrary time delay τ [9,10]. It is easy to know, the consensus states will satisfy

$$x_1^* = x_2^* = \cdots = x_n^*. \tag{4}$$

3 Adjustable Consensus of Mobile Agent System with Heterogeneous Delays

In this paper, we discuss a generally delayed consensus algorithm based on the algorithm in reference [6], where the agent i has a self-time delays τ_i and many parameters is set to adjust the consensus value.

$$r_i \dot{x}_i(t) = \sum_{j \in N_i} a_{ij}[x_j(t-\tau_{ij}) - x_i(t-\tau_i)], \tag{5}$$

where $r_i > 0, \sum_{i=1}^{n} r_i = 1$. We call $r = (r_1, r_2, ..., r_n)$ as a desired weighting vector. If $\tau_i = \tau_{ij}$, the algorithm can degenerate the system (2). if $\tau_i = 0$, the algorithm can degenerate the system (3). For the function $\alpha(t) = \sum_{i=1}^{n} r_i x_i(t)$,it is easy to know, the consensus states will satisfy $\dot{\alpha}(t) = \sum_{i=1}^{n} r_i \dot{x}_i(t) = 0$. So the consensus states have

$$x_1^* = x_2^* = \cdots = x_n^* = \alpha(0).$$

where $\alpha(0)=\sum_{i=1}^{n} r_i x_i(0)$ can be adjusted by setting the parameter $r=(r_1,r_2,...,r_n)$.

Following the consensus states of the system, suppose $\hat{x}_i(t)=x_i(t)-x_i^*$, the system (4) is changed to

$$r_i\dot{\hat{x}}_i(t)=\sum_{j\in N_i} a_{ij}(\hat{x}_j(t-\tau_{ij})-\hat{x}_i(t-\tau_i)). \tag{6}$$

Applying Laplace transformation, let $X_i(s)=L(\hat{x}_i(t))$ and $X_i(0)=0$, then

$$r_i s X_i(s)=\sum_{j\in N_i} a_{ij}(e^{-\tau_{ij}s}X_j(s)-e^{-\tau_i s}X_i(s))$$

Let $X(s)=(X_1(s),\cdots,X_n(s))^T$, we have

$$X(s)=-diag(\frac{e^{-\tau_i s}}{r_i s},i=1,...,n)(D-A(s))X(s)$$

where $D=diag(d_i)$, $A(s)=(a_{ij}e^{(\tau_i-\tau_{ij})s})$. The open-looped transformation function of the system

$$G(s)=diag(\frac{e^{-\tau_i s}}{r_i s},i=1,...,n)(D-A(s)). \tag{7}$$

Let $s=j\omega$, we get

$$G(j\omega)=diag(\frac{e^{-j\tau_i\omega}}{jr_i\omega},i=1,...,n)(D-A(j\omega)). \tag{8}$$

The system is local asymptotical stable at the equilibrium, if the Nyquist plots of the eigenvalues of the function matrix G do not enclose the point *-1+j0.*

Theorem 1. Suppose the topology of the system is fixed, undirected and connected network, then the system (4) will be consensus, if

$$\tau_i<\frac{r_i\pi}{4d_i}. \tag{9}$$

Proof. Based on the supposition that the topology of the system is fixed undirected and connected network, we can know that the adjacent matrix A is a positive matrix and satisfies $a_{ij}=a_{ji}$, and $D-A$ is a M matrix. Let

$$E(j\omega) = diag(\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}, i = 1,...,n),$$

Then

$$G(j\omega) = E(j\omega)B(j\omega), \quad (10)$$

where $B(j\omega) = diag(\frac{d_i\tau_i}{r_i}, i = 1,...,n)(I - D^{-1}A(j\omega))$.

When $\tau_i\omega = \frac{\pi}{2}$, the function $\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}$ will pass the real axis at the point $-\frac{2}{\pi} + j0$. Following the definition of the matrix, we can obtain the supreme radius of the matrix $D^{-1}A(j\omega)$ satisfying $\rho(D^{-1}A(j\omega)) \le 1$, then the supreme radius of the matrix $I - D^{-1}A(j\omega)$ satisfies $\rho(I - D^{-1}A(j\omega)) \le 2$. Let λ be the eigenvalue of the matrix $G(j\omega)$, then there exists a vector $v = (v_1, v_2, ..., v_n)^T$, $\|v\| = 1$, satisfying

$$\lambda = v^H G(j\omega)v.$$

Let $\bar{\rho} = \rho(B(j\omega)$, then

$$\bar{\rho} = \rho(diag(\frac{d_i\tau_i}{r_i}, i = 1,...,n)(I - D^{-1}A(s))) \le \rho(diag(\frac{2d_i\tau_i}{r_i})), \quad (11)$$

When $\tau_i < \frac{r_i\pi}{4d_i}$, there is $\frac{2d_i\tau_i}{r_i} < \frac{\pi}{2}$. We have

$$\bar{\rho} < \frac{\pi}{2}. \quad (12)$$

In the function matrix $G(j\omega) = E(j\omega)B(j\omega)$, when $\tau_i\omega = \frac{\pi}{2}$, the function $\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}$ will pass the real axis at the point $-\frac{2}{\pi} + j0$. Therefore, the Nyquist plot of the function $\bar{\rho}\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}$ will pass the real axis at the right side of the point $-1 + j0$, and the Nyquist plot will never encircle the point $-1 + j0$.

Since $v^H B(j\omega)v = v^H diag(d_i\tau_i, i=1,...,n)(I-D^{-1}A(s))v = \overline{\rho}$, we have

$$\lambda = v^H G(j\omega)v = v^H E(j\omega)B(j\omega)v$$

and obtain

$$\lambda \in Co(0 \cup \{\overline{\rho}\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}\}) .$$

Because the Nyquist plot of the function $\overline{\rho}\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}$ will never encircle the point $-1+j0$, the locus of $\lambda(G(j\omega)$ will never encircle the point $-1+j0$, too. Following the generalized Nyquist criterion and $(-1, j0) \notin Co(0 \cup \{\overline{\rho}\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}\})$, the system is local asymptotical stable at the equilibrium. We finish the proof of the Theorem 1.

Corollary 1. Suppose the topology of the system is fixed undirected and connected network, and there exist homogeneous communication delays, i.e. $\tau_i = \tau_{ij} = \tau$. Then the system (4) will be consensus, if

$$\tau < \frac{\pi \underline{r}}{2\rho} .$$

where ρ is the supreme radius of Laplace matrix L and $\underline{r} = \min_{i \in N}\{r_i\}$.

Proof. In the proof of theorem 1, if there exist homogeneous communication delays $\tau_i = \tau_{ij} = \tau$, the Eq.(10) will be changed to

$$G(j\omega) = E(j\omega)B ,$$

where $E(j\omega) = diag(\frac{e^{-j\tau\omega}}{j\tau\omega})$, $B = diag(\frac{d_i\tau}{r_i}, i=1,...,n)(I-D^{-1}A)$. Then the Eq.(11) will be changed to

$$\begin{aligned}\overline{\rho} &= \rho(B) = \rho(diag(\frac{d_i\tau}{r_i}, i=1,...,n)(I-D^{-1}A)) \\ &= \tau\rho(D-A)diag(r_i^{-1})) \\ &\le \frac{\tau\rho(L)}{\underline{r}} .\end{aligned}$$

When $\frac{\tau\rho}{\underline{r}} < \frac{\pi}{2}$, the Nyquist plot of the function $\overline{\rho}\frac{e^{-j\tau\omega}}{j\tau\omega}$ will never encircle the point $-1+j0$, the locus of $\lambda(G(j\omega))$ will never encircle the point $-1+j0$, too. Applying the generalized Nyquist criterion and since $(-1, j0) \notin Co(0 \cup \{\overline{\rho}\frac{e^{-j\tau_i\omega}}{j\tau_i\omega}\})$, the system is local asymptotical stable at the equilibrium. We finish the proof of the corollary 1.

4 Simulation Experiments

In this section, many computer simulations are used to verify our results. We apply the topology of the system as Fig. 1, which is an undirected graph with weighted

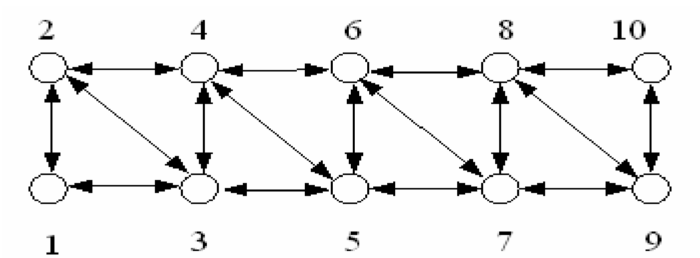

Fig. 1. The topology of the system

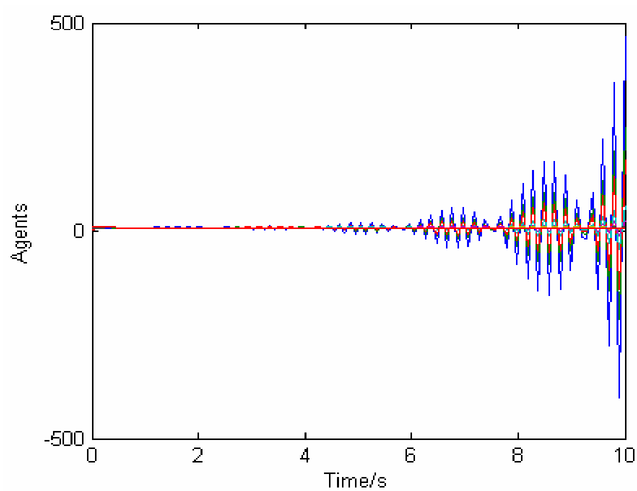

Fig. 2. The simulation of the system for delays $\tau_{ij} = 0.01$, $\tau_i = 0.03$

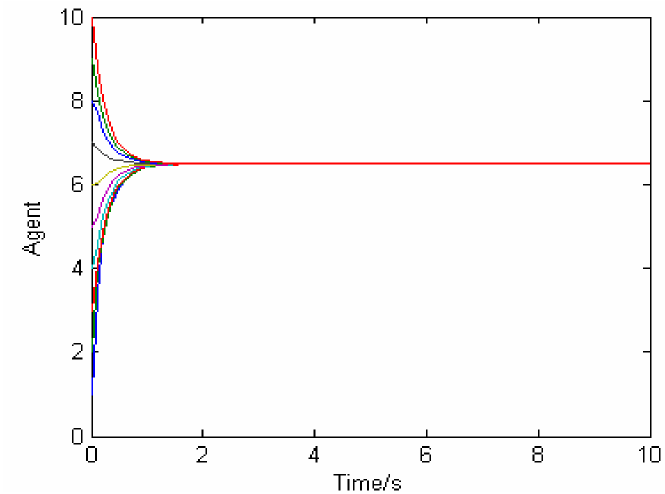

Fig. 3. The simulation of the system for delays $\tau_{ij} = 0.01$, $\tau_i = 0.02$

value 1. Suppose the state of the agent is one dimension, and the initialized values is 1,2,…,10, and the weighted parameters $r_i = \frac{i}{55}, i = 1,2,...,10$, respectively. Firstly, setting the communication delays as $\tau_{ij} = 0.01$, $\tau_i = 0.03$, the simulation of the system is shown in Fig.2, which the consensus of the delayed system have not been achieved from the effects of the communication delays. Then we decrease the communication delays $\tau_{ij} = 0.01$, $\tau_i = 0.02$. Following the theorem 1, the consensus of the system can be achieved, and the information states of the multi-agent systems are shown in Fig.3.

5 Conclusions

This paper solves the adjustable consensus problem of multi-agent systems with heterogeneous communication delays. Based on the information communicating process, a generalized consensus algorithm of multi-agent systems with time-delays is presented, whose consensus value can be adjusted by setting the parameters of the agents. Applying generalized Nyquist criterion, the multi-agent delayed algorithm is analyzed, and a sufficient condition is obtained to ensure the consensus of the system. It is proved that the driven multi-agent system asymptotically converges to the pre-specified consensus value while preserving the connectivity of the group topology. Finally, many computer simulations are used to show the validity of the results.

Acknowledgements

This work is supported by Chinese National Natural Science Foundation (under the grant 60774016, 60875039), the Science Foundation of Office of Education of

Shandong Province (under the grant J08LJ01) and the Natural Science Foundation of Ludong University (under the grant L20074102, Z0704) of China.

References

[1] Reynolds, C.W.: Flocks, herds, and schools: a distributed behavioral model. Computer Graphics 21(4), 25–34 (1987)
[2] Vicsek, T., Czirok, A., Ben-Jacob, E., et al.: Novel type of phase transition in a system of self-driven particles. Physical Review Letters 75(6), 1226–1229 (1995)
[3] Jadbabaie, A., Lin, J., Morse, A.S.: Coordination of groups of mobile agents using nearest neighbor rules. IEEE Trans. on Automatic Control 48(6), 988–1001 (2003)
[4] Moreau, L.: Stability of multi-agent systems with time-dependent communication links. IEEE Trans. on Automatic Control 50(2), 169–182 (2005)
[5] Ren, W., Beard, R.W.: Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans. on Automatic Control 50(5), 655–661 (2005)
[6] Olfati-Saber, R., Murray, R.M.: Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. on Automatic Control 49(9), 1520–1533 (2004)
[7] Olfati-Saber, R.: Flocking for multi-agent dynamic systems: Algorithms and theory. IEEE Trans. Automat. Control 51(3), 401–420 (2006)
[8] Fax, J.A., Murray, R.M.: Information flow and cooperative control of vehicle formation. IEEE Trans. on Automatic Control 49(9), 1465–1476 (2004)
[9] Moreau, L.: Stability of continuous-time distributed consensus algorithms: Proceedings of the 43rd IEEE Conference on Decision and Control, 2004, December 14-17 (2004)
[10] Wang, W., Slotine, J.J.E.: Contraction analysis of time-delayed communications and group cooperation. IEEE Trans. On Automatic Control 51(4), 712–717 (2006)
[11] Bauso, D., Giarre, L., Pesenti, R.: Nonlinear protocols for optimal distributed consensus in networks of dynamic agents. Systems and Control Letters 55(11), 918–928 (2006)
[12] Lawton, J.R., Beard, R.W.: Synchronized multiple spacecraft rotations. Automatica 38(8), 1359–1364 (2002)
[13] Caughman, J.S., Lafferriere, G., Veerman, J.J.P., Williams, A.: Decentralized control of vehicle formations. Syst. Control Lett. 54(9), 899–910 (2005)
[14] Ren, W., Beard, R.W., Atkins, E.M.: Information Consensus in Multivehicle Cooperative Control: Collective Group Behavior through Local Interaction. IEEE Control Systems Magazine 27(2), 71–82 (2007)
[15] Olfati-Saber, R., Fax, J.A., Murray, R.M.: Consensus and cooperation in networked Multi-agent systems. Proceedings of the IEEE 95(1), 215–233 (2007)